An Outlook of the μ-term Models

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Supersymmetry

- Gauge Hierarchy Problem
- Gauge Unification
- Consistent string theory
- Dark Matter Candidate (SUSY with R-parity)

SUSY is introduced only to stabilize $M_{\rm EW}$

Why $M_{\rm EW}$: $\mathcal{O}(10^2~{\rm GeV})$?

MSSM Yukawa Sector

$$W_{\text{MSSM}} = Qu^c H_u + Qd^c H_d + \ell e^c H_d + \mu \mathbf{H_u H_d}$$

- Superpotential is holomorphic.
- Anomaly cancellation

$$H_d = \begin{pmatrix} H_d^0 \\ H_d^- \end{pmatrix} \; H_u = \begin{pmatrix} H_u^+ \\ H_u^0 \end{pmatrix} : \langle H_d \rangle = \begin{pmatrix} v_d \\ 0 \end{pmatrix} \; \langle H_u \rangle = \begin{pmatrix} 0 \\ v_u \end{pmatrix}$$

vectorial under $U(1)_Y$

The μ-term Problem

Higgs mass parameter

$$\mathcal{W}_{\rm MSSM} \supset \mu H_u H_d$$

$$\mathcal{L}_{\rm soft} \supset B\mu H_u H_d + {\rm h.c.}$$

- $\mu = 0$: massless charged fermion (Higgsino)
- $\mu = M_{\rm Pl}$: Higgs Decoupled

$$\mu \sim 10^2\ {\rm GeV}(\sim {\rm M_{SUSY}})$$
$$B \sim 10^2\ {\rm GeV}(\sim {\rm M_{SUSY}})$$

Remarks

- M_{SUSY} scale Higgsino plays important role in SUSY gauge unification
- Large B-term in Split SUSY requires fine-tuning to get one Higgs light but $\mu \sim M_{\mathrm{SUSY}}$

  (Kingman Cheung, Cheng-Wei Chiang, hep-ph/0501265)

Questions in Model Building

- How to generate μ and B scale
- How gauge sector works out

Beyond-SM New Physics Scale

$M_{\rm Pl} \sim \mathcal{O}(10^{19}\rm GeV)$ $\not B$ and $\not L$

$M_{\rm GUT} \sim \mathcal{O}(10^{16}\rm GeV)$ $\not B$ and $\not L$

$M_{\rm R} \sim \mathcal{O}(10^{14}\rm GeV)$ $\not L$ (Close to $M_{\rm GUT}$)

$M_{\rm PQ} \sim \mathcal{O}(10^{11}\rm GeV)$ $\not{PQ}$ (Strong CP & invisible QCD axion)

$M_{\rm SUSY} \sim \mathcal{O}(10^{3}\rm GeV)$

Physics Scale in SM

$M_{\rm EW} \sim \mathcal{O}(10^{2}\rm GeV)$

$\Lambda_{\rm QCD} \sim \mathcal{O}(10^{-1}\rm GeV)$

Global R-symmetry and μ-term

Hall, Nomura & Pierce, 2000

$$W_\mu = \lambda X H_u H_d + f X (Y^2 - \Lambda^2)$$

$$V = |F_X|^2 + |F_Y|^2 = |Y^2 - \Lambda^2|^2 + |f X Y|^2$$

at SUSY limit $\langle X \rangle = \langle Y \rangle = 0$, $\langle F_X \rangle = \langle F_Y \rangle = 0$

$$\mathcal{L}_{\mathrm{soft},0} = -m_X^2 |X|^2 - m_Y^2 |Y|^2 - \left(a_f X Y^2 - a_\Lambda \Lambda^2 X + \mathrm{h.c.} \right)$$

$$\langle X \rangle \simeq (a_\Lambda^* - a_f^*)/4|f|^2 \sim M_{\mathrm{SUSY}}$$

$$\langle F_X \rangle \simeq [(a_\Lambda + a_f)(a_\Lambda^* - a_f^*)/4|f|^2 + m_Y^2]/2f \sim M_{\rm SUSY}^2$$

$$\mu = \lambda \langle X \rangle \simeq \frac{\lambda(a_\Lambda^* - a_f^*)}{4|f|^2}$$

$$\mu B = -\lambda \langle F_X \rangle + a_\lambda \langle X \rangle \simeq \frac{(2f a_\lambda - \lambda a_\Lambda - \lambda a_f)(a_\Lambda^* - a_f^*)}{8f|f|^2} - \frac{\lambda m_Y^2}{2f}$$

Why New Symmetry?

