Potential realizations

- Spin systems (or simulated by cold atomic systems).
- Defects in symmetry-protected topological phases.
- Fractional quantum Hall systems.

**FQHE**

2D electron gas

Strong magnetic field

(qeanches kinetic energy)

Physics dominated by electron-electron interaction.

**Filling fraction** \( n = \frac{\# \text{ of electrons}}{\# \text{ of flux quanta}} \)

**Hall conductance** \( \sigma_{xy} = \frac{I_x}{V_y} \) quantized at \( \nu \frac{e^2}{h} \) at

(FQHE) \( \nu = \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \ldots, \frac{2}{3}, \frac{4}{3}, \frac{5}{3}, \frac{7}{3}, \left( \frac{5}{2} \right), \frac{8}{3}, \ldots \)

(FQHE) \( 1, 2, 3, \ldots \)

\( \nu = \frac{5}{2} \) is the strongest candidate for the realization of anyons in real materials.
Universal quantum gates

A set of gates is said to be universal for quantum computation if any unitary operation may be approximated to arbitrary accuracy by a quantum circuit involving only those gates.

The standard set:

Hadamard \( H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \)

Phase \( \frac{\pi}{4} \) \( S = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/4} \end{pmatrix} \)

\( T = \begin{pmatrix} 1 & 0 \\ 0 & e^{-i\pi/4} \end{pmatrix} = e^{i\pi/8} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\pi/8} \end{pmatrix} \)

CNOT \( \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \)


Unfortunately, using Ising anyons in 2D cannot make a fault-tolerant \( \frac{\pi}{8} \)-gate. We thus need to search for systems that support, say, Fibonacci anyons. See, most recently Carrahan et al., arXiv:1511.00749 for the construction of quantum gates using Fibonacci anyon braiding.
Fibonacci Anyons

\[ \{1, 2\} \]

\[ 2 \times 2 = 1 + 1 \]

\[ 2 \times (2 \times 2) = 2 \times (1 + 1) = 2 + 1 + 1 \]

\[ \tau \quad \tau \quad \tau \]

\[ \tau \quad \tau \quad \tau \quad \frac{1}{\tau} \quad \frac{1}{\tau} \quad \frac{1}{\tau} \quad \text{a qubit} \]

R matrix

\[ R = \begin{pmatrix} e^{-i\frac{4\pi}{5}} & 0 \\ 0 & e^{i\frac{3\pi}{5}} \end{pmatrix} \]

\[ \frac{1}{\tau} = R_{bb} \frac{1}{\tau} = b \]

F matrix

\[ F = \begin{pmatrix} \phi^{-1} & \phi^{-\frac{1}{2}} \\ \phi^{-\frac{1}{2}} & -\phi^{-1} \end{pmatrix} \]

\[ F^2 = 1 \]

\[ \phi = \frac{\sqrt{5} + 1}{2} \quad (\phi^2 = 1 + \phi) \]

\[ \tau \quad \tau \quad \tau \]

\[ b = \frac{1}{\tau} \quad \frac{1}{\tau} \quad \frac{1}{\tau} \quad \text{F} \]

\[ b' \in \{0, 1\} \]

\[ \sum_{b' \in \{0, 1\}} F_{bb'} \frac{1}{\tau} = b' = \frac{1}{\tau} \]
Equivalence of 3-anyon qudit and 4-anyon qudit

\[ |0\rangle = \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \quad \iff \quad \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \quad \iff \quad \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \]

\[ |1\rangle = \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \quad \iff \quad \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \quad \iff \quad \begin{array}{c}
\begin{array}{c}
\bullet \\
\bullet
\end{array}
\end{array} \]

Hexagon Diagram:

\[ [R']^{-1} = e^{i4\pi/5} R \]

\[ = \begin{pmatrix}
1 & 0 \\
0 & e^{-i3\pi/5}
\end{pmatrix} \]

\[ R_{11} = e^{-i4\pi/5} \quad R_{12} = e^{i3\pi/5} \]
Braiding

$\sigma_1 = R$

$\sigma_2 = F R F$

Questions:

1. How to use $\sigma_1$ & $\sigma_2$ to construct an arbitrary single-qubit gate?

2. How to construct two-qubit gates?
The anyonic system has an exponentially large Hilbert space
whose states cannot be distinguished by local measurements.
Anyon world-lines forming braids carry out unitary
transformations on this Hilbert space. The so-called
topological quantum computation is robust against
local noises, which are gapped excitations.