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The role of inter-well tunnelling strength on coherence dynamics of two-species Bose–Einstein condensates

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Abstract

Coherence dynamics of two-species Bose–Einstein condensates in double wells is investigated in the mean-field approximation. We show that the system can exhibit decoherence phenomena even without the condensate–environment coupling and the variation tendency of the degree of coherence depends not only on the parameters of the system but also the initial states. We also investigate the time evolution of the degree of coherence for a Rosen–Zener form of tunnelling strength, and propose a method to get a condensate system with a certain degree of coherence through a time-dependent tunnelling strength.

(Some figures in this article are in colour only in the electronic version)

1. Introduction

The manipulating Bose–Einstein condensates (BECs) in double wells provide us with a versatile tool to explore the underlying physics in various nonlinear phenomena since almost each parameter can be tuned experimentally [1]. There have been many studies on the fascinating features of nonlinear effect, such as Rabi oscillation [2–5], Josephson oscillation [5–7], self-trapping [4, 7–10], and measure synchronization [11], in terms of BECs in double wells. Because the BECs in double wells can be regarded as a two-level system, it is also expected either to be employed as a possible qubit or to simulate certain issues [12] in quantum computation and information. Recently, the effect of decoherence of BECs in double wells was investigated experimentally by means of interference between BECs [13, 14] and studied theoretically in terms of a single-particle density matrix [15]. In order to exhibit the phenomena of decoherence, the authors of [13–15] need to introduce the condensate–environment coupling because one-species BECs in double wells were merely considered there. In comparison to the one-species BEC system, the two-species system can exhibit distinct decoherence phenomena due to the existence of the interspecies interaction. For example, the degree

of coherence in a two-species system can evolve with time without the application of condensate–environment coupling, which is useful for one to get a system with a desired degree of coherence. It is therefore worthwhile to study the coherence dynamics of two-species BECs in double wells.

In this paper, we study the effects of the inter-well tunnelling strength on the coherence dynamics for a system of two-species BECs in double wells. With the help of the reduced single-particle density matrix, we show that such a system can exhibit decoherence phenomena without condensate–environment coupling. We also propose an experimental strategy to prepare a BEC system with any desired degree of coherence through a time-dependent tunnelling strength. In the following section, we model the two-species BEC system and introduce the reduced single-particle density matrix. In section 3, we study the time evolution of the degree of coherence for a time-independent inter-well tunnelling strength. In section 4, we investigate the time evolution of the degree of coherence for a Rosen–Zener form of tunnelling strength and discuss the possibility of preparing a BEC system with any degree of coherence. Then we briefly give our conclusion in section 5.

2. The model and reduced density matrix

We consider a two-species Bose–Einstein condensate system confined in double wells. The Hamiltonian is given by

$$\hat{H} = -J_a(\hat{a}_1^\dagger\hat{a}_2 + \text{H.c.}) - J_b(\hat{b}_1^\dagger\hat{b}_2 + \text{H.c.}) + \sum_i \tilde{U}_{ab}\hat{n}_{ai}\hat{n}_{bi} + \sum_i \frac{1}{2}\tilde{U}_{aa}\hat{n}_{ai}\hat{n}_{ai} + \sum_i \frac{1}{2}\tilde{U}_{bb}\hat{n}_{bi}\hat{n}_{bi}, \quad (1)$$

where \hat{a}_i^\dagger (\hat{b}_i^\dagger) and \hat{a}_i (\hat{b}_i) create and annihilate a bosonic atom of species a (b) in the i th well, respectively; $\hat{n}_{ai} = \hat{a}_i^\dagger\hat{a}_i$ ($\hat{n}_{bi} = \hat{b}_i^\dagger\hat{b}_i$) denotes the particle number operator of species a (b). Here the parameters J_a and J_b denote the tunnelling strengths of species a and b between the two wells, \tilde{U}_{aa} and \tilde{U}_{bb} are the intraspecies interaction strengths, and \tilde{U}_{ab} is the interspecies interaction strength. The Hamiltonian (1) can describe a BEC mixture confined in a double-well potential consisting of different atoms, or different isotopes, or different hyperfine states of the same kind of atom. The coherence dynamics of the above model has not been investigated, although its dynamical properties, like Josephson oscillation, stability and measure synchronization, etc, have been studied in earlier works [5, 11, 16]. However, we know that the role of coherence of the system is very important since the first obstacle attempted to be avoided is the decoherence when a condensate in double wells is expected to be employed as a qubit. So the coherence dynamics of the model (1) is worthy of study.

