Phase separation in atom-molecule mixtures near a Feshbach resonance

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We study the phase separation of a Bose-Einstein condensate system with the conversion of one-species bosonic atoms into molecules via Feshbach resonance. Adopting the Thomas-Fermi approximation, we plot the phase diagrams of the system in two different parameter spaces for different atom-molecule interactions. In the presence of an isotropic trap, the density profiles in spatial space with fixed total number of atoms are also plotted. The clear phase separation in spatial space enables one to determine the location of atoms in the trap, which may be used as a possible experimental probe method.

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I. INTRODUCTION

There have been many studies on multicomponent Bose-Einstein condensates (BECs) without atom-to-molecule conversion. Such systems possess many interesting features, e.g., the well-known phase separation allowing different spatial regions to favor different phases [1–4]. This phenomenon originates from the interplay between intraspecies or interspecies interactions and the spatial variation in the trapping potential. The technique of Feshbach resonance (FR) [5], originally introduced to manipulate the interactions in atomic clouds by tuning the magnetic field across the resonance point [6], enables us to observe and control phase separations experimentally, which is otherwise impossible for traditional condensed-matter systems. Now FR has been widely used to convert fermionic or bosonic atoms into homonuclear [7,8] or heteronuclear [9,10] molecules. For homonuclear cases these systems exhibit both interesting dynamic and static properties. The so-called “superchemistry,” in which the giant collective oscillations between the atomic and molecular condensates have been studied in Ref. [11], and the stability of such oscillation were addressed later [12]. In Refs. [13,14], the authors studied the conversion dynamics of the many-body system in the classical approach. Additionally, the static properties of the system are also fascinating. The different symmetries between the atomic and molecular condensates induce a quantum phase transition characterized by an Ising order parameter, which has been already studied [15–17]. Recently, the thermal and dynamic stabilities of each phase were explored [18]. For the heteronuclear systems the atomic population imbalance introduces more interesting physics [19] while the phase diagrams become much richer [20].

Although the phase separation was investigated in the bare atomic BECs, the atom-to-molecule conversion makes such a phenomenon present more fascinating features which are worthwhile to be studied. However, here we focus on the phase separation in spatial space of the mixed system consisting of bosonic atoms and homonuclear molecules trapped in an isotropic harmonic potential. We find that whether the phase separation exists depends on the system parameters (such as the atom-molecule interaction or the molecular binding energy). Additionally, one can change the location of the atomic condensate in the trap through sweeping the magnetic field near the Feshbach resonance to tune the system parameters.

This paper is organized as follows. In Sec. II, we present the model for atom-molecule mixtures and derive the coupled mean-field Gross-Pitaevskii-like equations for each fraction by minimizing the free energy of the system. For simplicity, we first discuss the case of $U_a U_m=U_{am}$ in Sec. III where the corresponding phase diagrams in the parameter spaces of effective chemical potentials or molecular binding energy are given. Considering the case with fixed total number of atoms, we also plot the density profiles of each fraction in spatial space. In Sec. IV, we extend our analysis to the $U_a U_m\neq U_{am}$ case and discuss the effects of $U_{am}$. The summary with brief discussions is given in Sec. V.

II. MODEL AND GENERAL FORMULATION

We consider converting the same species of atoms into homonuclear molecules via Feshbach resonance. Near the resonance point, the Hamiltonian of our system can be modeled as

$$H = \int (H_a + H_m + H_{am})d^3r,$$

in which $H_a$ and $H_m$ denote the pure atomic and molecular contributions, and $H_{am}$ refers to the coupling between them, namely,

$$H_a = -\frac{\hbar^2}{2m_a} \nabla \psi_a^\dagger \nabla \psi_a + V_a \psi_a^\dagger \psi_a + \frac{U_a}{2} \psi_a^\dagger \psi_a^\dagger \psi_a \psi_a,$$

$$H_m = -\frac{\hbar^2}{2m_m} \nabla \psi_m^\dagger \nabla \psi_m + (V_m + \epsilon) \psi_m^\dagger \psi_m + \frac{U_m}{2} \psi_m^\dagger \psi_m^\dagger \psi_m \psi_m,$$

$$H_{am} = \frac{\alpha}{2} (\psi_a \psi_m^\dagger \psi_a \psi_m^\dagger + \text{H.c.}) + U_{am} \psi_a \psi_a^\dagger \psi_m \psi_m^\dagger.$$

