Dynamics for partially coherent Bose-Einstein condensates in double wells

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The dynamical properties of partially coherent Bose-Einstein condensates in double wells are investigated in three typical regimes. In the extreme Fock regime, the time evolution of the degree of coherence is shown to decay rapidly. In the Rabi regime, a relation between the amplitude of Rabi oscillation and the degree of coherence is obtained, which is expected to determine the degree of coherence by measuring the amplitude of Rabi oscillation. The study on the self-trapping phenomena in the Josephson regime exhibits that both the degree of coherence and the initial relative phase can affect the final particle distribution.

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I. INTRODUCTION

Quantum tunneling through a barrier, as a paradigm of quantum mechanics, has been observed in different systems, such as two superconductors separated by a thin insulator [1], two reservoirs of superfluid helium connected by nanoscopic apertures [2,3], and Bose-Einstein condensates (BECs) trapped in double wells [4]. Comparing with the former two systems, BECs in double wells offer a versatile tool to study the quantum tunneling phenomena due to the fact that almost each parameter, such as the interwell tunneling strength, the interparticle interaction, and the energy bias between the two wells, can be tuned experimentally. The competition between those parameters makes BECs in double wells exhibit several fascinating phenomena, like Rabi oscillation, self-trapping, and Josephson oscillation, which have been extensively studied [5–8]. When a condensate is expected to be employed as a qubit, the first obstacle attempted to be avoided is the decoherence, which was studied in experiment through the interference between BECs [9,10]. Theoretically, the effect of decoherence on the dynamics of BECs in double wells was recently discussed with the help of single-particle density matrix [11]. The problem of decoherence in qubit measurement was recently investigated [12] with a BEC in double wells. In current literature, the most of authors paid more attention on the time evolution of the atom distribution but less attention on the relative phase between the condensates in two wells. Meanwhile, the systems were mostly assumed to be completely coherent at the initial time. Since both relative phase and degree of coherence are believed to affect the dynamical properties of BECs in double wells, it is worthwhile to study the coherence dynamics with attention to the relative phase and the degree of coherence.

In this paper, we study a BEC system in double wells with different degrees of coherence. In mean-field approximation, we study the dynamical properties of the system in different regimes with the help of the single-particle density matrix and show that, in comparison to the completely coherent case, partially coherent BECs in double wells can exhibit richer physics. In the next section, we model the partially coherent system and give the mean-field dynamical equation for the elements of single-particle density matrix. In Sec. III, we study the evolution of the degree of coherence and discuss the dynamical property of the partially coherent system in Fock regime. In Sec. IV, we study the system in Rabi regime and discuss the influence of degree of coherence on the Rabi oscillation. In Sec. V, we study self-trapping phenomenon for the partially coherent system. Then brief summary and discussion are given in Sec. VI.

II. MODELING PARTIALLY COHERENT SYSTEMS

We consider a Bose-Einstein condensate confined in a double-well potential, where atoms can tunnel between the two wells. The Hamiltonian of such a system in the second-quantization form is given by

$$\hat{H} = \frac{\gamma}{2}(\hat{n}_1 - \hat{n}_2) - T(\hat{a}_1^\dagger \hat{a}_2 + \hat{a}_2^\dagger \hat{a}_1) + \frac{U}{4}(\hat{n}_1 - \hat{n}_2)^2, \qquad (1)$$

where the bosonic operators $\hat{a}_\mu^\dagger$ and $\hat{a}_\mu$ ($\mu = 1,2$) create and annihilate an atom in the μth well, respectively; and $\hat{n}_\mu = \hat{a}_\mu^\dagger \hat{a}_\mu$ is the particle number operator. Here the parameter γ denotes the energy bias between the two wells, T is the interwell tunneling strength, and U is the interaction strength between atoms. The Hamiltonian (1) can describe not only the BEC in double wells but also that in two hyperfine states [13,14]. This model was conventionally studied in a mean-field approach by replacing the expectation values of annihilators in two different wells with two complex numbers a_1 and a_2, respectively [15,16]. In those works, the system is essentially assumed to be of complete coherence, i.e., the condensate stays in a completely coherent superposition state $|\psi_{\text{coh}}\rangle = \frac{1}{N!}(a_1\hat{a}_1^\dagger + a_2\hat{a}_2^\dagger)^N|\text{vac}\rangle$. Whereas, a realistic system of condensates may be not always in complete coherence for various situations. It is therefore worthwhile to study the dynamical properties of partially coherent BECs in double wells.