As we know, under the semiclassical limit [2, 6, 9, 17], the dynamics of this system is conventionally studied in the mean-field approach by replacing the expectation values of annihilators with complex numbers, i.e. $\langle\hat{a}_i\rangle = \tilde{a}_i$ and $\langle\hat{b}_i\rangle = \tilde{b}_i$. The complex numbers a_i and b_i satisfy the conservation law $|\tilde{a}_1|^2 + |\tilde{a}_2|^2 = N/2$ and $|\tilde{b}_1|^2 + |\tilde{b}_2|^2 = N/2$ with N being the total particle number of species a and b . With the help of Heisenberg equation of motion for operators, one can easily get the dynamical equations for \tilde{a}_i and \tilde{b}_i . To simplify the calculation, one usually assumes $a_i = \tilde{a}_i/\sqrt{N}$ and $b_i = \tilde{b}_i/\sqrt{N}$. Then the aforementioned dynamical equations for \tilde{a}_i and \tilde{b}_i can be easily written as

$$i \frac{d}{dt} \begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix} = H_{\text{eff}} \begin{pmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{pmatrix} \quad (2)$$

with

$$H_{\text{eff}} = \begin{pmatrix} U_a|a_1|^2 + U_{ab}|b_1|^2 & -J_a & 0 & 0 \\ -J_a & U_a|a_2|^2 + U_{ab}|b_2|^2 & 0 & 0 \\ 0 & 0 & U_b|b_1|^2 + U_{ab}|a_1|^2 & -J_b \\ 0 & 0 & -J_b & U_b|b_2|^2 + U_{ab}|a_2|^2 \end{pmatrix}, \quad (3)$$

where $U_a = \tilde{U}_{aa}N$, $U_b = \tilde{U}_{bb}N$, and $U_{ab} = \tilde{U}_{ab}N$. For convenience in the following expression, we write the wavefunction $(a_1, a_2, b_1, b_2)^T$ as

$$|\psi\rangle = (a_1, a_2, b_1, b_2)^T = \sum_{\sigma,i} |\sigma\rangle \otimes |i\rangle, \quad (4)$$

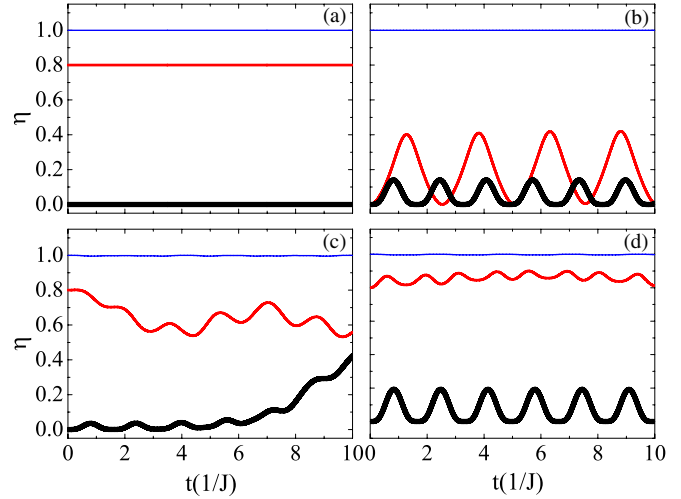


Figure 1. Time evolution of the degree of coherence for different initial states. The parameters are $U_a = U_b = U_{ab} = 3J$ (a), $U_a = U_b = 3J$ and $U_{ab} = 6J$ (b), $U_a = 3J$, $U_b = 2J$ and $U_{ab} = J$ (c), and $U_a = 3J$, $U_b = 2J$ and $U_{ab} = 6J$ (d).

