Fundamentals of Semiconductor Physics
Problem Set #3

Refer to the tables and figures in textbooks for needed quantities.

**pn junctions.**

1. A 0.6 Ω-cm, n-type silicon sample contains $10^{15}$ cm$^{-3}$ generation-recombination centers located at the intrinsic Fermi level with $\sigma_n = \sigma_p = 10^{-15}$ cm$^2$. Assume $v_{th} = 10^7$ cm s$^{-1}$.

   (a) Calculate the generation rate if the region is depleted of mobile carriers.

   (b) Calculate the generation rate in a region where only the minority-carrier concentration has been reduced appreciably below its equilibrium value.

2. Light is incident on a silicon bar doped with $10^{16}$ cm$^{-3}$ donors, creating $10^{21}$ cm$^{-3}$ s$^{-1}$ electron-hole pairs uniformly throughout the sample. There are $10^{15}$ cm$^{-3}$ bulk recombination centers at $E_i$ with electron and hole capture cross sections of $10^{-14}$ cm$^2$.

   (a) Calculate the steady-state hole and electron concentrations with the light turned on.

   (b) At time $t = 0$ the light is turned off. Calculate the time response of the total hole density and find the lifetime. The thermal velocity is $10^7$ cm s$^{-1}$, and there is no current flowing.

3. Consider an ideal, long-base, silicon abrupt p-n junction diode with uniform cross section and constant doping on either side of the junction. The diode is made from 1 Ω-cm p-type and 0.2 Ω-cm n-type materials in which the minority-carrier lifetimes are $\tau_n = 10^{-6}$ s and $\tau_p = 10^{-8}$ s, respectively. (“Ideal” implies that effects within the space-charge region are negligible and that minority carriers flow only by diffusion mechanisms in the charge neutral regions.)

   (a) What is the value of the built-in voltage?
(b) Calculate the density of the minority carriers at the edge of the space-charge region when the applied voltage is 0.589 V (which is $23 \times kT/q$).

(c) Sketch the majority- and minority-carrier currents as functions of distance from the junction on both sides of the junction, under the applied bias voltage of part b.

(d) Calculate the location(s) of the plane or planes at which the minority-carrier and majority-carrier currents are equal in magnitude for the applied bias voltage of part b.

4. Is Zener breakdown more likely to occur in a reverse-biased silicon or germanium $pn$ junction diode if the peak electric field is the same in both diodes? Discuss. (Consider the size of the bandgap of each material.)

Metal-oxide-silicon system.

1. Sketch the energy-band diagrams (i) at thermal equilibrium and (ii) at flat band for ideal MOS systems made with aluminum gates (a) to 1 Ω-cm $n$-type silicon, and (b) to 1 Ω-cm $p$-type silicon.

2. Repeat the sketches required in the previous problem for a MOS system with a polycrystalline silicon gate. Assume that the silicon gate has a band structure similar to single-crystal silicon but that (i) the gate over $n$-type silicon is doped with acceptors until it is just at the edge of degeneracy and (ii) the gate over $p$-type silicon is doped with donors until it is just at the edge of degeneracy.

3. Prove that the small-signal capacitance of an MOS capacitor $C$, biased into depletion, is given by

$$C = \frac{1}{1/C_{ox}} + \frac{1}{C_s} = \frac{1}{1/C_{ox} + x_d/\epsilon_s}.$$  

That is, show that $C$ is equal to the capacitance of a series connection of two capacitors: (1) a capacitor made with one plate in the bulk of the silicon and the other plate at the oxide-silicon interface, and (2) a capacitor that has its plates separated by the oxide. (*Hint. Use Gauss’ law to express the charge $\Delta Q = \epsilon_{ox}\Delta E_{ox}$. Then, show that the voltage*
across the capacitor is \[ \Delta V = \Delta \mathcal{E}_{ox} x_{ox} + \Delta \mathcal{E}_{ox} \xi_{ox} x_d / \xi_s \] and evaluate \( C = \Delta Q / \Delta V \).)

4. The value of \( \phi_s \) (the surface potential) is frequently needed for experimental studies of MOS systems.

(a) By using the results of the previous problem (MOS systems, Problem 3), show that when the gate voltage \( V_G \) is changed on an MOS capacitor biased in the depletion region, it is possible to find the corresponding change in \( \phi_s \) by using the measured capacitance of the MOS system. The change in \( \phi_s \) can be calculated from the relationship

\[
\phi_s(V_{G2}) - \phi_s(V_{G1}) = \int_{V_{G1}}^{V_{G2}} \left(1 - \frac{C}{C_{ox}}\right) dV_G \tag{2}
\]

This technique is known as Berglund’s method after its originator [C. N. Berglund, IEEE Trans. Electron Devices, ED-13, 701 (1966)]. It can be used conveniently if \( V_{G1} \) is taken to be \( V_F B \) at which point \( C \) is given by

\[
C = \frac{1}{1/C_{ox} + L_D / \xi_s} = \frac{1}{x_{ox} / \xi_{ox} + [kT / q^2 \xi_s N_a]^{1/2}}. \tag{3}
\]

(b) If \( V_{G1} \) is taken as \( V_{FB} \) sketch a low-frequency MOS capacitance curve for \( p \)-type silicon (normalized to \( C_{ox} \)) and indicate (by shading) an area on the curve equal to \( \Delta \phi_s \).

5. Sketch capacitance-voltage curves of the MOS structures shown in Figures 1a, 1b, and 1c. The capacitance is the small-signal value normalized to that of the oxide and measured at 100 kHz. In all cases, the gate dc bias is varied slowly. Show (by using dotted curves) what effect an increase in positive \( Q_f \) would have on the \( C-V_G \) curves. Label each region on the curves (accumulation, depletion, and inversion). Assume that the substrate resistivity is of the order of 10 \( \Omega \)-cm in each case and make your sketches qualitatively correct.