Inverse See-saw in Supersymmetry

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hep-ph/10xx.xxxx with Seong-Chan Park
See-saw is perhaps the most elegant mechanism for neutrino mass generation, $\nu_R$ is well motivated from $SO(10)$ and $SU(3)_H$. but..

$$y_\nu \ell_L n_R H_u + M_R \bar{n}_R^c n_R + M_S \bar{s}_L n_R$$

Then, in the basis of $\left(\nu_L, s_L, n_R^c\right)$

$$\mathcal{M} = \begin{pmatrix} 0 & 0 & M_D \\ 0 & 0 & M_S \\ M_D & M_S & M_R \end{pmatrix}$$

Lightest mass eigenstate remain massless.....

- Is there any exact chiral symmetry to protect $m_\nu$?
- Why is there an additional singlet? $E_6$?
- What is the Lepton number violation scale $\Lambda_L$? Can it be within weak scale?
In this talk...

Two examples that tree-level masses are suppressed but only arise radiatively.

- generate neutrino mass in a modified Wyler-Wolfenstein model from radiative corrections. *(with Seong-chan Park, hep-ph/1010.xxxx)*

- generate charged lepton masses $m_e$ and down-type quark masses $m_d$ radiatively from $\langle H_u \rangle$ in MSSM *(large $\tan \beta$ limit, see for example, Dobrescu-Fox, upper-lifted MSSM, hep-ph/1001.3147)*

So no unbroken chiral symmetry....
Lessons from Upper-lifted MSSM Dobrescu-Fox, 1001.3147

$\langle H_u \rangle \gg \langle H_d \rangle$, $m_e, m_d$ from $\langle H_u \rangle$

Accidental symmetries in SM lagrangian

$$iQ^i_L D Q^i_L + i\bar{u}_R^i D u_R^i + i\bar{d}_R^i D d_R^i + ...$$

$$Q^i_L \rightarrow U^i_Q Q^j_L, \quad u_R^i \rightarrow U^i_u u_R^j, \quad d_R^i \rightarrow U^i_d d_R^j$$

With three generations, $U(3)_Q \times U(3)_u \times U(3)_d \times U(3)_\ell \times U(3)_e$

$$-y^i_u \bar{Q}^i_L cH^\dagger u_R^j - y^i_d \bar{Q}^i_L H d_R^j + ...$$

break the above $[U(3)]^5$ into $U(1)_B \times U(1)_{Lep}$

$$Q^i_L \rightarrow e^{i\theta/3} Q^i_L, \quad u_R^i \rightarrow e^{i\theta/3} u_R^i, \quad d_R^i \rightarrow e^{i\theta/3} d_R^i$$

$$\ell^i_L \rightarrow e^{i\phi} \ell^i_L, \quad e^i_R \rightarrow e^{i\phi} e^i_R$$

Inverse Seesaw in SUSY Kai Wang, IPMU, U-Tokyo
Fermion mass is not only a electroweak symmetry breaking (EWSB) effect.

- If $y \to 0$, $U(3)$ symmetry will be restored and the corresponding fermion will be massless up to all loops.
- $m_t \, (\text{or } m_u) \neq 0 \to m_d, m_e$ must break the $U(3)$s. (for instance, topcolor model)

To eliminate the tree-level contribution, tune the vev .....possible in 2HDM (large $\tan \beta$)

$$\langle H_u \rangle \gg \langle H_d \rangle$$

Non-zero Yukawa couplings ensure that the chiral symmetries have been broken. The masses can be generated radiatively.
MSSM is a natural 2HDM

- Superpotential is holomorphic and $\epsilon H^*$ is forbidden in superpotential.
- $\tilde{H}_u$, $\tilde{H}_d$ contributes to anomaly $[SU(2)_L]^2U(1)_Y$, .... and Witten Anomaly

$$W = y_u Qu^c H_u + y_d Qd^c H_d + y_e \ell e^c H_d + \mu H_u H_d$$
2HDM has Peccei-Quinn symmetry (DFSZ axion, 1981).

\[ A_{[SU(3)_C]^2U(1)} = 3\alpha + \frac{3}{2}(2(q - \alpha) + (u - \alpha) + (d - \alpha)) \]

\[ = 3\alpha - \frac{3}{2}(h_u + h_d) \]

\[ q + u + h_u = 2\alpha, \quad q + d + h_d = 2\alpha \]

\[ h_u + h_d \neq 2\alpha \rightarrow A_3 \neq 0 \]
$$10 \cdot 10 H_u + 10 \cdot \bar{5} H_d$$