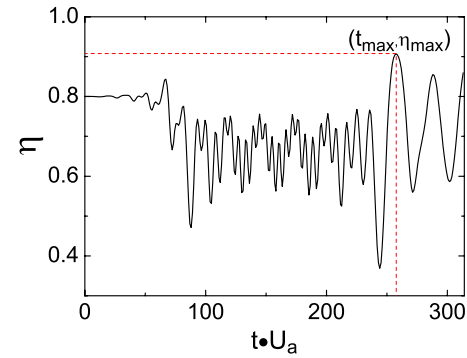


Figure 2. Time evolution of the degree of coherence for the Rosen–Zener form of inter-well tunnelling strength. The parameters are $U_b = \frac{2}{3}U_a$, $U_{ab} = \frac{1}{3}U_a$, $J_0 = \frac{1}{3}U_a$ and $\omega = 0.01U_a$.

where the state $|\sigma\rangle = |a\rangle$ or $|b\rangle$ specifies the two different species, while $|i\rangle = |1\rangle$ or $|2\rangle$ specifies the two wells. Note that the dynamical properties of the system can be determined by equation (2) if the system is in a completely coherent state (i.e. a pure state). Whereas, if the system is in a mixed state, the equation (2) becomes insufficient.

In order to study the coherence dynamics of the system, we introduce the single-particle density matrix $\hat{\rho} = |\psi\rangle\langle\psi|$ whose elements are $\rho_{\mu\nu}$ ($\mu, \nu = 1, 2, 3, 4$). From this definition, we can see that, as a 4×4 matrix, $\hat{\rho}$ describes a pure state. Their diagonal elements ρ_{11} (ρ_{22}) and ρ_{33} (ρ_{44}) represent the population of species a (b) in the first and second well, respectively. In this paper, we only focus on the distribution of the total particle numbers in the two wells but do not distinguish the particle species, so the system can be described by the reduced density matrix

$$\hat{\rho}^r = \sum_{\sigma} \langle\sigma|\hat{\rho}|\sigma\rangle, \quad (5)$$

whose elements are $\rho_{11}^r = \rho_{11} + \rho_{33}$, $\rho_{12}^r = \rho_{12} + \rho_{34}$, $\rho_{21}^r = \rho_{21} + \rho_{43}$, and $\rho_{22}^r = \rho_{22} + \rho_{44}$. Clearly, the matrix $\hat{\rho}^r$ can describe a mixed state. Its diagonal elements ρ_{11}^r

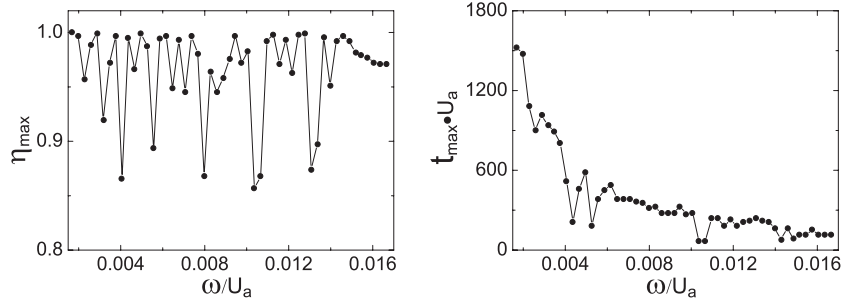


Figure 3. The dependence of the maximum value of the degree of coherence (left panel) and the corresponding time (right panel) on the period of the inter-well tunnelling strength. The parameters are $U_b = \frac{2}{3}U_a$, $U_{ab} = \frac{1}{3}U_a$ and $J_0 = \frac{1}{3}U_a$.