Under $SU(3)_c \times SU(2)_L \times U(1)_Y$,

$$L = (1, 2, -\frac{1}{2}); H_d = (1, 2, -\frac{1}{2})$$

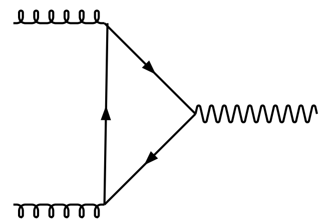
(even under $SU(5)$, both arise from $\bar{\mathbf{5}}$ unless they originate from $SO(10)$, $L \in \mathbf{16}$ while $H_d \in \mathbf{10}$)

$$W = \mu H_u H_d + \mu' L H_u$$

where LH_u violates R-parity.

$H_u + H_d$ can NOT be vectorial

Flavor Independent case



$$
\begin{aligned}
A_3 &= 3\alpha + \frac{3}{2}(2(q-\alpha) + (u-\alpha) + (d-\alpha)) \\
&= 3\alpha - \frac{3}{2}(h_u + h_d)
\end{aligned}
$$

$$q + u + h_u = 2\alpha, q + d + h_d = 2\alpha$$

Induce mixed QCD anomaly $A_{[SU(3)_C]^2 \times G}$

The symmetry that ensures the absence of bare μ-term in the superpotential carries mixed QCD anomaly and is broken explicitly by anomaly thus a ”Goldstone”.

- Flavor dependent: Naturalness problem Dreiner, Muraryama & Thormeier, 2003
- NMSSM: Global Z_3 in Higgs effective potential, may lead to Domainwall near TeV
- Gauge this symmetry
  - $U(1)'$: Extra Z', exotic quark
  - Anomalous $U(1)_A$ Symmetry (broken near M_{string})
    Guidice-Masiero Mechanism with Anomalous $U(1)_A$ subgroup
- QCD Axion and the accidental Peccei-Quinn Symmetry.
  PQ symmetry is explicitly broken by the mixed QCD anomaly and it will induce axion. PQ arises as an accidental symmetry. Anomalous $U(1)_A$ subgroup stabilizes the axion solution.

$$M_{\text{PQ}}(f_a) \sim 10^{11}\ \text{GeV}$$

Kim & Nilles, 1984; Babu, Gogoladze, KW 2003

$$M_{\rm EW} \sim \frac{M_{\rm PQ}^2}{M_{\rm Pl}}$$

Anomalous $U(1)$ Symmetry

4D Effective Theory arising from String

$$\mathcal{L} = -\int d^4\theta \ln(S + S^\dagger - \delta_{GS} V_X) + \int d^2\theta \left[\frac{S}{4} (\sum_a k_a \mathrm{Tr} W_a^\alpha W_{a\alpha} + k_X \mathrm{Tr} W_X W_X) + h.c. \right]$$

S is the string dilaton which is a massless particle carries coupling.

$$g_X^2 = 2/k_X (S + S^\dagger)$$

k_a: Kac–Moody levels where a: $SU(3)_C$, $SU(2)_L$ or $U(1)_Y$ Kac-Moody level $k_i \in \mathcal{Z}^+$ for non-Abelian Group, k_1 positive rational number

$$[T^a_m, T^b_n] = \imath f^{abc} T^c_{m+n} + d^{ab}_{mnj}\, \mathbf{k^j}$$

CFT s: Unification: String Theory so central charges of Virasoro algebra add up implies k_i should not be large.

