Physics of the Piano

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All is about the frequency of the sound.

PITCH, COLOR AND MUSICAL SCALE
Octave

[Diagram of a piano keyboard showing notes and intervals]
Pitch and Tone

![Graphs of sound pressure over time and power spectrum of frequencies](image)

- **Pitch**: Refers to the perceived highness or lowness of a sound.
- **Tone**: Refers to the quality or timbre of a sound, which can be influenced by factors such as overtones and harmonics.
Spectrum of middle C: linear scales

- $f_1$
- $f_2$
- $f_3$
- $f_4$
- $f_5$
- $f_6$
- $f_7$
- $f_8$

Power (arb. units)

Frequency (Hz)
Musical Scale

*Pythagoras*

- **Frequency (Hz):**
  - C4: 261.6 Hz
  - C5: 523.3 Hz
  - C6: 1047 Hz

- **Segments:**
  - (a) C4 to C5
  - (b) C4 to C5
  - (c) C4 to C5

- **Multiplication Factors:**
  - C4 to D4: x1.5
  - D4 to E4: x0.5
  - E4 to G4: x1.5
  - G4 to A4: x1.5
  - A4 to C5: x0.5
  - C5 to D5: x1.5
  - D5 to E5: x1.5
  - E5 to C6: x1.5
Musical Scale

*Equal Temperament*

- A4, fundamental frequency of 440 Hz
- \(2^{1/12} = 1.05946\ldots\)
- Perfect 5\(^{th}\), \(2^{7/12} = 1.4983\ldots \approx 1.5\)
Vibrating strings
Hammers that hit the strings
Soundboard producing sound

PHYSICS IN PIANO WHEN MAKING MUSIC
Vibrating Strings

\[ f = \frac{v}{\lambda} \]
\[ \lambda = \frac{L}{n} \]
\[ f \propto \frac{v}{L} \]

\[ f \propto f(T, \rho, L) \]
Vibrating Strings

f = v/\lambda
\lambda = L/n
f \propto v/L
f \propto \sqrt{T/\rho/L}
Vibrating Strings

copper windings

steel core

agrazze

tuning pins

50 mm
Hammers that Hit the Strings

- Middle C
- Sound pressure vs. time

(a) key pressed
(b) key released
Hammers that Hit the Strings
Hammers that Hit the Strings

(a) nut

(b) bridge

Hammer force vs. time hitting a rigid target

Hammer string force: Collision with a string
Hammers That Hit the Strings

\[ F_h (N) = k z^\alpha \]

- Force-compression relation
- Hammer striking a rigid plate

Effect of hammer hardness on strength of partials

Relative power as a function of partial number

Strength of partials

Note A4
Hammers That Hit the Strings

(a) damper felt

(damper rod)

(hammer)

(string)

(hammer butt)

(hinge)

(keypress)

(jack)

(intermediate lever)

(balance point for key)

(jack pushes up on intermediate lever which pushes up on hammer butt)

(b) damper felt

(damper rod)

(hinge)

(keypress)

(jack)

(balance point for key)

(jack begins to slide off intermediate lever)

(c) damper felt

(damper rod)

(hinge)

(keypress)

(jack)

(balance point for key)

(jack slips off edge of intermediate lever and lever drops away from hammer)

(right side)

Force-compression relation

(Hammer striking a rigid plate)

(F_N (N))

(z (mm))

(100)

(1)

(0.01)

(0.0001)

(n = partial number)

Strength of partials

(Note A4)
Soundboard Producing Sound

(a)

(b)

hitchpin

T

T

soundboard

bridge

change in force on bridge

T

T
Soundboard Producing Sound

(a)

(b)
Soundboard Producing Sound

end view of log

Y/ρ

bass bridge

treble bridge

ribs

bass strings

treble bridge

bass bridge

treble strings

(keyboard)
Soundboard Producing Sound

(a)

(b)

bridges

ribs