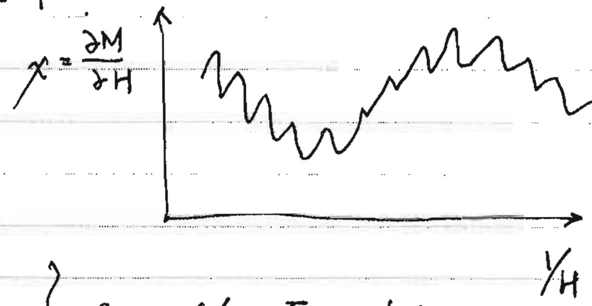
## I. Overview

- Measure magnetization M as a function of field for large H and low T.
- Find oscillations periodic in $1/H$

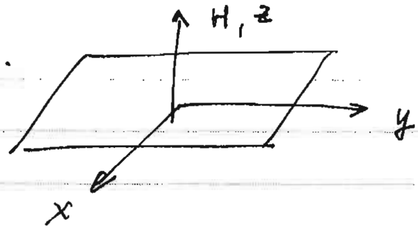



- Similar oscillations in
  - sound attenuation
  - thermoelectric effect
  - magnetoresistance
  - conductivity, etc.

  See A/M Fig 14.3. p. 267–268

## II. Landau levels in 2-D. (Landau & Lifshitz, QM)

Electrons move in a plane $\perp \vec{H}$.



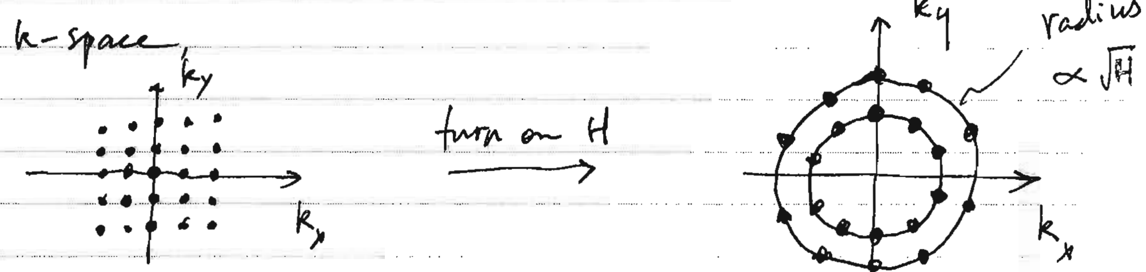

We are interested in high enough field, (i.e. $\omega_c \tau \gg 1$. In real crystal, need pure single crystal and very low temperature).

<u>Qualitative picture</u>: Electrons go in circles. The quantization of the orbits leads to

$$\varepsilon_\nu = \left(\nu + \tfrac{1}{2}\right)\hbar\omega_c \qquad \omega_c = \frac{eH}{mc}$$

$\Rightarrow$ discrete levels.

In k-space




All states between the circles collapse onto one of the circles. large degeneracy! $\propto H$

Free electron theory (Landau)

Schrödinger Eq. $H\psi = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 \psi = E\psi$

We choose Landau gauge $\vec{A} = H(0, x, 0)$; hence

$$H = \frac{1}{2m}\left[p_x^2 + (p_y - m\omega_c x)^2\right] \qquad \omega_c = \frac{eH}{mc}$$

- Classical eq. of motion

$$\dot{x} = \partial H/\partial p_x = p_x/m \qquad \dot{y} = \partial H/\partial p_y = (p_y - m\omega_c x)/m$$

$$\dot{p}_x = -\partial H/\partial x = +(p_y - m\omega_c x)\omega_c \qquad \dot{p}_y = 0$$

$p_y = \hbar k_y$ constant of motion.

$$\Rightarrow m\ddot{x} = -m\omega_c^2\left(x - \frac{\hbar k_y}{m\omega_c}\right)$$

which is the equation of a linear harmonic oscillator of freqency $\omega_c$ with origin at

$$x_0 = \frac{\hbar k_y}{m\omega_c}$$

A typical solution is

$$x = \frac{\hbar k_y}{m\omega_c} + \rho\cos\omega_c t, \quad y = y_0 + \rho\sin\omega_c t$$