As we know, it is convenient to introduce the single-particle density matrix $\tilde{\rho}$ with entities $\tilde{\rho}_{\mu\nu}(t) = \langle \hat{a}_\mu^\dagger(t)\hat{a}_\nu(t)\rangle$, where the expectation value is taken for the initial state of the system. Clearly ρ_{11} and ρ_{22} represent the population in the first and in the second well, respectively. With the help of Heisenberg equation of motion for operators, one can derive the dynamical equations for the elements of the aforementioned density matrix by the mean-field approach in the semiclassical limit,

$$i\frac{d\rho_{11}}{dt} = -i\frac{d\rho_{22}}{dt} = -T(\rho_{12} - \rho_{21}),$$

$$i\frac{d\rho_{12}}{dt} = -\gamma\rho_{12} + T(\rho_{22}-\rho_{11}) - UN(\rho_{11}-\rho_{22})\rho_{12},$$

$$i\frac{d\rho_{21}}{dt} = \gamma\rho_{21} - T(\rho_{22}-\rho_{11}) + UN(\rho_{11}-\rho_{22})\rho_{21}, \tag{2}$$

where $\hbar$ is set to unit and $\rho_{\mu\nu}=\tilde{\rho}_{\mu\nu}/N$. The conservation of particle number requires that $\rho_{11}+\rho_{22}=1$.

As the 2×2 density matrix can be expanded as $\rho=(I+\vec{P}\cdot\vec{\sigma})/2$ with σ's the Pauli matrices, and $\vec{P}$ a vector inside the so-called Bloch sphere. Since $|\vec{P}|=1$ refers to a pure state (completely coherent superposition) while $|\vec{P}|<1$ refers to a mixed state (partial coherence), one can measure the degree of coherence by $|\vec{P}|^2\equiv\eta$. Thus a natural definition of the degree of coherence is given by [17]

$$\eta = 2\,\mathrm{Tr}\,\rho^2 - 1, \tag{3}$$

which is an important quantity that affects the dynamical features, such as Rabi oscillation, self-trapping, etc. With the help of the mean-field dynamical Eqs. (2) for the single-particle density matrix, the dynamics of the system can be investigated. We know that the ratio of the interaction strength U to the tunneling strength T determines three distinct regimes, namely, the Rabi regime, $UN/T\ll 1$, the Josephson regime, $1\ll UN/T\ll N^2$, and the Fock regime, $N^2\ll UN/T$. The system manifests different dynamical features in different regimes, which will be given in the following sections.

III. EVOLUTION OF THE DEGREE OF COHERENCE IN THE FOCK REGIME

To study the evolution of the degree of coherence, we can conveniently introduce the pseudospin operators,

$$\hat{J}_z = \frac{1}{2}(\hat{a}_1^\dagger\hat{a}_1 - \hat{a}_2^\dagger\hat{a}_2),$$

$$\hat{J}_x = \frac{1}{2}(\hat{a}_1^\dagger\hat{a}_2 + \hat{a}_2^\dagger\hat{a}_1),$$

$$\hat{J}_y = -\frac{i}{2}(\hat{a}_1^\dagger\hat{a}_2 - \hat{a}_2^\dagger\hat{a}_1),$$

which obey commutation relations for the angular momenta $[\hat{J}_j,\hat{J}_k]=i\epsilon_{jkl}\hat{J}_l$, and fulfill

