The Search for Non-Abelian Anyons in Fractional Quantum Hall Systems
– The Past Five Years

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Mini-Workshop on Topological Quantum Computation  
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Theory oriented; FQHE + TQC (6 talks); 17 researchers and 10 students

Recording available at http://zimp.zju.edu.cn/~xinwan/topo06/
Collaborators

- **Fractional quantum Hall effect in 2DEGs**
  
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  Ki Hoon Lee (Pohang), Ed Rezayi (Los Angeles)
  Peter Schmitteckert (Karlsruhe), Kun Yang (Tallahassee)

- **Universal topological quantum gate construction**
  
  Haitan Xu (U Maryland)
  Michele Burrelle (Trieste), Giuseppe Mussardo (Trieste)

- **Quantum Hall effect in rotating ultracold fermion systems with dipolar interaction**
  
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Outline

- Motivation: Topological quantum computation
- A simple picture for the 5/2 FQH state
- Experimental progress on the 5/2 FQH state
  - Shot noise, and
  - Conductance, for charge tunneling across a narrow constriction
  - Charge tunneling into localized states in the bulk
  - Quasiparticle interference (and my understanding)
- Outlook
Motivation: Topological Quantum Computation

Fault-tolerant. Information is stored globally, while environmental noises are local. Thus decoherence due to noises is protected against. *(Need non-Abelian anyons.)*

(1) $n$ qubits
(2) initial state
(3) quantum gates
(4) classical control
(5) readout

Kitaev
Freedman
Motivation: Topological Quantum Computation

Fault-tolerant. Information is stored globally, while environmental noises are local. Thus decoherence due to noises is protected against. (*Need non-Abelian anyons.*)
Topological Quantum Gate Construction

• In topological quantum computing, tensor decomposition is unnecessary and inconvenient – a leakage error occurs when a tensor decomposition is forced.
  

• Topological quantum gate construction, also known as topological quantum compiling:
  
  – Given a set of fundamental gates (braids), finding a sequence approaching an arbitrary gate is a hard question.


• $O(\log(1/\epsilon))$ in time and $O(\log^2(1/\epsilon))$ in length – beats currently the most efficient algorithms (i.e. the Solovay-Kitaev algorithm, c.f. the Nielsen & Chung book)

• Cited by Zhenghan Wang in *Topological Quantum Computation* (Published by American Mathematical Society, June 2010)
Non-Abelian FQH States

- Ising CFT: Moore & Read (1991); Morf (1998); Rezayi & Haldane (2000); Read & Green (2000)
- Parafermion CFTs: Read & Rezayi (1999)
- Das Sarma, Freedman & Nayak (2005)
- Experimental candidate: $\nu = 5/2$ FQHE (Willett, 1987)
  
  Moore-Read Pfaffian \(\sim p+ip\) superconductor? 
  
  e/4 quasiparticles \(\sim\) flux $\hbar c/2e$ vortices 
  
  Majorana condition 
  
  \[ \gamma_i = \gamma_i^+ \]  

Microsoft Station Q @ UCSB

Xia et al., PRL 93, 176809 (2004)
FQH Condensates

- Condensate of composite bosons ($\nu = 1/3$)

- Condensate of composite fermions ($\nu = 5/2 = 2 + 1/2$)

\[
\Psi_{\text{qh}}^{e/4} = \sigma e^{i \phi / 2 \sqrt{2}}
\]

\[
\Psi_{\text{qh}}^{e/2} = e^{i \phi / \sqrt{2}}, \quad \Psi e^{i \phi / \sqrt{2}}
\]
Even though we fixed all the positions of the excitations, there are still internal degrees of freedom.

A, B, C are not linearly independent – though it is not obvious [Nayak & Wilczek, Nucl. Phys. B (1996)]

We use two of their linear combinations as a basis set.
Braiding = Quantum Evolution
= Quantum Computation
Non-Abelian Statistics

Excited states

Gap $\Delta$

ground state manifold

| $\psi_f \rangle = \begin{bmatrix} \tilde{\alpha} \\ \tilde{\beta} \end{bmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{bmatrix} \alpha \\ \beta \end{bmatrix} |

| $\psi_i \rangle = \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$

Matrices form a non-Abelian representation of the braid group.