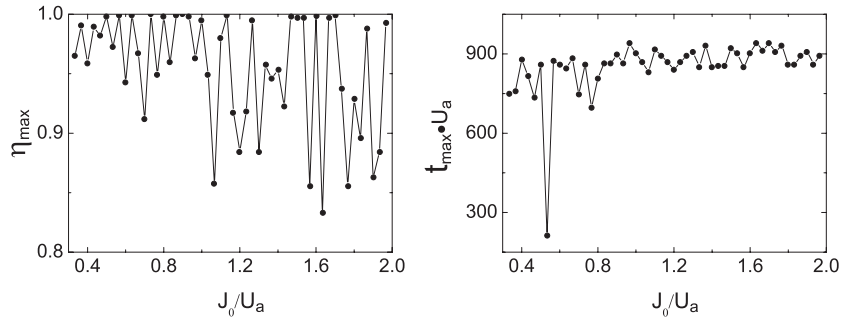


Figure 4. The dependence of the maximum value of the degree of coherence (left panel) and the corresponding time (right panel) on the maximum value of the inter-well tunnelling strength. The parameters are $U_b = \frac{2}{3}U_a$, $U_{ab} = \frac{1}{3}U_a$ and $J_0 = \frac{1}{3}U_a$.

and ρ_{22}^r represent the total population probability in the first and second well, respectively. Note that the reduced density matrix can be given by summing up the single-particle density matrices for two one-species BEC systems in double wells, i.e. $\hat{\rho}^r = \hat{\rho}^a + \hat{\rho}^b$, with

$$\hat{\rho}^a = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix}, \quad \hat{\rho}^b = \begin{pmatrix} \rho_{33} & \rho_{34} \\ \rho_{43} & \rho_{44} \end{pmatrix}. \quad (6)$$

Here, $\hat{\rho}^a$ refers to the single-particle density matrix for species a and $\hat{\rho}^b$ for species b . The degree of coherence for the one-species BEC system described by $\hat{\rho}^a$ or $\hat{\rho}^b$ does not change with time without the condensate–environment coupling, but that for the two-species BEC system described by $\hat{\rho}^r$ can obviously change with time due to the sum of $\hat{\rho}^a$ and $\hat{\rho}^b$. To investigate the coherence dynamics of the system, we can introduce the definition of the degree of coherence according to [4, 17],

$$\eta = 2\text{Tr}(\hat{\rho}^r)^2 - 1. \quad (7)$$

From equations (5) and (7), we can see that the time evolution of the degree of coherence depends on that of the elements $\rho_{\mu\nu}$ of the single-particle density matrix $\hat{\rho}$. Since $\hat{\rho}$ describes a pure state, the time evolution of $\rho_{\mu\nu}$ can be determined by equation (2). Thus one can solve equation (2) to get the evolution of a_i and b_i firstly, and then gives the time evolution of $\rho_{\mu\nu}$ according to the definition of $\hat{\rho} = |\psi\rangle\langle\psi|$. Then in the following sections, we will study the coherence dynamics of the system with the help of equation (2). We know from equation (7) that the degree of coherence is expected to be determined by measuring the particle distribution and the relative phases of species a and b between the two wells experimentally. Note that in the following calculations, we take $J_a = J_b = J$ for simplicity.

3. Evolution of the degree of coherence

We know that the degree of coherence does not change for an isolated BEC system [4]. To study the effect of decoherence of the BEC system, several authors considered the condensate–environment coupling [15, 18], whereas we will show that the degree of coherence of the two-species BEC system in double wells can still change with time even without the condensate–environment coupling. Here, we consider the case of time-independent inter-well tunnelling strength, i.e. J is a constant in the calculation. Due to the fact that equation (2) cannot be analytically solved, we solve equation (2) numerically to get the time evolution of a_i and b_i . Then we give the time evolution of the degree of coherence with the help of the definition of $\hat{\rho}$ and $\hat{\rho}^r$ and equation (7). The corresponding results are summarized in figure 1.