$$\hat{\mathbf{J}}^2 = \hat{J}_x^2 + \hat{J}_y^2 + \hat{J}_z^2 = \frac{N}{2}\left(\frac{N}{2}+1\right),$$

for systems of N bosons. In terms of these pseudospin operators, the Hamiltonian (1) can be written as

$$\hat{H} = \gamma\hat{J}_z + U\hat{J}_z^2 - 2T\hat{J}_x. \tag{4}$$

This implies that the dynamical properties are determined only by the direction of the pseudospin $\mathbf{J}$ since its magnitude is fixed on $N/2$.

We know that the eigenvalue of $\hat{J}_z$ is $(n_1-n_2)/2$, which refers to the population imbalance between the two wells. For convenience, let us introduce the canonically conjugated operator $\hat{\varphi}$ of $\hat{J}_z$ to characterize the relative phase, which can be regarded as the angle of $\hat{\mathbf{J}}$ in the x-y plane in the angular-momentum picture. According to Ref. [17], the operator $\hat{\varphi}$ can be defined through $\hat{E}\equiv\exp i\hat{\varphi}$, where

$$\hat{E} = [(N/2-\hat{J}_z)(N/2+\hat{J}_z+1)]^{-1/2}(\hat{J}_x+i\hat{J}_y). \tag{5}$$

Such a definition satisfies $[\hat{J}_z,\hat{E}]=\exp i\hat{\varphi}$, which is consistent with the condition $[\hat{J}_z,\hat{\varphi}]=-i$, so that $\hat{J}_z$ and $\hat{\varphi}$ are canonically conjugated to each other. Then, in terms of $\hat{J}_z$ and $\hat{\varphi}$, Eq. (4) can be rewritten as

$$\hat{H} = U\hat{J}_z^2 + \gamma\hat{J}_z - TN\sqrt{1-\frac{4\hat{J}_z}{N^2}}\cos\hat{\varphi}. \tag{6}$$

In the Fock regime, the tunneling strength is much smaller than the interaction one between atoms, i.e., $U/(NT)\gg 1$, hence the last term in Eq. (6) can be neglected. Such a condition can be satisfied in experiment through increasing the distance between the two wells or enhancing the height of the potential barrier separating the two wells. Then the dynamical equations in this regime become

$$\frac{d\hat{J}_z}{dt}=0, \quad \frac{d\hat{\varphi}}{dt} = 2U\hat{J}_z + \gamma,$$

which means that the difference in particle numbers between the two wells is fixed but the relative phase $\hat{\varphi}=2U\hat{J}_z t+\gamma t$ evolves with time. According to Eq. (3) and the definition of pseudospin operators, the degree of coherence can be written as

$$\eta = \frac{4}{N^2}(\langle\hat{J}_z\rangle^2 + |\langle\hat{a}_1^\dagger\hat{a}_2\rangle|^2).$$

From the definition of $\hat{\varphi}$, one can also get

$$\langle\hat{a}_1^\dagger\hat{a}_2\rangle = \frac{N}{2}\left\langle\left(1-\frac{4}{N^2}\hat{J}_z^2\right)^{1/2}\exp i\hat{\varphi}\right\rangle.$$

Since the expectation value of $\hat{J}_z$ is fixed in the extreme Fock limit $T=0$, the evolution of the degree of coherence only depends on that of $\hat{\varphi}$. For example, if the particle numbers in the two wells are the same at the initial time, the expectation value of $\hat{J}_z$ will be always fixed on zero in the extreme Fock limit. In this case, the degree of coherence becomes $\eta=4|\langle\hat{a}_1^\dagger\hat{a}_2\rangle|^2/N^2\approx|\langle\exp i\hat{\varphi}\rangle|^2$, which reflects that the evolution of the degree of coherence is determined by the interaction strength U, the detuning γ, and the initial state of the system.