$$\begin{aligned} A_\mu &\rightarrow A_\mu + \partial_\mu \alpha(x) \\ S &\rightarrow S + \frac{\imath}{2}\delta_{GS}\alpha(x) \end{aligned}$$

Anomalous Symmetry

$$\mathrm{Tr} Q_i \neq 0$$

Anomaly Induced Fayet-Illiopoulos term

$$D = -(\sum_i q_i |\tilde{Q}_i|^2 + \xi)$$

where

$$\xi = \frac{g^2 \mathrm{Tr} Q_i}{192\pi^2} M_{\mathrm{P}l}^2$$

Green-Schwarz Anomaly Cancellation Mechanism

Cancellation of anomalies by the modified transformation law of p-form potential in Chern-Simons terms.

$$\frac{A_3}{k_3} = \frac{A_2}{k_2} = \frac{A_1}{k_1} = \frac{A_{\mathrm{Gravity}}}{24} = \delta_{GS}$$

A_n: mixed anomalies of $[SU(n)]^2 \times U(1)_A$

Guidice-Masiero Mechanism & Anomalous $U(1)$

SUGRA mediated SUSY Breaking

Gaugino Mass

$$\int d^2\theta W_\alpha W_\alpha Z / M_{\rm Pl}$$

$$\int d^4\theta H_u H_d Z^* / M_{\rm Pl}$$

$$\langle F_Z \rangle / M_{\rm Pl} \sim \mu$$

R-symmetry: gaugino being charged to distinguish the superpotential from Kälher potential.

- Absence of μ-term in the superpotential
- Only arises in Kälher potential

Z_4	q	u	d	l	e	n	h	$\bar{h}$	α	$(A_2,\ A_3)$
Assignment	1	1	1	1	1	1	0	0	1	(3, 1)

Babu, Gogoladze, KW 2002

- Why discrete? Low energy effective theory only contains this gauge freedom.
- Automatic gauged R-parity (Z_2 subgroup of the I_{3R})
- Compatible with $SO(10)$

Dine–Fischler–Srednicki–Zhitnitskii (DFSZ) Axion

$$W_{\mathrm{DFSZ}} = H_u H_d S^2 / M_{\mathrm{Pl}}$$

- A New Physics scale: M_{PQ}, $M_{\mathrm{SUSY}} \sim M_{\mathrm{PQ}}^2 / M_{\mathrm{Pl}}$
- Explanation of μ and B
- Axion: the Strong CP problem and Dark matter candidate

Questions needed to be addressed

- How to naturally generate M_{PQ}: Dynamical or Higgs
- A Complete Model with the appropriate symmetry
- Phenomenology Implication

$$W_{\rm DFSZ} = \lambda_1 \frac{H_u H_d S^2}{M_{\rm Pl}} + \lambda_2 \frac{(S\tilde{S})^2}{M_{\rm Pl}} + \frac{S^n}{M_{\rm Pl}^{n-3}}$$

$$V = (\lambda_2 C \frac{(S\tilde{S})^2}{M_{\rm Pl}} + h.c) + {m_S}^2 |S|^2 + {m_{\tilde{S}}}^2 |\tilde{S}|^2 + 4\lambda_2 \frac{|S\tilde{S}|^2}{M_{\rm Pl}^2} (|S|^2 + |\tilde{S}|^2)$$

$$f_a^2 = \frac{C \pm \sqrt{C^2 - 12{m_S}^2}}{12\lambda_2} M_{\rm Pl} : f_a = \langle |S| \rangle = \left\langle |\tilde{S}| \right\rangle \sim \sqrt{M_{\rm Pl} M_{\rm SUSY}} \sim 10^{11}\ {\rm GeV}$$

$\mathcal{O}({\rm TeV})$ axino and saxino

$$\langle F_S \rangle \sim M_{\rm PQ} M_{\rm SUSY}$$

- $\mu \sim M_{\rm PQ}^2 / M_{\rm Pl} \sim M_{\rm SUSY}$
- $B\mu \sim \langle S \rangle \langle F_S \rangle / M_{\rm Pl} \sim M_{\rm SUSY}^2$

