# Understanding spin transport from motion in SU(2)×U(1) fields

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Starting from a continuum constituted by charged tops, we formulate the classical counterpart of a previously obtained covariant continuitylike equation for the spin current. Such a formulism provides an intuitive picture to elucidate the nonconservation of the spin current and to interpret the condition for the emergence of an infinite spin relaxation time. It also facilitates the discussion on the spin precession in a one-dimensional quantum wire with the Dresselhaus and Rashba spin-orbit couplings. Furthermore, we derive the diffusion equations for both the charge and spin densities and find that they couple to each other due to the *Zitterbewegung*.

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## I. INTRODUCTION

Recently, the coherent manipulation of the spin degree of freedom in semiconductor, one of the main subjects in spintronics,[1] has attracted much attention for its potential applications in future information processing and storage technologies. Understanding on the basic properties of the spin transport, spin dynamics, and spin relaxation in semiconductors is required for the design of spin-based devices. In this field, the intrinsic spin-orbit coupling (SOC) in semiconductors is considered to be an effective route for manipulating the spin degree of freedom and generating the spin current. The former aspect is involved in the proposal of the Datta–Das spin transistor,[2] while the latter one both theoretically[3–6] and experimentally[7–11] leads to the numerous studies of the spin Hall effect. However, the SOC also brings about the nonconservation of the spin current, which makes the definition of the spin-current cumbersome.[12–14] A covariant form for the continuitylike equation for the spin current was given[12] in the terminology of SU(2) gauge potentials. It was shown to play an essential role in guaranteeing the consistency of a generalized Kubo formula for the linear response to non-Abelian fields with different gauge fixings.[15] As the work mentioned above are all based on quantum mechanics, one may ask what is the classical interpretation for the nonconservation of spin current and whether this classical picture can bring new clues to the understanding on the spin transport.

In this paper, we investigate the motion of electrons in the presence of SOCs [regarded as SU(2) fields] as well as the U(1) electromagnetic fields. Considering a continuum constituted by charged tops, we obtain the classical counterpart of the continuitylike equation for the spin current. It takes the same form as that proposed in the view of the quantum mechanics[12] and exhibits an intuitive picture for the nonconservation of the spin current. In the present paper, this classical picture of the motion of electrons is the basis of our discussions on the spin transport. The whole paper is organized as follows. In Sec. II, the classical counterpart of the continuitylike equation for the spin current is formulated. In Sec. III, we discuss the precession of the spin orientation in a ballistic quantum wire with both the Dresselhaus and Rashba SOCs. In Sec. IV, we derive the equations of motion for a single spin from which we can get the condition for the emergence of infinite spin relaxation time. Starting with the semiclassical Boltzmann equation, we derive the diffusion equations for both the charge and spin densities in Sec. V. Finally, a brief summary is given in Sec. VI.

## II. CLASSICAL COUNTERPART OF CONTINUITYLIKE EQUATION

We start with considering a moving top (classical analogy of spin) which rotates at a certain rate. A continuum constituted by such kind of tops is completely characterized by a local velocity field $\mathbf{v}(\mathbf{r},t)$ and a local particle-density field $\rho(\mathbf{r},t)$ together with a local alignment field $\vec{N}(\mathbf{r},t)$.[16] Hereafter, a letter in boldface denotes for a vector in the conventional spatial space with indices $i,j=1,2,3$ labeling its components, e.g., $\mathbf{r}=x_i\hat{e}_i$, where $\hat{e}_i$ are the bases and repeated indices are summed over, while the one with an arrow over head is a vector in the spin space, e.g., $\vec{N}=(N^x,N^y,N^z)$.

The time evolution of $\vec{N}(\mathbf{r},t)$ is determined by comparing $\vec{N}$ at different times at the same place, i.e., $\vec{N}(\mathbf{r},t+\Delta t)-\vec{N}(\mathbf{r},t)=\vec{\omega}\times\vec{N}(\mathbf{r},t)\Delta t$, while its spatial deviation is simultaneously determined by comparing $\vec{N}$ at different places i.e., $\vec{N}(\mathbf{r}+\Delta\mathbf{r},t)-\vec{N}(\mathbf{r},t)=\vec{\Omega}_i\times\vec{N}(\mathbf{r},t)\Delta x_i$, as shown in Fig. 1. The vector fields $\vec{\omega}$ and $\vec{\Omega}_i$ are natural consequences of $\vec{N}$ being a vector with constant module; then, we have