As we know, the Fock bases $|l\rangle$ composing the $(N+1)$-dimensional Hilbert space of the system are

$$|l\rangle = \left|\frac{N}{2}+l,\frac{N}{2}-l\right\rangle, \quad l=-\frac{N}{2},-\frac{N}{2}+1,\ldots,\frac{N}{2}, \tag{7}$$

where l corresponds to the quantum number of $\hat{J}_z$ denoting the half of the difference in the particle numbers between the two wells. Since any initial state can be written as

$$|\Psi(0)\rangle = \sum_{l=-N/2}^{l=N/2} \psi_l |l\rangle,$$

the degree of coherence can be obtained as long as ψ_l is given. Taking $\psi_l = e^{-(l-\delta)^2/l_0^2}/(\pi l_0^2/2)^{1/4}$ as an example and replacing the sum over l by an integral, one can obtain the degree of coherence. If keeping the lowest-order term in the Taylor expansion in the calculation of the expectation value of $(1-4/N^2\hat{J}_z^2)^{1/2}\exp i\hat{\varphi}$, we have

$$\eta(t) \simeq \frac{4}{N^2}\delta^2 + \exp(-l_0^2U^2t^2). \quad (8)$$

This expression is valid when the width of the Gaussian distribution is much smaller than N, i.e., $l_0 \ll N$. Clearly, the degree of coherence decays with time as long as there is interaction between atoms. Such result tells us that one can prepare systems with different degrees of coherence through changing the evolution time t in the extreme Fock regime. From Eq. (8), we can also find that the larger the l_0 is, the faster the degree of coherence will decay. Then one can suppress the decay through decreasing the value of l_0. Note that in the above calculation of $\eta(t)$, we used $\hat{\varphi} = 2U\hat{J}_z t + \gamma t$, which is valid only for the extreme Fock regime. In the other regimes, one needs to solve Eq. (2) either analytically or numerically without any assumption.

IV. INFLUENCE OF DEGREE OF COHERENCE ON THE RABI OSCILLATION

Rabi oscillation is an important phenomenon reflecting the coherent property of a system, which has been discussed recently [8,18,19]. In Ref. [8] the initial state was assumed to be a pure state, i.e., $\rho_{11}=1$, implying $\eta=1$. Whereas, according to our formulation in Sec. III, one can prepare states with different degrees of coherence, which makes Rabi oscillation worthwhile to be studied from a new angle of view. Now we study the dynamics of Rabi oscillation in terms of the density matrix and give the relation between the amplitude of the Rabi oscillation and the degree of coherence for BECs in double wells.

In the extreme Rabi limit, $U=0$, Eqs. (2) are reduced to Bloch equations that can be solved analytically. For example, we consider a initial state described by the density matrix $\rho_{11}(0)=\rho_{22}(0)=1/2$ and $\rho_{12}(0)=\rho_{21}(0)=c$, we can obtain the solution of Eqs. (2),

$$\rho_{11} = \frac{1}{2} - \frac{2c\gamma T}{\Omega^2} + \frac{2c\gamma T}{\Omega^2}\cos(\Omega t),$$

$$\rho_{22} = \frac{1}{2} + \frac{2c\gamma T}{\Omega^2} - \frac{2c\gamma T}{\Omega^2}\cos(\Omega t),$$

$$\rho_{12} = c - \frac{c\gamma^2}{\Omega^2} + \frac{c\gamma^2}{\Omega^2}\cos(\Omega t) + i\frac{c\gamma}{\Omega}\sin(\Omega t),$$

$$\rho_{21} = c - \frac{c\gamma^2}{\Omega^2} + \frac{c\gamma^2}{\Omega^2}\cos(\Omega t) - i\frac{c\gamma}{\Omega}\sin(\Omega t), \quad (9)$$

