Measuring Charge and Statistics of FQH Quasiparticles through Edge State Transport

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Review talk based mostly on other people’s work.
Measuring Fractional Charge

Early experiment by Goldman and Su (Science 95) using equilibrium, electrostatic measurements obtained evidence suggestive of e/3 charge quasiparticles at $v=1/3$; needed assumptions about electrostatic configurations/Coulomb blockade arguments about the system.

More direct measurement of fractional charge uses non-equilibrium shot noise generated by backscattering of quasiparticles at the edge of a fractional quantum Hall liquid [Saminadayar et al. (Saclay group) PRL 97; de-Picciotto et al (Weizmann group) Nature 97].
Shot (Poison) Noise in Current at Fixed Voltage

Due to discreteness in charge, actual current made of a sequence of delta-function spikes; this is shot-noise first found by Schottky in 1918.

Average Current: \[ \langle I \rangle = e \langle N \rangle / t \]

Fluctuation in Current (Noise):
\[ S = 2e^2 (\langle N^2 \rangle - \langle N \rangle^2) / t = 2e \langle I \rangle \]

There are other sources of noise; shot noise dominant in the low-T limit, in certain frequency ranges.
Basic Geometry for Noise Measurement at a Single Point Contact

Without quasiparticle back-scattering: \( I = G_h V \) 
\( (G_h = \nu e^2 / h) \);

With quasiparticle back-scattering: 
\( I = G_h V - I_B \).

Measure shot noise in backscattering current, which is directly proportional to its charge \( e^* \). 
(Kane and Fisher; Chamon, Freed and Wen; Fendley, Ludwig and Saleur, 94-96).
Situation not nearly as clear for hierarchy states.
Two-path interference for quasiparticles; setup similar to Aharonov-Bohm.

Can measure $e^*$ if one can change flux without changing number of quasiparticles in the liquid; can measure statistical angle if one can change number of quasiparticles without changing flux. Neither is easy!
Difficulty can be seen from a simpler and more standard A-B interference setup:

More specifically, there are three topologically distinct ground states with 0, 1, and 2 quasiparticles in the hole; the flux induce transitions among them. Thus fractional charge and statistics conspire to give us “trivial” interference.

Q: What is the expected periodicity in $\Phi$?
A: $\Phi^* = 2\pi/e^* = 3\Phi_0$, right?
Wrong!
$\Phi^* = \Phi_0$

The reason for $\Phi^* = \Phi_0$ is gauge invariance. At the end, the system is “made” of real electrons, hence adiabatically adding $\Phi_0$ should bring the system back to its ground state (Byers and Yang, 1961).

More specifically, there are three topologically distinct ground states with 0, 1, and 2 quasiparticles in the hole; the flux induce transitions among them. Thus fractional charge and statistics conspire to give us “trivial” interference.
Something that looks like non-trivial interference pattern was seen recently:

Camino, Zhou, Goldman (2005)
Superperiod oscillation with $\Delta \Phi = 5\phi_0$.

Their conclusion (with a number of non-trivial assumptions): mutual statistics between $1/3$ and $1/5$ quasiparticles is $-1/15$. Interpretation unclear and controversial at this point.
The situation may simplify considerably for interference among non-Abelian quasiparticles!

Origin: braiding of non-Abelian quasiparticles can change the internal state in addition to giving an Abelian phase to the original state (Fradkin et al. 98; Stern and Halperin 06; Bonderson, Kitaev and Shtengel 06).

\[ I_B \propto \left| (t_1 U_1 + t_2 U_2) \right|^2 \]

\[ = |t_1|^2 + |t_2|^2 + 2 \text{Re} \left\{ t_1^* t_2 \langle \Psi | U_1^{-1} U_2 | \Psi \rangle \right\} \]

The interference term involves unitary transformation generated by a quasiparticle circling a loop; a phase in the Abelian case but a nontrivial transformation in the non-Abelian case.
Applying to Moore-Read State:

Odd number of quasiparticles in the loop:

\[ \langle \Psi | U_1^{-1} U_2 | \Psi \rangle = 0; \text{ no interference at all!} \]

Physics: a new state orthogonal to the original one is generated after a quasiparticle circles around an odd number of other quasiparticles.

Even number of quasiparticles in the loop:

\[ \langle \Psi | U_1^{-1} U_2 | \Psi \rangle \neq 0; \text{ there is interference pattern.} \]

Dramatic difference between even and odd number of quasiparticles in the loop is indication of non-Abelian statistics (Stern and Halperin 06; Bonderson, Kitaev and Shtengel 06); being looked for experimentally.
One step further: a Topological Qubit

Das Sarma, Freedman and Nayak PRL 05
Measuring statistics using noise correlation in T-shaped point contacts

\[ S(t) \equiv \frac{\langle \Delta I_1(t) \Delta I_2(0) \rangle}{\langle I_1 \rangle \langle I_2 \rangle} \]

\[ \tilde{S} \left( \frac{\omega}{\omega_0} \right) = A \left( \frac{\omega}{\omega_0}, \frac{T}{T_0}, K \right) + \cos \theta \ B \left( \frac{\omega}{\omega_0}, \frac{T}{T_0}, K \right) \]

Kim, Lawler, Vishveshwara and Fradkin PRL 05