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<th>$H_u$</th>
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$$10 \cdot \bar{5} H_u^*: \quad R: \quad \frac{1}{5} + \frac{3}{5} - \frac{4}{5} = 0 \neq 2, \quad PQ$$
\[ W = \mu H_u H_d \]

\[ \mathcal{L}_{\text{soft}} \ni m_f^2 | \tilde{f} |^2 \quad R - \text{invariant} \]

\[ + M_{\frac{1}{2}} \lambda \lambda + A_u \tilde{Q} \tilde{u} H_u + \ldots \quad R \]

\[ + B \mu H_u H_d \quad R, P\bar{Q} \]

If \( P\bar{Q}, R \) and \([U3]_5\),

\[ 10 \cdot \tilde{5} H_u^* \rightarrow m_e, m_d \neq 0 \]

Another \( P\bar{Q} \) source, (proportional to \( \mu \))

\[ F_{H_d} = \frac{\partial W}{\partial H_d} = y_d Q d^c + y_e \ell e^c + \mu H_u \]

\[ V \ni | F_{H_d} |^2 = y_d \mu^* H_u^* \tilde{Q} \tilde{d} + y_e \mu^* H_u^* \tilde{\ell} \tilde{e} \]
Inverse Seesaw in SUSY

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Inverse Seesaw in SUSY

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If all Yukawa couplings in MSSM are perturbative at $M_{\text{GUT}}$, $2 \lesssim \tan \beta \lesssim 50$.

But what if $m_b$ arise from $\langle H_u \rangle$ radiatively.....
\( \nu_L \) does not carry any unbroken gauge symmetry 
\((SU(3)_C \times U(1)_{EM})\)...

\[
- \frac{1}{2} M^i_{ij} \nu^i L C \nu^j_L
\]

Type-I see-saw

\[
y \nu \ell_L n_R H_u + M_R n^c_R n_R + h.c.,
\]

For one generation \( y_\nu \) break \( U(1)_\ell \times U(1)_n \rightarrow U(1)_{\text{Lep}} \)

\[
M_R \rightarrow U(1)_{\text{Lep}}
\]

\[
m_\nu = M^T_D M^{-1}_R M_D
\]

- Without tuning dimensionless \( y_\nu \), tiny \( m_\nu \) from \( M_{\text{GUT}} \)

- \( U(1)_{B-L} \) becomes anomaly free, easily embedded into \( SO(10) \)
In basis \((\nu_L, s_L, n^c_R)\)

\[
M = \begin{pmatrix}
0 & 0 & M_D \\
0 & 0 & M_S \\
M_D & M_S & 0
\end{pmatrix}
\]

\(m_{\nu} = 0\)
Inverse see-saw (Mohapatra, Valle)

\[ y_\nu \ell_L n_R H_u + M_S s_L n_R + \epsilon s^c_L s_L \]

In basis \((\nu_L, s_L, n^c_R)\)

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & M_D \\
0 & \epsilon & M_S \\
M_D & M_S & 0
\end{pmatrix}
\]

\[
m_\nu \simeq \epsilon \frac{M_D^2}{M_D^2 + M_S^2}
\]
Tuning: Dimensionless $y_{\nu}$ or dimension-one $M$

$y_e \sim 10^{-6}, y_{\nu} \sim 10^{-12}$?

Dimension One: see-saw vs inverse

- In see-saw, $M_R$ breaks $U(1)_{B-L}$ gauge symmetry at ultra-high scale, for instance, $M_{\text{GUT}}$.

- Now $n, s$ are both SM gauge singlet..., the scale vanishes to restore the $U(1)_{\text{Lep}}$, can be identified as soft breaking of $U(1)_{\text{Lep}}$. 
\[ y_{\nu} \ell_L n_R H_u + M_S s_L n_R + M_R n_R^c n_R \]

In basis \((\nu_L, s_L, n_R^c)\)

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & M_D \\
0 & 0 & M_S \\
M_D & M_S & M_R
\end{pmatrix}
\]

\[
\nu = -\frac{M_S}{\sqrt{M_D^2 + M_S^2}} \nu_L + \frac{M_D}{\sqrt{M_D^2 + M_S^2}} s_L
\]

\[
N_{\pm} = \frac{1}{\sqrt{M_{\pm}^2 + M_D^2 + M_S^2}} (M_D \nu_L + M_S s_L - M_{\pm} n_R^c)
\]

with mass eigenvalues as

\[
m_{\nu} = 0, \quad M_{\pm} = \frac{1}{2} \left( M_R \pm \sqrt{4M_D^2 + M_R^2 + 4M_S^2} \right)
\]
\[ U(1)_\nu \times U(1)_n \times U(1)_s \]