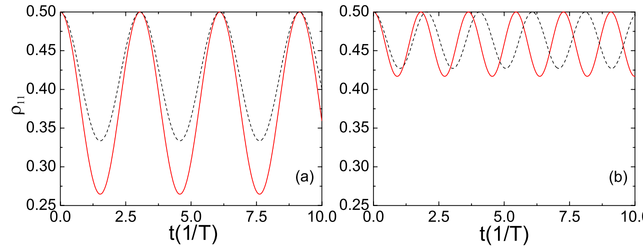

FIG. 1. (Color online) The Rabi oscillation of the particle distribution with different degrees of coherence for (a) $U/T=0$ and for (b) $UN/T=4$. The parameters are $\gamma/T=0.5$ and $\eta=0.5$ (dot line), 1 (solid line).

where $\Omega=(4T^2+\gamma^2)^{1/2}$ and c is a real number related to the degree of coherence, i.e., $c=\sqrt{\eta}/2$. From Fig. 1, we can see that the particle numbers in the two wells both periodically oscillate with time. The oscillation amplitude $c\gamma T/\Omega^2$ diminishes with the decrease in the degree of coherence. Here the relative phase between the condensates in two wells is taken as zero, i.e., $\rho_{12}(0)$ is real at the initial time. If the degree of coherence is fixed but the relative phase is not zero, saying $\rho_{12}(0)=c\exp(i\phi)$ and $\rho_{21}(0)=c\exp(-i\phi)$, we can also solve Eqs. (2) analytically [the expression of $\rho_{ij}(t)$ is omitted for saving space]. One can find that the amplitude of Rabi oscillation is affected by the initial values of the degree of coherence as well as the relative phase. For the sake of comparison, we plot a numerical result of the Rabi oscillation in the presence of interaction ($U\neq 0$) in Fig. 1(b). Comparing the two panels in Fig. 1, we can see that the interaction between atoms will make both the amplitude and the period of the Rabi oscillation decrease for the same initial states.

The above discussions tell us that one can obtain the degree of coherence through measuring the amplitude of the Rabi oscillation in experiment. For example, like in the experiment [20], prepare a BEC in one hyperfine state and transfer one half of atoms into the other hyperfine state through a two-photon pulse; turn off the pulse to allow the system to evolve freely without the interstate tunneling (i.e., $T=0$) until $t=\tau_0$. Then turn on a pulse which makes the atoms tunnel between the two states and measure the amplitude of the Rabi oscillation. Because the system is in the extreme Fock regime when $t<\tau_0$, based on the calculation given in Sec. III, we can have $\rho_{11}=\rho_{22}=1/2$ and $\rho_{12}=\frac{1}{2}\exp(i\gamma\tau_0)\exp(-l_0^2U^2\tau_0^2/2)$ at the time τ_0. Such a state can be actually prepared as the initial state of the subsequent Rabi oscillation procedure. Note that the expression of ρ_{12} can be also rewritten as $\rho_{12}=\sqrt{\eta}\exp(i\gamma\tau_0)/2$ considering the definition of η. Since the relative phase $\gamma\tau_0$ is determined, the amplitude of Rabi oscillation only depends on the degree of coherence. The analytical result of Eqs. (2) cannot be obtained due to the existence of interaction terms in the above example, so one can solve it numerically. The degree of coherence can be determined through fitting the experimental Rabi oscillation profiles with the theoretical ones as the degree of coherence is evolved.

V. SELF-TRAPPING FOR THE PARTIALLY COHERENT SYSTEM

As we know, the most prominent feature of atomic tunneling between two wells is the nonlinear dynamics arising