(Related to the Jones Polynomial, TQFT (Witten), Conformal Field Theory (Moore, Seiberg), etc.)
Key Issues to Prove

- Spin fully polarized
- Quasiparticles carry charge e/4 (not necessarily non-Abelian)
- Non-Abelian statistics (interferometry)
Noise, not clean
Dolev et al., Nature 452, 829 (2008)
Tunneling fits, but not without an effort

Radu et al., Science 320, 899 (2008)

\[ g_T = A T^{(2g-2)} F\left(g, \frac{e^* I_{dc} R_{xy}}{kT}\right). \]
Local Incompressibility

\[ F = \sum_i (c_i - V_{BG})Q_i + \frac{1}{2} \sum_i U_i Q_i (Q_i - 1) + \sum_{i<j} V_{ij} Q_i Q_j - \sum_i \Delta \left| \frac{Q_i}{2} \right| \]

With comparable gap, the disorder potential not altered. In the limit of an isolated compressible puddle surrounded by an incompressible fluid, incompressibility scales with local charge.

\[ \left( \frac{\langle C_1(x)C_2(x) \rangle}{\sqrt{\langle C_1(x)^2 \rangle \langle C_2(x)^2 \rangle}} \right) \]
FQH Quasiparticle Interferometry

Gates controlling the strength of tunneling

path via point contact 1

path via point contact 2

Edge of the $\frac{1}{2}$ droplet

Side gate controlling the number of quasiparticles on the central antidot

\[ G \propto \left| t_1 U_1 + t_2 U_2 \right| \Psi \right|^2 = \left| t_1 \right|^2 + \left| t_2 \right|^2 + 2 \Re \left\{ t_1^* t_2 e^{i\phi} \right\} \left( \Psi \right| M_n \left| \Psi \rangle \right) \]

Das Sarma, Freedman & Nayak (05)
Stern & Halperin (06); Bonderson, Kitaev & Shtengel (06)
Observing Non-Abelian Statistics

\[ \gamma_i \rightarrow -\gamma_j, \quad \gamma_j \rightarrow \gamma_i \]


\[ U_{ij} = \frac{1}{\sqrt{2}} (1 + \gamma_j \gamma_i) \]

In general, \( U \)s do not commute.

\[ \psi \rightarrow U_{ij} \psi \]

e.g. \[ [\gamma_a \gamma_1, \gamma_b \gamma_1] \neq 0 \]

Dependent of the circling anyon!

Odd:
\[ \gamma_{1a} \Rightarrow [U_{1a}]^2 = \gamma_{a} \gamma_{1} \]

Even:
\[ \gamma_{1a} \Rightarrow [U_{2a}]^2 [U_{1a}]^2 = \gamma_{a} \gamma_{2} \gamma_{a} \gamma_{1} = \gamma_{1} \gamma_{2} \]

no interference pattern
To be or not to be

Even number of non-Abelian quasiparticles inside the interference loop

Odd number of non-Abelian quasiparticles inside the interference loop
Model Experimental Systems

\[ \Phi = \frac{N\Phi_0}{\nu} \]

- Electron layer (-Ne)
- Background charge (+Ne)

\[ H = \frac{1}{2} \sum_{mnl} V_{mn}^l c_{m+l}^+ c_n^+ c_{n+l} c_m + \sum_m U_m c_m^+ c_m \]


Advantage of disk geometry: interplay of edge modes and bulk quasiholes.
Pfaffian Stable in $V_{\text{Coulomb}} + U_{\text{Confining}}$

Coulomb Interaction ($\lambda = 0$)

$$H = (1 - \lambda) H_C + \lambda H_{3B}$$

XW, Hu, Rezayi & Yang, PRB 77, 165316 (2008)
Simple Pictures for FQH Edge Excitations

Ground state

Excited state

Integer QH edge: chiral Fermi liquid

$\nu = 1/3$ FQH edge: chiral Luttinger liquid

Number of edge states: 1 1 2 3 5 7 11 ...
Edge Spectrum Analysis

\[ N = 12; \quad M_{gs} = 126 \]

\[ E = \sum n_b(l_b)l_b + \sum n_f(l_f)l_f \]

\[ \Delta E = \sum n_b(l_b)\epsilon_b(l_b) + \sum n_f(l_f)\epsilon_f(l_f) \]

Right panels:

\[ H = \frac{1}{2} H_C + \frac{1}{2} H_{3B} \]

Bulk, bosonic, and fermionic edge excitations clearly distinguishable.

Left panel: Coulomb only.
Bulk and edge excitations mixed up!

Conclusion: Bose-Fermi separation

Fermionic edge-mode velocity is much lower than the bosonic edge-mode velocity.

XW, Hu, Rezayi & Yang, PRB (2008)
Non-Abelian Signatures at the edge

\[ \Delta E(M) = e^2/\epsilon_B \]

\[ H_w = W c_0^+ c_0 \]

Confirmation: The edge Majorana fermion mode has **anti-periodic boundary condition** (due to the $2\pi$ spinor rotation) in the presence of even number of charge $e/4$ quasiholes in the bulk, but **periodic boundary condition** in the presence of odd number of charge $e/4$ quasiholes.