Here field operators $\psi_a(r)$ and $\psi_m(r)$ annihilate atoms and molecules at position $r$ (which is omitted in the above equations for simplicity), respectively. $V_i=m_i\omega_i^2r^2/2 \ (i=a,m)$ are the isotropic harmonic trap potentials for atoms and molecules with $m_a$ and $m_m$ being the atomic and molecular masses, and $\omega_a$ and $\omega_m$ are the corresponding characteristic
frequencies. In our present work we consider \( m_a = 2m_r = 2m \) and \( \omega_a = \omega_m = \omega \). \( U_a \) and \( U_m \) are the atomic and molecular interactions while \( U_{am} \) the atom-molecule interaction. We assume that \( U_a \) and \( U_m \) are both positive here while \( U_{am} \) can be either positive or negative. Parameter \( \varepsilon \leq 0 \) is the binding energy of a molecule, and \( \alpha \) represents the Feshbach coupling strength between atoms and molecules.

In the mean-field approximation, the field operators \( \psi_a(r) \) and \( \psi_m(r) \) are replaced by c-numbered order-parameter fields, i.e., \( \psi_a = \phi_a e^{i\theta_a}, \psi_m = \phi_m e^{i\theta_m} \). Here \( \theta_a \) and \( \theta_m \) (real valued) denote the phases of each species, while the squares of \( \phi_a \) and \( \phi_m \) represent the atomic and molecular densities, respectively. Since a minus sign can be retracted in the phase variable \( \theta \)'s, we can always regard both real \( \phi_a \) and \( \phi_m \) to be non-negative. Note that all these four quantities are spatially dependent.

Owing to the conservation of the total number of atoms, i.e., \( \int (\phi_a^2 + \phi_m^2) d^3r = N \), the free energy of our system is given by

\[
K = \int \left[ (H_a + H_m + H_{am}) d^3r - \mu N \right]
\]

\[
= \left[ -\mu_a \phi_a^2 - \mu_m \phi_m^2 + \frac{1}{2} U_a \phi_a^4 + \frac{1}{2} U_m \phi_m^4 + U_{am} \phi_a \phi_m^3 + \alpha \phi_m^2 \phi_a \cos \theta \right] d^3r,
\]

where \( \theta = \theta_a - 2\theta_m \) is the phase difference, and \( \mu_a(r) = \mu - m_o \omega^2 r^2/2 \) and \( \mu_m(r) = 2\mu - m_o \omega^2 r^2 - \varepsilon \) denote two effective local chemical potentials. Clearly, it is the phase difference \( \theta \) between the atomic and molecular condensates rather than their individual values that plays a role in the energy of the system and the value of \( \mu \) that tunes the total number of atoms (including those bounded into molecules). Note that in deriving the free energy above, we have adopted the Thomas-Fermi approximation (TFA) [21] that neglects all terms with \( \nabla \) in Eq. (1).

To determine the ground state of the system, we minimize the free energy Eq. (2) with respect to \( \phi_a \), \( \phi_m \), and \( \theta \). Here \( \delta K / \delta \phi = \sin \theta = 0 \) simply solves that \( \theta = 0 \) or \( \pi \). Moreover the other two coupled equations for \( \phi_a \) and \( \phi_m \) are

\[
\frac{\delta K}{\delta \phi_a} = 2\phi_a(\mu_a + U_a \phi_a^2 + U_{am} \phi_m^3 + \alpha \eta \phi_m^2) = 0,
\]

\[
\frac{\delta K}{\delta \phi_m} = 2\phi_m(\mu_m + U_{am} \phi_a^3 + U_m \phi_m^3 + \alpha \eta \phi_a^2) = 0,
\]

where \( \eta = \pm 1 \). For nonvanishing Feshbach coupling \( \alpha \neq 0 \), there are three classes of solutions for the above equations [15,16]: (i) the vacuum state (V) corresponding to the trivial solution \( \phi_a = \phi_m = 0 \), (ii) the pure molecular superfluid state (MSF), \( \phi_a = 0 \) but \( \phi_m = \sqrt{\mu_m / U_m} \), and (iii) the mixed atomic-molecular superfluid state (AMSF), \( \phi_a \neq 0 \) and \( \phi_m \neq 0 \) (which was denoted as "ASF" in Refs. [15,16]).