In figure 1, we plot the time evolution of the degree of coherence for different initial states and parameters. The initial states are $a_1 = \sqrt{1/2}$, $a_2 = 0$, $b_1 = \sqrt{1/2}$ and $b_2 = 0$ for the top (blue) line, $a_1 = \sqrt{2/5}$, $a_2 = \sqrt{1/10}$, $b_1 = \sqrt{1/2}$ and $b_2 = 0$ for the middle (red) line, and $a_1 = \sqrt{1/2}$, $a_2 = 0$, $b_1 = 0$ and $b_2 = \sqrt{1/2}$ for the bottom (black) line. Here, we choose the initial states randomly. We think one can prepare the initial states aforementioned according to [8] where the authors showed how to prepare the initial state with different population imbalance between the double wells in detail. Although the double-well system studied in [8] consists of one species, one can prepare the initial states for the two-species system by using the same method. Note that at the initial time, one needs to distinguish the two species. From figure 1(a), we can find that the degree of coherence of the two-species BEC system in double wells does not

change for any initial states when $U_a = U_b = U_{ab}$. That can be easily understood because this two-species system is equivalent to the one-species system once $U_a = U_b = U_{ab}$. And according to ([4]), the degree of coherence of the one-species BEC system in double wells does not change without the condensate–environment coupling. So the result shown in figure 1(a) is reasonable. Figure 1(b) shows that when $U_a = U_b \neq U_{ab}$, the degree of coherence does not change for the case of $a_i = b_i$ at the initial time, i.e. the initial population distributions of species a and b are the same, but that changes for the other initial states. Note that for the case of $U_a = U_b \neq U_{ab}$, the time evolutions of species a and b are the same if $a_i = b_i$ at the initial time. So the degree of coherence does not change due to the two species having the same symmetry. Additionally, from figures 1(c) and (d), we can find that the degree of coherence changes for any initial states for the case of $U_a \neq U_b \neq U_{ab}$. Comparing figures 1(c) and (d), we can see that for the same initial state, the variation tendency of the degree of coherence depends on the parameters of the system. In summary, whether the degree of coherence can change with time depends on both the parameters and the initial states. So does the variation tendency of the degree of coherence. This fact indicates that one can control the degree of coherence without introducing the effect of environment, which will be discussed in the following section.

4. Controlling the degree of coherence

In the previous section, we showed that the time evolution of the degree of coherence depends on the initial states and the parameters of the system, so one can change it by varying either the initial states or the parameters of the system. In the following section, we will show how to control the degree of coherence through a Rosen–Zener form of J [19, 20]:

$$J = J_0 \sin^2 \omega t \quad (0 \leq t \leq \pi/\omega), \quad (8)$$

i.e. J increases from zero to its maximum value J_0 and then decreases to zero again in the end of the calculation. In the numerical calculation in this section, we take the initial state $a_1 = \sqrt{2/5}$, $a_2 = \sqrt{1/10}$, $b_1 = \sqrt{1/2}$, and $b_2 = 0$.

The time evolution of the degree of coherence for the Rosen–Zener form of inter-well tunnelling strength is plotted in figure 2. We can see that the degree of coherence changes with time and can reach the maximum value η_{\max} at time $t = t_{\max}$. Note that the values of η_{\max} and $t = t_{\max}$ depend on both the period ω and the maximum value J_0 of the inter-well tunnelling strength. In figure 3, we plot the dependence of η_{\max} and t_{\max} on ω for a fixed J_0 in the left and right panel, respectively. Meanwhile, we plot the dependence of η_{\max} and t_{\max} on J_0 for a fixed ω in figure 4.