In basis \((\nu_L, s_L, n^c_R)\)

\[
\mathcal{M} = \begin{pmatrix}
0 & 0 & M_D \\
0 & 0 & M_S \\
M_D & M_S & M_R
\end{pmatrix}
\]

With \(M_D, M_S\)

\[ U(1)_\nu \times U(1)_n \times U(1)_s \rightarrow U(1)_{\nu-s} \times U(1)_{\text{Lep}} \]

Under \(U(1)_{\text{Lep}}\)

\[
\begin{align*}
\nu_L & \rightarrow e^{i\alpha} \nu_L \\
s_L & \rightarrow e^{i\alpha} s_L \\
n^c_R & \rightarrow e^{-i\alpha} n^c_R
\end{align*}
\]

With \(M_R\)

\[ U(1)_{\text{Lep}} \rightarrow U(1)_{\nu-s} \]

unbroken

Inverse Seesaw in SUSY \hspace{1cm} \text{Kai Wang, IPMU, U-Tokyo}
Why no $M_R^* S^c_L s_L$? SUSY

Supersymmetry does not forbid lepton number violation but only stabilize the model. So it is just “Technically Natural”.

$$W = y_\nu \ell n^c H_u + M_S s n^c + M_R n^c n^c$$

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<th>$R$-charge of $m$</th>
<th>$U(1)_L$ charge</th>
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New gauge interaction?

Under $E_6$

$$27 = 16 + 10 + 1$$

$s_L$ is completely gauge singlet and any term involving $s_L$ will be only gravitationally induced in

- Kähler potential
- Lepton number violation $B$-terms in soft-breaking lagrangian
$R$-invariant piece

$U(1)_{B-L}$ becomes anomalous so the leading is Yukawa interaction induced $\ell\ell H_u H_u$ Non-SUSY contribution

\[
(M_{\nu}^{1\text{-loop}})_{ij} = \sum_{k=1}^{3} \frac{1}{16\pi^2} M_R^k \sum_{\phi=h,H} \frac{Y_{ik}^* Y_{jk}^* M_{\phi}^2}{M_{\phi}^2 - M_R^k} \ln \left( \frac{M_{\phi}^2}{M_R^k} \right) \\
- \frac{Y_{ik}^* Y_{jk}^* M_A^2}{M_A^2 - M_R^k} \ln \left( \frac{M_A^2}{M_R^k} \right)
\]
\[ M_R \frac{M^2_\phi}{M^2_\phi - M^2_R} \ln \left( \frac{M^2_\phi}{M^2_R} \right) \]

- \( M_R \gg M_{h,H} \) restore the see-saw, \( M_R < 10^{12} \) GeV
- \( M_R \ll M_{h,H} \), inverse see-saw limit, \( M_R \sim \text{KeV} \)

We take the inverse see-saw limit (non-canonical Kählher potential) to ensure light neutrino mass.
\[ V = \left| \frac{\partial W}{\partial n_R} \right|^2 = \left| (\ell H_u + M_s s_L + M_R n_R) \right|^2 \]
\[ = M_R^* \tilde{n}_R^* \tilde{\ell} H_u + M_R^* M_s \tilde{n}_R^* \tilde{s}_L \]

After \( M_{\text{susy}} \),

\[ m_{\tilde{n}}^2 \sim M_{\text{susy}}^2 + M_R^2 \]

- \( M_R \gg M_{\text{susy}} \)
- \( M_R \ll M_{\text{susy}} \)
The effective operators do not break $R$-symmetry

\[ \mathcal{L}_{\text{soft}} \ni B_R M_R \tilde{n}_R \tilde{n}_R + B_S \epsilon_s \tilde{s}_L \tilde{s}_L + A_s M_R \tilde{\ell}_L \tilde{s}_L H_u + B_\nu M_\nu \tilde{\nu}_L \tilde{\nu}_L \]

$m_\nu$ only arise with gaugino mass insertion. Soft SUSY breaking terms that also violate $U(1)_{\text{Lep}}$ but $1/M_{\text{Pl}}$ suppression.
Conclusions

- I present two examples that the fermion masses arise from radiative correction where the tree level masses are suppressed but all the chiral symmetries are broken.
- Supersymmetry plays an important role in stabilizing the suppressed tree level masses.