Due to the asymmetric form of Feshbach coupling term, atomic condensation will inevitably lead to the formation of molecular condensates; however, the reverse procedure is forbidden. This is different from the heteronuclear molecule conversion case [20] where the pure atomic condensate is possible. We need to emphasize that the MSF phase can exist only when \( U_m \neq 0 \).

We now focus on the AMSF solutions, i.e., \( \phi_a \neq 0 \) and \( \phi_m \neq 0 \), then Eq. (3) gives rise to

\[
P \frac{\delta^3 K}{\delta \phi_a \delta \phi_m \delta \phi_a} + Q \frac{\delta^3 K}{\delta \phi_m \delta \phi_m \delta \phi_m} + R \frac{\delta \phi_m}{\delta \phi_m} + S = 0,
\]

\[
\phi_a^2 = \mu_a - U_{am} \phi_m^3 - \alpha \phi_m,
\]

where we denoted \( \phi_m = \eta \phi_m \) and

\[P = 2U_a U_m - 2U_{am}^2,\]

\[Q = -3U_{am}\alpha,\]

\[R = 2\mu_a U_{am} - 2\mu_m U_a - \alpha^2,\]

\[S = \alpha \mu_a.\]

\( P \) is called the net interaction here. Note that the newly defined quantity \( \phi_m \) could be either positive or negative, which is unlike \( \phi_a \). However, its square plays the same role as \( \phi_m \) does. The thermodynamical stability of each solution is guaranteed only if the solution itself corresponds to the local minimum of free energy. The Hessian matrix, defined as \( M_{ij} = \delta^2 K / \delta \phi_i \delta \phi_j \) (\( i, j = a, m \)), can be introduced to identify the local minimum of \( K \) if the following conditions are satisfied:

\[M_{ii} \geq 0, \quad \text{det}(M_{ij}) \geq 0.\]

The stability analysis will help us determine the phase diagrams in Secs. III and IV.

In order to make quantities dimensionless [12], hereafter, we measure the spatial coordinates and the density in units of \( (h/m_o)^{1/2} \) and \( (h/m_o)^{3/2} \); the energy, the interaction, and the Feshbach coupling parameter \( \alpha \) are in units of \( h_o, (h_o)(h/m_o)^{1/2}, \) and \( (h_o)(h/m_o)^{3/4}, \) respectively. To specify the model parameters, we choose \( \omega^2/2\pi = 100 \) Hz and consider the case of \(^8\text{Rb}\) atoms whose realistic values of parameters can be obtained from experimental data directly. In this case, the Feshbach coupling parameter \( \alpha \) is about 1.08 [11] and the atomic interaction strength \( U_a \) is about 0.1. Additionally, the molecular interaction strength \( U_m \) is regarded to be smaller than \( U_a \). We also choose \( U_{am} = 0.36U_a \) in the present paper, for which we have verified that the finite value of \( U_m \) (only if \( U_m < U_a \)) will not make any qualitative difference. Both \( U_a \) and \( U_m \) are kept as constants throughout this paper. Strictly speaking, the concrete magnitudes of those quantities depend on the value of the applied magnetic field in experiments. A renormalization scheme was derived [22] to build up the connections. It is worthwhile to emphasize that our simplification does not affect the validity of the feature exhibited.

III. GROUND STATE FOR ZERO NET INTERACTION

For simplicity, we first consider the case \( U_a U_m = U_{am}^2 \) in this section and then discuss the \( U_a U_m \neq U_{am}^2 \) case in Sec. IV.
For the former case, we have $P=0$ in Eq. (4), which makes the picture easily captured. The phase diagrams and density profiles in spatial space are given successively in the following.

A. Phase diagram

The mean-field phase diagram as a function of $\mu_a$ and $\mu_m$ can be mapped out by solving Eq. (4). Since the first one in Eq. (4) becomes a second-order algebraic equation when $P=0$, the explicit solutions exhibit simple analytical expressions,

$$
\delta_{m,z} = \frac{R \pm \sqrt{R^2 - 12U_{am}\mu_a\alpha^2}}{6U_{am}\alpha},
$$

and

$$
\delta_{a,z}^2 = \mu_a - \frac{U_{am}\delta_{m,z}^2 - \alpha\delta_{m,z}}{U_{am}}.
$$

