Large Momentum Transfer in Slow Atoms by Traversing a Sequence of Propagating Light Wave in Optical Ring Cavities

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By considering a beam of two-level atoms traversing a sequence of propagating light wave tuned to atomic transition in optical ring cavities, a large momentum transfer is obtained. This is a new scheme for realization of atomic splitter and it can be used to construct atom interferometer.

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Interference of neutral atoms is a focus of research in recent years. It is essential to construct atomic splitter and mirror for atom interferometer. An atomic splitter splits the atom wave packet into two wave packets propagating in different directions. An atomic mirror, on the other hand, deflects the two wave packets and bring them to interference. Certainly, the interference will happen only if the scattering process is coherent. In some experiments for interference of neutral atoms, laser field plays the role of "optical elements", such as atomic splitters and mirrors. Various laser fields have been applied to split and recombine the atoms. The key point in the design of interferometer is to obtain large enough scattering angles that the atom wave packet is truly spatially separated. The theory of an atomic beam splitter in which a monochromatic beam of two level atoms is incident normally to a classical standing-wave light field is developed based on velocity-tuned resonances. The incident atomic wave packet can be split into two coherent wave packets with transverse momenta ±(2n+1)ħk using velocity-tuned resonances, where n is the order of the resonance. In recent work, the transverse momentum with 4ħk in atom is obtained by adiabatic passage in circularly polarized laser beams. In this paper, we consider a single two-level atom normally crossing several optical ring cavities. Inside each cavity, one propagating mode which we treated classically is brought to oscillation by an external laser. The longitudinal motion of the atom is described by a classical velocity vz, whereas the transverse degree of freedom z is treated quantum mechanically.

We assume cavities to be positioned very close to each other, so that the transverse spreading of the wave packet between two successive cavities can be neglected. The wave vector of the mode in first cavity is directed along +z, whereas that of second cavity along −z, the third along +z and so on. The interaction time in each cavity is equal to T. The frequency of light waves in cavities is all equal to that atomic transition. The Hamiltonian while the cavity field is treated classically is

\[
\hat{H} = \frac{p^2}{2m} + \sum_{n=1}^{N} h\omega [\sigma_+ e^{(-1)^n i k z} + \sigma_- e^{(-1)^n i k z}] |\chi_n(t)\rangle.
\]

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Here, $\hat{p}$ and $z$ denote, respectively, the $z$ component of the momentum and position operator of the atomic center of mass, and $\sigma_\pm$ denote the raising and lowering Pauli operators of the two level atom with electronic levels $|1\rangle$, $|2\rangle$. The atom-field coupling is $g$ and $k$ is the wave number of light field. The window functions

$$\chi_n(t) = \begin{cases} 1 & \text{for } t \in [(n-1)T, nT] \\ 0 & \text{elsewhere} \end{cases}$$

switch the interaction with $n$th cavity on and off ($n = 1, 2, ..., N$).

Now, we consider the atom-field interaction in the first cavity, the Hamiltonian becomes

$$\hat{H}_1 = \frac{\hat{p}^2}{2m} + \hbar g (\sigma_+ e^{-ikz} + \sigma_- e^{-ikz}).$$  \hspace{1cm} (2)

It can be solved exactly by invoking the unitary transformation

$$W(z) = e^{-iksz/2}.$$  

The corresponding effective Hamiltonian is

$$\hat{H}_c = \frac{\hat{p}^2}{2m} + \frac{\hbar^2 k^2}{2m} + \hbar \Omega (\cos \alpha \sigma_z + \sin \alpha \sigma_x),$$  \hspace{1cm} (3)

where

$$\Omega = \Omega (\hat{p}) = \sqrt{g^2 + \frac{k^2 \hat{p}^2}{4m^2}}, \quad \cot \alpha = -\frac{k \hat{p}}{2mg}.$$  \hspace{1cm} (4)

The evolution operator corresponding to $\hat{H}_1$

$$U_1(t) = e^{-i\hat{H}_1 t} = W(z) e^{-i\hat{H}_c t} W^+(z) = e^{-iksz/2} \cdot e^{-\frac{i \hat{p}^2 t}{2m}} \cdot e^{-\frac{\hbar k^2 z^2}{4m^2} [\cos \Omega t - i \sin \Omega t (\cos \alpha \sigma_z + \sin \alpha \sigma_x)]} \cdot e^{-iksz/2}. $$  \hspace{1cm} (5)

In the second cavity, the propagating light wave is counterpropagating with respect to the one in the first cavity. The evolution operator can be easily obtained by substituting $k$ with $-k$ in $U_1(t)$:

$$U_2(t) = e^{iksz/2} \cdot e^{-\frac{i \hat{p}^2 t}{2m}} \cdot e^{-\frac{\hbar k^2 z^2}{4m^2} [\cos \Omega t - i \sin \Omega t (\cos \beta \sigma_z + \sin \beta \sigma_x)]} \cdot e^{-iksz/2}, $$  \hspace{1cm} (6)

where $\cot \beta = k \hat{p}/2mg$. The evolution operator when the atom interacts with field in the third cavity is the same as $U_1(t)$, and in the fourth cavity is $U_2(t)$ and so on.