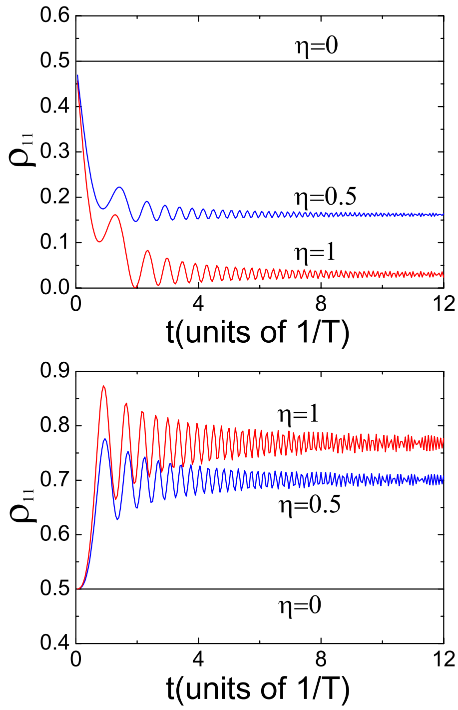

FIG. 2. (Color online) Time evolution of the population probability ρ_{11} for different degrees of coherence and initial relative phases ($\phi=\pi/3$ for the top panel and $\phi=\pi$ for the bottom panel). The other parameters are $UN/T=4$ and $\alpha/T^2=5$.

from the interaction of atoms. Whereas, Eqs. (2) cannot be analytically solved once the interaction terms are taken into account. In this case, we solve Eqs. (2) numerically. In the numerical calculation, we adopt a linearly time-dependent energy bias $\gamma=\alpha t$, where α is a constant characterizing the rate of the change of the energy bias γ. The initial values are $\rho_{11}=\rho_{22}=1/2$ and $\rho_{12}=\frac{1}{2}\sqrt{\eta}\exp i\phi$, where ϕ is the phase difference between the condensates in the two wells at initial time. Our results manifest that the initial relative phase and the degree of coherence affect the dynamical properties of self-trapping.

In Fig. 2, we plot the time evolution of the population ρ_{11} for different initial values of the degree of coherence and relative phase. From this figure, we can see that the system exhibits the phenomenon of self-trapping if the system is coherent, i.e., $\eta\neq 0$, but the atoms do not tunnel between the two wells for the incoherent system, i.e., $\eta=0$. We can also find that most of the atoms favorite to stay in the right well (i.e., $\rho_{22}>\rho_{11}$) at the final time for $\phi=\pi/3$, which is contrast to the case for $\phi=\pi$. The difference between the two panels of Fig. 2 implies that the initial relative phase can affect the phenomenon of self-trapping. It depends on the energy bias and the initial relative phase that in which well the atoms prefer to stay at the final time. When the initial relative phase is fixed, Fig. 2 shows that the larger the degree of coherence is, the larger the population difference between the two wells at the final time will be. This is confirmed by Fig. 3 where the final population probability versus the degree of coherence is plotted for three different values of ϕ. In Fig. 4, we plot the relation between the final population probability and the initial relative phase for different degrees of coherence.

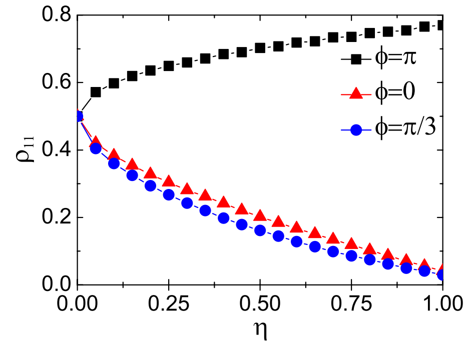

FIG. 3. (Color online) The dependence of the final population probability on the degree of coherence. The parameters are the same as in Fig. 2.

This figure confirms that the initial relative phase can affect the final distribution of atoms.