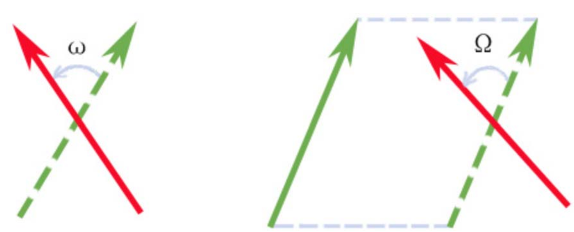


FIG. 1. (Color online) The left scheme depicts the time evolution of $\vec{N}$ at a point $\mathbf{r}$; the right scheme illustrates the spatial deviation of $\vec{N}$ by comparing the fields at two neighborhood points $\mathbf{r}$ and $\mathbf{r}+\Delta\mathbf{r}$ for which the parallel displacement is inevitable.

 

$$\frac{\partial}{\partial t}\vec{N}(\mathbf{r},t) = \vec{\omega}(\mathbf{r},t) \times \vec{N}(\mathbf{r},t),$$

$$\frac{\partial}{\partial x_i}\vec{N}(\mathbf{r},t) = \vec{\Omega}_i(\mathbf{r},t) \times \vec{N}(\mathbf{r},t). \quad (1)$$

By making use of these two relations together with the density conservation $\partial\rho/\partial t + \partial j_i/\partial x_i = 0$, we can easily derive a continuitylike equation,

$$\left(\frac{\partial}{\partial t} - \vec{\omega}\times\right)\vec{\sigma} + \left(\frac{\partial}{\partial x_i} - \vec{\Omega}_i \times\right)\vec{J}_i = 0, \quad (2)$$

as long as the natural definitions of the spin density $\vec{\sigma} = \rho\vec{N}$ and the spin-current density $\vec{J}_i = \rho v_i \vec{N}$ are employed. Compared to the quantum mechanical results,[12] one can recognize that $\vec{\omega}$ and $\vec{\Omega}_i$ correspond to the SU(2) gauge potentials $\eta\vec{\mathcal{A}}_0$ and $-\eta\vec{\mathcal{A}}_i$, with $\eta = \hbar$, respectively. For a two-dimensional electron gas in narrow gap zinc-blende III-V semiconductors, the SU(2) gauge potentials have been shown[12] to be related to the Rashba[17] and Dresselhaus[18] SOCs, concretely,

$$\vec{\mathcal{A}}_x = \frac{2m}{\eta^2}(\beta,\alpha,0), \quad \vec{\mathcal{A}}_y = -\frac{2m}{\eta^2}(\alpha,\beta,0), \quad \vec{\mathcal{A}}_0 = 0. \quad (3)$$

In terms of these gauge potentials, the SU(2) "electric" and "magnetic" fields can be expressed as

$$\vec{\mathcal{E}}_i = -\partial_0\vec{\mathcal{A}}_i - \partial_i\vec{\mathcal{A}}_0 + \eta\vec{\mathcal{A}}_0 \times \vec{\mathcal{A}}_i,$$

$$\vec{\mathcal{B}}_i = \epsilon_{ijk}\partial_j\vec{\mathcal{A}}_k + \frac{\eta}{2}\epsilon_{ijk}\vec{\mathcal{A}}_j \times \vec{\mathcal{A}}_k, \quad (4)$$

which provides a "spin-related" force,[12]

$$\mathcal{F}_i = \vec{\mathcal{E}}_i \cdot \vec{\sigma} + \epsilon_{ijk}\vec{J}_j \cdot \vec{\mathcal{B}}_k. \quad (5)$$

In the above, we have employed the notations $\partial_0 \equiv \partial/\partial t$ and $\partial_i = \partial/\partial x_i$ for simplicity.

Equations (1) and (2) are the main relations of this section which depicts a classical picture for the motion of electrons in both U(1) and SU(2) fields. In the following, we will carry on the discussions on the spin transport in which these results will be employed.

## III. PRECESSION OF THE SPIN ORIENTATION

As an immediate application of Eq. (1), we investigate the precession of the spin orientation $\vec{N}$ in a one-dimensional quantum wire. For example, a spin-polarized current is injected at $x=0$ and ballistically transported through the wire. We consider the case that the Dresselhaus SOC is homogeneous, while the Rashba SOC can be either homogeneous or inhomogeneous.