- Quantum equation of motion

$$i\dot{x} = [x, H] = i\,p_x/m \qquad i\dot{y} = [y, H] = i(p_y - m\omega_c x)/m$$

$$i\dot{p}_x = [p_x, H] = -i(p_y - m\omega_c x) \qquad i\dot{p}_y = 0$$

Let $p_y = \hbar k_y$, $x = \frac{\hbar k_y}{m\omega_c} + q = x_0 + q$

$$\Rightarrow \quad \mathcal{H} = \frac{1}{2m}\left(p_x^2 + m^2\omega_c^2 q^2\right) \qquad [q, p_x] = i\hbar$$

which has eigenvalues

$$E = \left(\nu + \tfrac{1}{2}\right)\hbar\omega_c \qquad \nu = 0, 1, 2, \dots$$

eigenfunctions

$$\psi(\vec{r}) = e^{i k_y y} \underbrace{H(x - x_0)}_{\text{harmonic oscillator function}}$$

Range of $X_0$: $\quad 0 < x_0 < L_x$

$$\Rightarrow \quad 0 < k_y < m\omega_c L_x/\hbar$$

neglecting spin, total # of allowed states

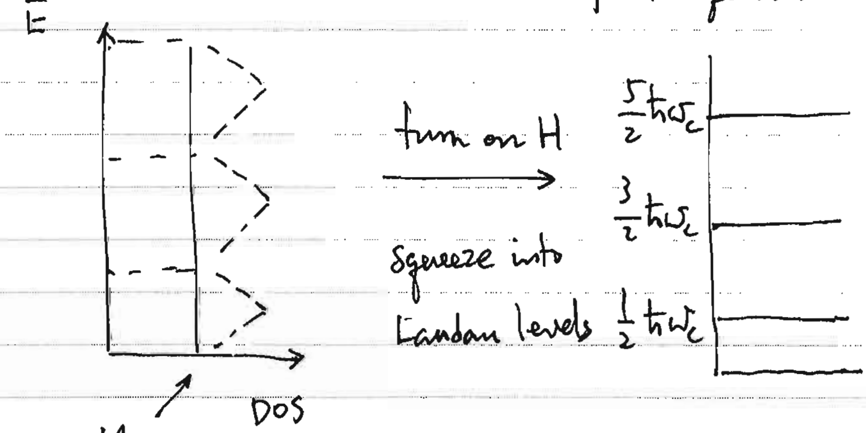
$$\#\text{deg} = \frac{m\omega_c L_x/\hbar}{2\pi/L_y} = \frac{eH}{2\pi\hbar c}\cdot L_x L_y = \frac{L_x L_y}{2\pi l_B^2}$$

$l_B$: magnetic length.

$$= \frac{\text{total Area}}{\text{quantized area for each orbit}}$$

Alternatively, $\#\text{deg} = \dfrac{H \cdot L_x L_y}{hc/e} = \dfrac{\Phi}{\Phi_0}$

$$= \frac{\text{total magnetic flux}}{\text{flux quantum}}$$




$L_x L_y \dfrac{m}{\pi\hbar^2}$ (with spin)

DOS w/out spin

$$\#\text{deg} = \hbar\omega_c \cdot \boxed{\frac{m}{2\pi\hbar^2}} L_x L_y$$

$\times 2$ ← spin

Generalization to 3D

$$\mathcal{E}_\nu = \frac{\hbar^2}{2m} k_z^2 + \left(\nu + \frac{1}{2}\right)\hbar\omega_c \qquad k_z = \frac{2\pi n_z}{L_z}$$