The emergence of the self-tapping phenomena is dependent on the initial state and the systems parameters, e.g., the interaction strength U, the interwell tunneling strength, and the energy bias γ between two wells. Such a phenomenon was investigated in a symmetric double-well potential [5], i.e., $\gamma=0$, and also in a double-well potential with a periodic modulation, i.e., $\gamma\propto\sin\omega t$ [21]. According to Ref. [5], we can find that the self-trapping phenomena occur only when the interaction strength is larger than the critical value $U_c \propto[\sqrt{1-z(0)^2}\cos\phi(0)+1]T/[Nz(0)^2]$ in the case of $\gamma=0$, where $z(0)$ refers to the initial population difference $\rho_{11}(0)-\rho_{22}(0)$ and $\phi(0)$ the initial relative phase between the condensates in the two wells. Therefore, for the initial states we considered afore, if there is no energy bias, the system cannot exhibit the self-trapping phenomena due to the critical interaction strength $U_c\to\infty$ for $z(0)=0$, $\phi(0)\neq\pi$ [22]. For the initial states with population imbalance (i.e., $z(0)\neq 0$), we plot the numerical solutions of Eqs. (2) for a symmetric double-well potential with different degrees of coherence in Fig. 5. Such a figure shows that the degree of coherence affects the critical interaction strength U_c above which the system can exhibit the self-trapping phenomena in a symmetric double-well potential. The system investigated in Ref. [5] corresponds to the case when the degree of coherence $\eta=1$ in our discussion (see the dot line in Fig. 5).

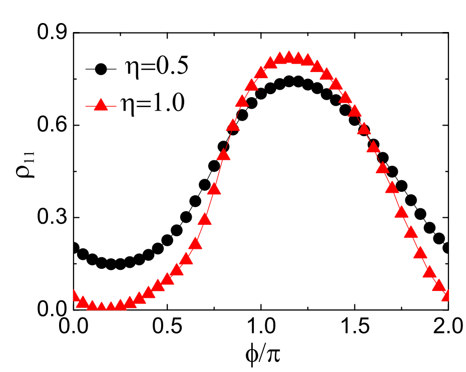

FIG. 4. (Color online) The dependence of the final population probability on the initial relative phase ϕ. The parameters are the same as in Fig. 2.

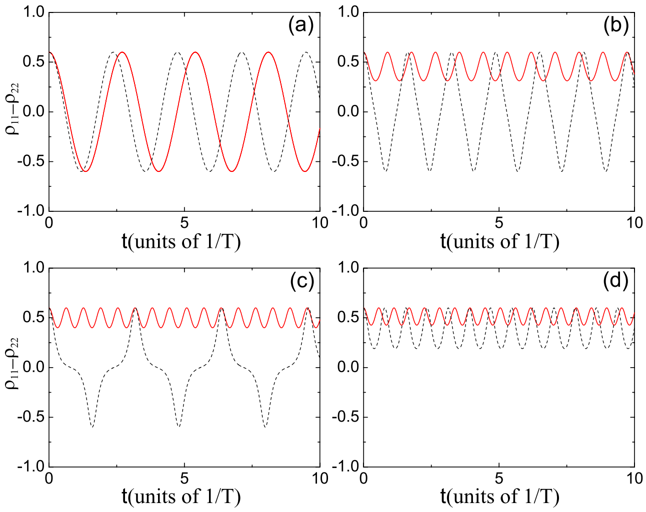

FIG. 5. (Color online) Time evolution of population imbalance for a symmetric double-well potential with different degrees of coherence. The initial states are $\rho_{11}=0.8$, $\rho_{22}=0.2$, and $\rho_{12}=0.4$ (dot line), $\rho_{12}=0.2$ (solid line). The parameters are (a) $UN/2T=1$, (b) $UN/2T=8$, (c) $UN/2T=9.99$, and (d) $UN/2T=11$.