The definitions of $\mu_a$ and $\mu_m$ tell us that the two effective chemical potentials can be regarded as independent parameters. At any point of the parameter space $(\mu_a, \mu_m)$, there exist four possible solutions at most, i.e., two possible AMSF solutions we obtained in Eq. (7), one MSF and one V solution. Selecting the solution with the lowest free energy, we can easily plot the phase diagram of the system. For $U_{am} = \sqrt{U_a U_m}$, as shown in Fig. 1(a), the MSF state only exists when $\mu_m > 0$ which is clear from the second one in Eq. (3). However, when $\mu_a$ increases, the AMSF state acquires lower energy, indicating the appearance of the atomic BEC fraction. When $\mu_m < 0$, the positive $\mu_a$ favors the AMSF state too. Note that the AMSF state also exists in a region with slight negative values of $\mu_a$ and $\mu_m$. The fact is guaranteed by the nonzero Feshbach coupling strength $\alpha$, which is distinguished from the pure two-species BEC mixtures without conversion. The $U_{am} = \sqrt{U_a U_m}$ case shown in Figs. 1(c) and 1(d) can be analyzed in the same way.

Equivalently, $\epsilon$ and $\mu_a$ can also be viewed as a pair of independent quantities due to the relation $\mu_m(r) = \epsilon = 2\mu_a(r)$. In order to identify the role of the binding energy $\epsilon$, we can also map the phase diagram into the $(\epsilon, \mu_a)$ space. Since the positive $\epsilon$ usually involves the dressed Feshbach molecules which cannot be simply described by Eq. (1), $\epsilon \neq 0$ is chosen here. From Fig. 1(b) for $U_{am} = \sqrt{U_a U_m}$ we can draw several basic conclusions. (i) There exists a tricritical value $\epsilon_1$ below which the MSF region emerges in between the V and AMSF regions. (ii) When $\epsilon > \epsilon_2$, the phase transition between AMSF and V (or MSF) is discontinuous. (iii) As $\epsilon$ decreases from zero, the width of the AMSF region in $\mu_a$ direction shrinks, which is related to the reduction in the atomic BEC contribution. When $\epsilon$ decreases further to be lower than a critical value $\epsilon_3$, the contribution of the atomic BEC vanishes, leaving a pure molecular condensate which is energetically more favorable. As shown in Fig. 1(d), the case for $U_{am} = \sqrt{U_a U_m}$ is much simpler. For any values of $\epsilon$, as
Note that in determining the phase diagrams in Fig. 1, we also have identified the stability of each phase by checking condition (6). It shows that they are all thermodynamically stable, except the MSF phase colored in light blue (shaded) in Fig. 1(c). However, we need to emphasize that the choice $\epsilon = 0$ here excludes the physical meaning of this unstable phase. That is why there is no unstable phase shown in Fig. 1(d).

The discontinuous transition may also happen between the AMSF and MSF phases for $U_{am} = \sqrt{U_a U_m}$ when $\epsilon$ decreases. However, when $\epsilon$ is smaller than a critical value $\epsilon_c$, the above discontinuous transition becomes continuous. Here as an example, the density profile for $\epsilon = -12$ is plotted in Fig. 2(b) (“T2” in Fig. 1). We find that the AMSF phase locates near the boundary of the trap. In other words, besides the molecular condensates existing everywhere in the trap, the small atomic fraction exists only in the thin outer shell of the molecular condensate existing everywhere in the trap, locates near the boundary of the trap. In other words, besides the physical meaning of this unstable phase. That is why there is no unstable phase shown in Fig. 1(d).

B. Density profile

We are in the position to construct the density profiles of the atomic and molecular condensates in spatial space, i.e., $\rho_i(r) = \delta_i(r)$ ($i = a, m$). Here we follow the strategy of Ho and Shenoy [1]. As $\mu_a$ and $\mu_m$ depend on coordinate variable $r$, i.e., $\mu_a(r) = \mu - r^2/2$ and $\mu_m(r) = 2\mu - r^2 - \epsilon$ (the expressions are renormalized to dimensionless quantities), there is a correspondence between a path in spatial space and a trajectory line in the $(\mu_a, \mu_m)$ or $(\epsilon, \mu_a)$ parameter space. For definite $\mu$ and $\epsilon$, a radial path starting from the center of the isotropic trap ($r = 0$) images a straight line from the point $(\mu, 2\mu - \epsilon)$ in the $\mu_a$-$\mu_m$ plane or from $(\mu, \epsilon)$ in the $\epsilon$-$\mu_a$ plane (see the dotted lines and circles in Fig. 1). With this in mind, the density profiles of both atomic and molecular condensates can be easily worked out from the phase diagrams. In our calculation we take $N = 2 \times 10^5$ which is the typical number of atoms involved in BEC experiments.