**Edge-mode Velocities**

\[ H_\lambda = (1 - \lambda) H_C + \lambda H_{3B} \]

\[ v_c = 5 \times 10^6 \text{ cm/s} \]
\[ v_n = 4 \times 10^5 \text{ cm/s} \]

Neutral velocity is significantly smaller!

Experimentally,
\[ v_c = 8 \sim 15 \times 10^6 \text{ cm/s} \]
\[ v_c = 4 \times 10^6 \text{ cm/s} \]

Marcus group
arXiv:0903.5097
Goldman group
PRB (2006)

\[ v_c = 5 \times 10^6 \text{ cm/s} \]
\[ v_n = 4 \times 10^5 \text{ cm/s} \]

XW, Hu, Rezayi & Yang, PRB (2008)

Depends on interaction & confinement!

\[ v_c = 5 \times 10^6 \text{ cm/s} \quad (\nu = 1/2) \]

\[ v_c \sim \nu \frac{e^2}{\epsilon \hbar} = \nu \frac{\alpha}{\epsilon} c \]
Interferometry Analysis

$n$ bulk non-Abelian $e/4$ quasiparticles

\[ \Psi_{qh}^{e/4} = \sigma e^{i\Phi/2\sqrt{2}} \]

\[ \Psi_{qh}^{e/2} = e^{i\Phi/\sqrt{2}}, \psi e^{i\Phi/\sqrt{2}} \]

odd-even effect:

\[ s_{e/4} = \begin{cases} \pm 1/\sqrt{2} & n \text{ even} \\ 0 & n \text{ odd} \end{cases} \]

\[ s_{e/2} = 1 \]

\[ I_{12} \propto \sum_q s_q |t_1||t_2| e^{-|x_1-x_2|/L_\Phi} \cos \left( 2\pi \frac{q}{e} \frac{\Phi}{\Phi_0} + \phi_q + \text{arg}(t_1^* t_2) \right) \]

coherence length due to thermal smearing

\[ L_\Phi = \frac{1}{2\pi k_B T} \left( \frac{g_c}{v_c} + \frac{g_n}{v_n} \right)^{-1} \]

favors $e/2$ qps
The Realistic Expectation (Low T)

Even number of non-Abelian quasiparticles inside the interference loop

Odd number of non-Abelian quasiparticles inside the interference loop

XW, Hu, Rezayi & Yang, PRB (2008)
At Higher Temperatures

Even number of non-Abelian quasiparticles inside the interference loop

Odd number of non-Abelian quasiparticles inside the interference loop

XW, Hu, Rezayi & Yang, PRB (2008)
Predictions Observed

- At 10 mK, e/4 pattern observable only when device size < 4 µm
- Both e/4 and e/2 interference patterns observable
- At higher temperatures, e/4 pattern suppressed


- 25 mK, size ~ 1 µm

Willett, Pfeiffer & West, PNAS (2009)
Temperature Dependence

Parameters:

\[ B = 6 \, \text{T} \]
\[ \varepsilon = 13.1 \]
\[ |x_1 - x_2| = 1 \, \mu\text{m} \]

period lines in the swept side-gate data. (C) Data indicate temperature dependence of e/4 and e/2 oscillations: e/2 oscillations may be made more prevalent with an increase in temperature. The temperature of the sample was taken from

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Hu, Rezayi, XW & Yang, PRB (2009)

\[ \frac{e}{2} \text{ oscillations} \]
\[ \frac{e}{4} \text{ oscillations} \]

Opposite trend for anti-Pfaffian
Willett *et al.*, PRB (2010)


Outlook

• The field is more exciting than ever
• Fractional quantum Hall effect in 2DEGs
  – Independent experimental checks
  – Samples & devices: innovating design
• Other systems
  – Cold atomic systems (with Kou, Qiu, Yi)
  – Graphene (with Bhatt, Hu, Yang)
  – p-wave superconductors
  – Noncentrosymmetric superconductors

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FQHE References

- Non-Abelian Fractional Quantum Hall States
  - Model non-Abelian FQH states with Coulomb interaction; non-Abelian signatures in edge excitations; predictions on quasiparticle interferometry
  - Estimate edge-mode velocities → coherence length/temperature
    Hu, Rezayi, XW & Yang, Phys. Rev. B 80, 235330 (2009)
  - Estimate quasiparticle tunneling amplitudes
  - Scaling in quasiparticle tunneling amplitude → conformal dimensions
  - Explore edge physics using Jack polynomials
    Lee, Hu & XW, in preparation