From figures 3 and 4, we can see that the values of ω and J_0 affect the maximum value of the degree of coherence sensitively. In order to get a system with a large value of the degree of coherence, one must choose the form of J with a suitable period ω and maximum value J_0 . Note that although one can get a system with a large value of the degree of coherence through tuning the values of ω and J_0 , it is difficult to

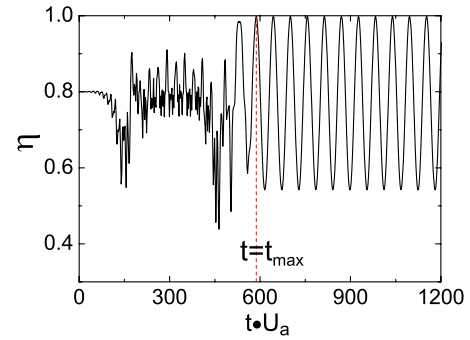


Figure 5. Time evolution of the degree of coherence for the inter-well tunnelling strength given in equation (9). The parameters are $U_b = \frac{2}{3}U_a$, $U_{ab} = \frac{1}{3}U_a$, $J_0 = \frac{1}{3}U_a$ and $\omega = 0.005U_a$.

control the time evolution of the degree of coherence due to the fact that the degree of coherence does not evolve periodically, which can be confirmed by figure 2. In order to overcome the aforementioned problem, we consider the following form of J :

$$J = \begin{cases} J_0 \sin^2 \omega t & (0 \leq t \leq t_{\max}) \\ 0 & (t > t_{\max}), \end{cases} \quad (9)$$

where t_{\max} can be obtained from figures 3 and 4.

For the form of J given in equation (9), the time evolution of the degree of coherence is plotted in figure 5. From this figure, we can find that the degree of coherence reaches its maximum value which is determined by the values of ω and J_0 , and then oscillates periodically at the following time. Since equation (2) can be analytically solved for the case of $J = 0$, we can give the oscillation period in the analytical method. Solving equation (2), we obtain $a_i(t) = a_i(t_{\max}) \exp(-iA_i t)$, and $b_i(t) = b_i(t_{\max}) \exp(-iB_i t)$ where $A_i = U_a |a_i(t_{\max})|^2 + U_{ab} |b_i(t_{\max})|^2$, and $B_i = U_b |b_i(t_{\max})|^2 + U_{ab} |a_i(t_{\max})|^2$. Substituting the expressions of a_i and b_i into equation (7), we can get the oscillation period of the degree of coherence $T = 2\pi/|A_2 - A_1 + B_1 - B_2|$. For the parameters taken in figure 5, we find $TU_a = 56.5$ which is consistent with the numerical result. From the above discussion, we know that the degree of coherence oscillates periodically after time t_{\max} , and both the maximum value of the degree of coherence and the oscillation period can be changed by tuning the values of ω and J_0 . Once the values of ω and J_0 are fixed, the time evolution of the degree of coherence is well defined, so that we can know the degree of coherence of the system at any time. Then one can easily get a system with any desired degree of coherence through controlling the evolution time.

5. Summary

In the above sections, we investigated the coherence dynamics for a system of two-species BEC in double wells. In the mean-field approximation, we studied the influence of the inter-well tunnelling strength on the coherence features with the help of the reduced single-particle density matrix. Since we do not need to distinguish particle species in the system, we only focused on the distribution of the total particle

numbers in the two wells, which can be described by a 2×2 reduced density matrix. After studying the time evolution of the degree of coherence for a time-independent inter-well tunnelling strength, we found that the degree of coherence of the two-species BEC system changes for some parameters and initial states even without the condensate–environment coupling, which differs from the case of [15, 18]. Motivated by the fact that the variation tendency of the degree of coherence depends on both the parameters and the initial states of the system, we considered a system with a Rosen–Zener form of inter-well tunnelling strength to control the degree of coherence. Although its tendency is not periodical, the degree of coherence can reach a maximum value η_{\max} at time t_{\max} which are dependent on the period ω and the maximum value J_0 of the inter-well tunnelling strength. The dependence of η_{\max} and t_{\max} on ω and J_0 that we obtained is helpful for one to get a system with a large value of the degree of coherence utilizing a Rosen–Zener form of inter-well tunnelling strength. We also gave a useful form of the inter-well tunnelling strength for one to easily get a system with any degree of coherence by controlling the evolution time.

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