For $U_{am} = \sqrt{U_a U_m}$, the corresponding density profiles at $\epsilon = 0$ are plotted in Fig. 2(a) (“T1” in Fig. 1). For the chosen parameters in our calculation, the restriction of the total number of atoms here excludes the existence of the MSF phase. However, when $N$ is increased to a certain value, the pure molecular condensate may also take place from the center of the trap. Due to the repulsive atom-molecule interaction, the atomic condensate concentrates near the trap edge while the molecular fraction in the center. The distribution owns the same feature of fermionic Feshbach model at the exact resonance point [23]. Note that at the boundary of the trap, the densities of the atomic and molecular condensates both drop to zero sharply, indicating the discontinuous transition from the AMSF to V state [18].

FIG. 2. (Color online) Density plot for $U_a U_m = U_{am}^2$: (a) $\epsilon = 0$ and $\mu = 10.59$, and (b) $\epsilon = -12$ and $\mu = 4.76$. Here $\mu$ and $\epsilon$ are in units of $\hbar \omega$ while $r$ is in units of $(\hbar/m\omega)^{1/2}$. The chosen chemical potentials are to make sure the total atom number is $2 \times 10^5$. Here $U_{am} = 0.6 U_a$ for both cases.

FIG. 3. (Color online) Phase diagram for $U_a U_m \neq U_{am}^2$ in the parameter space $(\mu_a, \mu_m)$ (left) and $(\epsilon, \mu_a)$ (right). Here $\mu_a$, $\mu_m$, and $\epsilon$ are all in units of $\hbar \omega$. $U_{am} = 0.0 U_a$. Each phase is stable here.
the isotropic trap, such as the mantle of earth. Moreover the lower the value of $\mu_a$, the lesser contribution the atomic condensate offers. When $\mu_a$ is small enough, i.e., $\mu_a \sim \hbar \omega$, the density profile is solely contributed by the molecular condensate, which is in the typical TFA form.

For $U_{am} = -\sqrt{U_a U_m}$, the corresponding density profiles can be plotted too. However, they are omitted here for saving space. Still, from Fig. 1(d) we can draw the conclusion that the atomic condensate always locates at the center of the trap. This is in accordance with the fact that $U_{am}$ is negative, which means that the coexistence of both fractions helps to lower the system energy. The value of $\epsilon$ plays no qualitative difference here.

**IV. GROUND STATE FOR NONZERO NET INTERACTION**

Now let us consider a more general case, i.e., $P \neq 0$. As expected, three sets of possible AMSF solutions exist due to the nonvanishing of the third order of Eq. (4). The lengthy analytic expressions for these solutions are omitted here for saving space. Following the same strategy as the $P=0$ case in Sec. III, the phase diagrams can be plotted in the same manner.

First let us take a look at the specific case for $U_{am} = 0$ where $P > 0$. The corresponding phase diagrams are shown in Fig. 3. This is an intermediate case between $U_{am} > 0$ and $U_{am} < 0$. Despite of the zero atom-molecule interaction, the nonzero $\alpha$ couples the atomic and molecular condensates,
which plays the role of effective interspecies interaction.

The phase diagrams for \( U_{am} \neq 0 \) are plotted in Fig. 4. It is clear that the system behaves differently around the \( P=0 \) point for \( U_{am} > 0 \) and \( U_{am} < 0 \). From Figs. 4(d)–4(f), we can find that the phase diagrams exhibit a continuous change across the \( P=0 \) (i.e., \( U_{am}=0.6U_{a} \)) point for \( U_{am} > 0 \). In the parameter space of \((\epsilon, \mu_{a})\), the increase in \( U_{am} \) only continuously shrinks the lobelike AMSF phase located near the edge of the trap but does not suddenly change the distribution of those phases. Note that each phase is stable for \( U_{am} > 0 \). We emphasize that the stability of AMSF phase when \( P < 0 \) is ensured by the nonzero Feshbach coupling while the coexistence phase of pure two-species BEC mixtures without conversion will definitely collapse when \( P < 0 \).