Area between circles in k-space. $(\nu_2 = \nu_1 + 1)$

$$\left.\begin{aligned} \frac{\hbar^2 k_1^2}{2m} &= \left(\nu_1 + \frac{1}{2}\right)\hbar\omega_c \\ \frac{\hbar^2 k_2^2}{2m} &= \left(\nu_2 + \frac{1}{2}\right)\hbar\omega_c \end{aligned}\right\} \Rightarrow \Delta A = \pi\left(k_2^2 - k_1^2\right) = \frac{2\pi m}{\hbar}\omega_c$$

independent of $\nu$

Alternatively, $\# \text{deg} = \hbar\omega_c \cdot \frac{m}{2\pi\hbar^2} \cdot L_x L_y = \frac{L_x L_y}{2\pi \ell_B^2}$ ← magnetic length

k-space area/state without H: $\frac{(2\pi)^2}{L_x L_y}$

$$\Delta A = \# \text{deg} \cdot \frac{(2\pi)^2}{L_x L_y} = \frac{2\pi m}{\hbar}\omega_c$$

(area for all states in a Landau level)

Typically $H = 1$ KGauss
$L = 1$ cm
$\Rightarrow \# \text{deg} = 10^{10}$

Note: $\ell_B \approx \frac{250}{\sqrt{H(T)}}$ Å

IV Levels of Bloch electrons in a uniform H-field. (Onsager)

Quantization of the semiclassical motion on the orbit specified by $k_z$ and $\mathcal{E}$. $(\vec{H} \parallel \hat{z})$

By correspondance principle,

$$\mathcal{E}_{\nu+1}(k_z) - \mathcal{E}_\nu(k_z) = \frac{h}{T(\mathcal{E}_\nu(k_z), k_z)} \leftarrow \text{period.}$$

$$T = \frac{\hbar^2 c}{eH} \frac{\partial A(\mathcal{E}, k_z)}{\partial \mathcal{E}} \qquad \text{A/M Eq. (12.42)}$$

$$A(\varepsilon_{\nu+1}) - A(\varepsilon_\nu) = (\varepsilon_{\nu+1} - \varepsilon_\nu)\frac{\partial A}{\partial \varepsilon}$$

$$= \frac{h}{T} \cdot \frac{eHT}{\hbar^2 c} = \frac{2\pi e H}{\hbar c}$$

"same as the free case"

$$\frac{2\pi m}{\hbar}\omega_c$$

$$\Rightarrow A(\varepsilon_\nu(k_z), k_z) = (\nu + \lambda)\Delta A$$

Onsager's result

indep. of $\nu$

$\frac{1}{2}$ for the free case.

IV. The de Haas-van Alphen effect : Origin

sharp peak in the density of states when an extremal ~~orbit~~ satisfies the quantization condition, i.e.

$$(\nu + \lambda)\Delta A = A_e(\varepsilon_F)$$

A/M Fig 14.5.

period $\Delta\left(\frac{1}{H}\right) = \frac{2\pi e}{\hbar c}\frac{1}{A_e(\varepsilon_F)}$

extremal area of Fermi surface.

Advatages: (over ARPES)

1. Bulk measurement
2. Low temperatures
3. Large effective mass (resolution low w/ ARPES)
   (narrow band!)

V. Conditions to observe dHvA

A. Nonzero temperature averages over a range of energies within $k_B T$ of $\varepsilon_F$. Hence

$$k_B T \lesssim \hbar \omega_c = \frac{\hbar e H}{m^* c}$$

High-$H$ for large effective mass (heavy fermions)
low $T$ " " "

B. Finite scattering time also mixes different Landau levels. Hence

$\omega_c \tau \gtrsim 1$ ← same as the high-field condition for semiclassical transport theory.

VI. Integer quantum Hall effect in 2DEG ← two-dimensional electron gas.

extreme conditions:

large $H$: electrons only occupy very few LLs.

low $T$: $k_B T \ll \hbar\omega_c$
effectively single-LL physics.

Very clean sample: $\omega_c \tau \gg 1$
LL broadening $\Gamma \ll \hbar\omega_c$

$\frac{\hbar}{\tau}$