More on Anomalous $U(1)_A$

- Stabilize the axion solution
- Flavor physics
  - ϵ parametrization naturally from $U(1)_A$ breaking
  - D-term splitting
- Anomalous $U(1)$ mediated SUSY Breaking Dvali & Pomarol, Mohapatra & Riotto
  - D-term breaking
  - additional μ-term problem

SUSYGUTs

Conventional Doublet-triplet Splitting in SUSYGUTs

Fine-tuned SUSY $SU(5)$ GUT

matter $\mathbf{10} + \bar{5} + 1$

Higgs $\mathbf{24}_H, \{\mathbf{5}_H, \bar{5}_H\}$ contain color triplets $(p \to \bar{\nu} K^+)$

Yukawa

$$\mathbf{10}_i \mathbf{10_j} \mathbf{5}_H + \mathbf{10}_i \bar{5}_j \bar{\mathbf{5}}_H$$

Doublet-triplet Splitting

$$W = \bar{\mathbf{5}}_H (\lambda \mathbf{24}_H + M) \mathbf{5}_H$$

$$\langle \mathbf{24}_H \rangle = \mathrm{diag}(1, 1, 1, -\frac{3}{2} - \frac{3}{2}) V$$

$$
\begin{aligned}
M_{H_c} &= \lambda V + M \sim \mathcal{O}(M_{\mathrm{GUT}}) \\
M_H &= -\frac{3}{2}\lambda V + M
\end{aligned}
$$

- Economic Higgs sector
- Fine tuning
- $d = 5$ Proton decay (Color Higgsino)

SUSY $SO(10)$ GUT with Non-renormalizable Yukawa

matter $\mathbf{16}_i$ (contains ν_R and seesaw)

Higgs $\mathbf{45}_H, \mathbf{10}_H, \mathbf{16}_H, \mathbf{\overline{16}}_H$

Yukawa

$$16_i 16_j 10_H + 16_i 16_j \overline{16}_H \overline{16}_H / M_{\mathrm{Pl}}$$

seesaw $m_D \sim m_t, m_R \sim M_{\mathrm{GUT}}^2 / M_{\mathrm{Pl}}$,

$$m_\nu \sim m_D^2 / m_R$$

Doublet-triplet Splitting (Dimopolous-Wilczek):

$$W = \lambda 10_H 45_H 10'_H + M 10'_H 10'_H$$

$$\langle 45_H \rangle = \mathrm{diag}(a, a, a, 0, 0) \otimes i\tau_2$$

- $U(1)_{B-L}$ breaking $\longrightarrow$ triplets only
- Proton Decay (10_H couples to matter)
- Unification ($10'_H$ mass M)

Anomalous $U(1)_A$ and GUT Compatibility : $G_{\rm GUT} \times U(1)_A$

Gauge Anomaly Cancelation: A GUT compatible symmetry automatically satisfies GS.

Spontaneous Symmetry breaking does not induce gauge anomalies.

$SU(5) \times U(1)_A$

$$W_{\rm Yukawa} = \Gamma_u^{ij} \mathbf{10}_i \mathbf{10}_j H_u + \Gamma_d^{ij} \mathbf{10}_i \bar{5}_j H_d$$

$$\begin{aligned} A_3 &= 3\alpha + \frac{N_g}{2}[2(\mathbf{10}-\alpha) + (\mathbf{10}-\alpha) + (\bar{\mathbf{5}}-\alpha)] \\ A_2 &= 2\alpha + \frac{N_g}{2}[3(\mathbf{10}-\alpha) + (\bar{\mathbf{5}}-\alpha)] + \frac{1}{2}(h_u + h_d - 2\alpha) \end{aligned}$$

From GSM

$$h_u + h_d = 4\alpha, \mathcal{L} \supset \int d^2\theta W, Q_W = 2\alpha$$

Absence of bare μ-term in W unless a non-R symmetry

PQ symmetry and R symmetry