For \( U_{am} < 0 \), the phase diagrams show a distinct transition across the \( P=0 \) (i.e., \( U_{am}=-0.6U_{a} \)) point. As \( U_{am} \) decreases from zero while restricted to be above the \( P=0 \) point, the AMSF phase expands accordingly, which can be found through comparing Fig. 3(b) with Fig. 4(a). However, when \( U_{am} \) is slightly below the \( P=0 \) point, the system exhibits a transition which can be proven by the distinct difference between Figs. 4(a) and 4(b). Once the value of \( P \) becomes smaller than zero, the AMSF phase shrinks quickly until it absolutely disappears with the decrease in \( U_{am} \). Moreover the unstable MSF phase enters the center of the trap. Though there is no atomic fraction inside the pure molecular condensate, its instability is quite an interesting issue. It indicates that the system will eventually collapse if the total atom number \( N \) is large enough.

The different behaviors around two \( P=0 \) points \((U_{am} = \pm \sqrt{U_{a}U_{m}})\) can also be confirmed by the phase diagram in the \((U_{am}, \mu_{a})\) parameter space in Fig. 5 with \( \epsilon=0 \) fixed. For positive \( U_{am} \), the system shows a smooth crossover around one \( P=0 \) point. Whereas, for negative \( U_{am} \), an abrupt transition obviously exists around the other \( P=0 \) point. Note that once \( U_{am} \leq U_{a}\sqrt{U_{m}} \), the AMSF state disappears for negative \( \mu_{a} \) at \( \epsilon=0 \) even with nonzero \( \alpha \) since it is not energetically favorable.

### V. SUMMARY

We considered a BEC system consisting of bosonic atoms and their homonuclear compounded molecules. Due to the interplay between different intraspecies and interspecies interactions and the spatially varying trapping potential, three possible phases are distinguished in the parameter space \((\mu_{a}, \mu_{m})\) or \((\mu_{a}, \epsilon)\). With the total number of atoms fixed, we plotted the spatial density profiles of atomic and molecular condensates, respectively. The phase diagrams, in accordance with the density profiles, all indicate the existence of possible phase separations. A shell structure of the atomic condensate distribution in the isotropic trap can emerge for some values of the molecular binding energy \( \epsilon \). Moreover its location can be tuned by varying the parameters of the system. We also explored the effect of the atom-molecule interaction \( U_{am} \) on the phase diagrams and the density profiles. For positive atom-molecule interactions (i.e., \( U_{am} > 0 \)), we found that as \( U_{am} \) increases, the fraction of atomic condensates smoothly shrinks both in phase and spatial space. Whereas, for attractive atom-molecule interactions (i.e., \( U_{am} < 0 \)), there is a sharp transition of the phase distribution across the \( P=0 \) point.

The method we adopted here can be directly generalized to the heteronuclear molecular case only if TFA is applicable. The validity of TFA requires the slowly spatially varying of atomic and molecular condensates which fails at the boundary of two phases (e.g., labeled by the two gray vertical lines in Fig. 2(b)). The boundary effects in two-component BEC mixtures without Feshbach coupling have been addressed in Ref. [2]. However, the exact density profiles require the numerical solutions of Eq. (1), which is beyond the TFA adopted here and also beyond the scope of this paper.

Phase separation is quite a general phenomenon for trapped mixtures. For the considered atom-molecule mixture coupled via Feshbach resonance, the spatial phase separation between atomic and molecular condensates has been observed in experiment [24]. Due to the possible three-body inelastic collisions or molecular spontaneous emission, the atom-to-molecule conversion efficiency is limited. For this reason the size of molecular condensate and the overlap between the two fractions in Ref. [24] are smaller than our expectations. Recently it has been proposed that the phenomenon of phase separation can be applied as a probe to detect weak force [25] and the Fermi superfluid [26]. The phase separation of mixed systems with Feshbach coupling in our work is expected to offer another possible routine to experi-
mentally detect the molecule-molecule and atom-molecule interactions which are still in lack of effective measurement. For example, one can fit the experimental density profiles with theoretical ones which involve the magnitudes of $U_m$, $U_{am}$, and other parameters of the system. Since there are two fitting parameters $U_m$ and $U_{am}$, there may exist a case where several pairs of them give rise to the same theoretical profile. In this occasion, one needs to vary the trapping potential while keeping the magnetic field fixed and make another fittings within the candidate solutions obtained above. One can repeat the above procedure until a unique pair of interactions is left.

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