Having assumed the interaction time $T$ is same in each cavity, we can get the wave function after interacting with the first two cavities while the atom is in the ground state $|1\rangle$ with atomic center momentum $p_0$ at initial time $0$.

$$\psi(2T) = e^{-\frac{[\sigma_0 - (k/\hbar)^2]T}{2\hbar}} e^{-\frac{ik^2 z^2}{8m}} \cdot (e^{-\frac{[\sigma_0 + (k/\hbar)^2]T}{2\hbar}} - i \cos \Omega,T + i \sin \Omega,T \cos \alpha_1) (\cos \Omega,T + i \sin \Omega,T \cos \beta_2) \cdot |1\rangle,$$

$$\cdot e^{-\frac{[\sigma_0 - (k/\hbar)^2]T}{2\hbar}} e^{-\frac{ik^2 z^2}{8m}} \cdot i \cos \Omega,T - i \sin \Omega,T \cos \beta_2) \cdot \sin \Omega,T \sin \alpha_1 |p_0 + \h k \rangle \otimes |2\rangle$$

$$+ \sin \Omega,T \sin \alpha_1 |\Omega,T \sin \beta_2 |p_0 - 2h k \rangle \otimes |1\rangle \rangle. $$  \hspace{1cm} (7)
where $\Omega_1 = \Omega(p_o + \hbar k/2)$, $\Omega_2 = \Omega(p_o - \hbar k/2)$, $\Omega_3 = \Omega(p_o - 3\hbar k/2)$, $\alpha_1 = \alpha(p_o - \hbar k/2)$, $\beta_0 = \beta(p_o + \hbar k/2)$, $\beta_2 = \beta(p_o - 3\hbar k/2)$. The probability of finding the atom in the state $|p_o - 2\hbar k\rangle \otimes |1\rangle$ which has momentum shift $-2\hbar k$ is

$$\sin^2 \Omega_1 T \sin^2 \alpha_1 \sin^2 \Omega_2 T \sin^2 \beta_2.$$  

(8)

In order to make the probability large, we use the slow atom which can be obtained by laser cooling technique and intense laser light. The atom is slow means that the oscillation frequency due to Doppler effect is neglected from Eq. (4). Often the oscillation frequency related to the atomic recoil is negligible in the optical range. So the oscillation frequency in each cavity is nearly at Rabi frequency $g$. Let $T = \pi/2g$, the terms $\sin^2 \Omega_1 T$, $\sin^2 \Omega_2 T$ in expression (8) are nearly 1 from the above analysis. According to the same reason, the terms $\sin^2 \alpha_1$, $\sin^2 \beta_2$ are also nearly 1. So the momentum shift $-2\hbar k$ with large probability is obtained. Because the laser light is intense, that is, $g$ is large, the interaction time $T$ is small, so we can neglect the spontaneous emission and the coherence of atom beam is preserved. If the initial state of atom is

$$\psi(0) = \frac{1}{\sqrt{2}}(|1\rangle + |2\rangle) \otimes |p_o\rangle,$$  

(9)

we can get momentum shifts $\pm 2\hbar k$ with large probability. In this case, the atom beam is well split into two beams with large momentums. Similarly, after time $2nT$, large momentum transfer $\pm 2n\hbar k$ can be obtained.

From Eq. (7), after passing through the first two cavities, the atomic beam will be split into four beams, but only the beam with momentum shift $-2\hbar k$ has preponderant probability. If the initial state is Eq. (9), the atomic beam will be split into five beams and the probability of finding the atomic beam with momentum shift $\pm 2\hbar k$ is large. So we get two beams with momentum shifts $\pm 2\hbar k$ respectively, and they can be recombined to interfere.

In conclusion, a novel effective beam splitter is proposed using a sequence of propagating light wave in optical ring cavities and can be used to split and recombine atomic beams. It couples a single incoming momentum state to a coherent superposition of different momentum states which contain two large momentum states with preponderant probability and opens up a new method for developing atomic interferometry.

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REFERENCES