VI. SUMMARY AND DISCUSSION

In the above, we considered Bose-Einstein condensates in double wells with different degrees of coherence. With the help of single-particle density matrix, we studied the dynamical properties of partially coherent Bose-Einstein condensates in double wells and showed that the degree of coherence is a useful parameter that affects the dynamical features. We investigated the system in different regimes and found that the degree of coherence can affect the dynamical properties of the system significantly. In the Fock regime, we mainly studied the time evolution of the degree of coherence by introducing the pseudospin operators and showed that the degree of coherence decays in exponential form of the square of time. In the Rabi regime, we studied the effects of the initial relative phase ϕ and degree of coherence η on Rabi oscillation and showed that the amplitude of Rabi oscillation is in proportional to the square root of η in the case of $\phi=0$. According to the relevant result, the degree of coherence is expected to be determined through measuring the amplitude of Rabi oscillation. Because the existence of nonlinear terms of interaction makes the dynamical equations in the Josephson regime not solvable analytically, we solved those equations numerically and found that the self-trapping phenomenon also exists for partially coherent BEC systems in double wells. A more fascinating feature is that which well the particles will stay in at the final time largely depends on the initial relative phase. This result is different from that in the previous works [11], where the particles are in the same well at the initial time such that the initial relative phase between BECs in two wells does not affect the dynamical property of systems explicitly.

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[1] K. K. Likharev, Rev. Mod. Phys. **51**, 101 (1979).
[2] S. V. Pereverzev, A. Loshak, S. Backhaus, J. C. Davis, and R. E. Packard, Nature (London) **388**, 449 (1997).
[3] A. Sukhatme, Y. Mukharsky, T. Chui, and D. Pearson, Nature (London) **411**, 280 (2001).
[4] M. Albiez, R. Gati, J. Fölling, S. Hunsmann, M. Cristiani, and M. K. Oberthaler, Phys. Rev. Lett. **95**, 010402 (2005).
[5] A. Smerzi, S. Fantoni, S. Giovanazzi, and S. R. Shenoy, Phys. Rev. Lett. **79**, 4950 (1997).
[6] G. J. Milburn, J. Corney, E. M. Wright, and D. F. Walls, Phys. Rev. A **55**, 4318 (1997).
[7] S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, Phys. Rev. A **59**, 620 (1999).
[8] J. M. Choi, G. N. Kim, and D. Cho, Phys. Rev. A **77**, 010501(R) (2008).
[9] T. Schumm, S. Hofferberth, L. M. Andersson, S. Wildermuth, S. Groth, I. Bar-Joseph, J. Schmiedmayer, and P. Krüger, Nat. Phys. **1**, 57 (2005).
[10] S. Hofferberth, I. Lesanovsky, B. Fischer, T. Schumm, and J. Schmiedmayer, Nature (London) **449**, 324 (2007).
[11] W. Wang, L. B. Fu, and X. X. Yi, Phys. Rev. A **75**, 045601 (2007).
[12] D. Sokolovski and S. A. Gurvitz, Phys. Rev. A **79**, 032106 (2009).
[13] M. R. Matthews, B. P. Anderson, P. C. Haljan, D. S. Hall, M. J. Holland, J. E. Williams, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **83**, 3358 (1999).
[14] Y. Q. Li, S. J. Gu, Z. J. Ying, and U. Eckern, Europhys. Lett. **61**, 368 (2003).
[15] B. Wu and J. Liu, Phys. Rev. Lett. **96**, 020405 (2006).
[16] C. Lee, Phys. Rev. Lett. **102**, 070401 (2009).
[17] A. J. Leggett, Rev. Mod. Phys. **73**, 307 (2001).
[18] B. R. da Cunha and M. C. de Oliveira, Phys. Rev. A **75**, 063615 (2007).
[19] N. Syassen, D. M. Bauer, M. Lettner, D. Dietze, T. Volz, S. Dürr, and G. Rempe, Phys. Rev. Lett. **99**, 033201 (2007).
[20] D. S. Hall, M. R. Matthews, C. E. Wieman, and E. A. Cornell, Phys. Rev. Lett. **81**, 1543 (1998).
[21] G. F. Wang, L. B. Fu, and J. Liu, Phys. Rev. A **73**, 013619 (2006).
[22] For the initial state of $z(0)=0$ and $\phi(0)=\pi$, if there is no the energy bias, the system will always stay at the initial state, which implies that the system can not exhibit the self-trapping phenomena.