(a) We first consider homogeneous $\alpha$, the precession of $\vec{N}$ can be analytically solved and its three components are given by

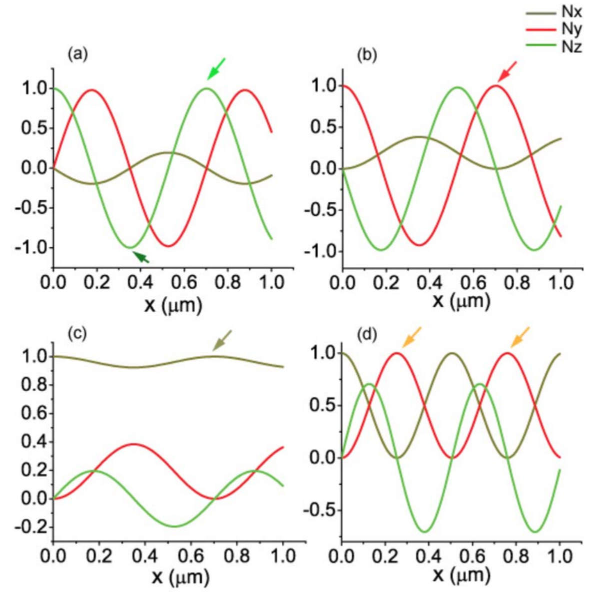


FIG. 2. (Color online) $N^x$, $N^y$, and $N^z$ are plotted in unit of $\hbar/2$ as functions of $x$ with $\alpha = 10^{-12}$ eV m and $\beta = 5\times10^{-12}$ eV m in panels (a)–(c). The initial direction $\vec{N}_0$ is along the $z$ direction in panel (a) and $\vec{N}$ can be antiparallel (parallel) to $\vec{N}_0$ as marked by the dark (light) green arrow. In panels (b) and (c), the initial direction $\vec{N}_0$ is along the $y$-axis and $x$-axis directions, respectively. $\vec{N}$ can only return its initial direction. In panel (d), $\alpha = \beta = 10^{-12}$ eV m and $\vec{N}$ can rotate to the $y$-axis direction at $x=0.25$ and 0.76 $\mu$m (marked by orange arrows) with $\vec{N}_0$ points to the $x$-axis direction.

$$N^x = -\frac{\gamma N_0^z}{\sqrt{1+\gamma^2}\cos\theta}\sin(\kappa x + \theta) + \frac{N_0^x + \gamma N_0^y}{1+\gamma^2},$$

$$N^y = \frac{N_0^z}{\sqrt{1+\gamma^2}\cos\theta}\sin(\kappa x + \theta) + \frac{\gamma(N_0^x + \gamma N_0^y)}{1+\gamma^2},$$

$$N^z = \frac{N_0^z}{\cos\theta}\cos(\kappa x + \theta), \quad (6)$$

where $\vec{N}_0 = (N_0^x, N_0^y, N_0^z)$ is the initial spin orientation, $\gamma = \alpha/\beta$, $\kappa = 2m\sqrt{\alpha^2+\beta^2}/\eta$, and $\tan\theta = (N_0^y - \gamma N_0^x)/\sqrt{1+\gamma^2}N_0^z$. Figure 2 shows the variation of $\vec{N}$ with respect to $x$ axis with $\alpha = 10^{-12}$ eV m and $\beta = 5\times10^{-12}$ eV m. The initial direction $\vec{N}_0$ affects the precession of $\vec{N}$. When $\vec{N}_0$ is along the $z$-axis direction, $\vec{N}$ can either precess to the opposite direction of $\vec{N}_0$ [e.g., at $x=0.35$ $\mu$m marked by the dark green arrow in Fig. 2(a)] or go back to its initial direction [e.g., at $x=0.7$ $\mu$m marked by the light green arrow in Fig. 2(a)]. However, when $\vec{N}_0$ is along the $x$-axis or $y$-axis direction, $\vec{N}$ can only return to its initial direction and never achieve the opposite direction of $\vec{N}_0$, as illustrated in Figs. 2(b) and 2(c). This can be understood as follows. If the initial direction is in the plane perpendicular to the revolution axis $\vec{\mathcal{A}}_x$, $\vec{N}$ will precess in this plane and definitely experience the opposite direction of $\vec{N}_0$. Besides, the position where $\vec{N}$ rotates to its initial direction is exclusively determined by $\kappa$. It can be seen from Eq. (6) that the three components of $\vec{N}$ oscillate with the same frequency $\kappa$. A special situation is $\alpha=\beta$ where $N^x$ and $N^y$ are formally equivalent. Thus, $\vec{N}$ can also precess to the $y$-axis direction ($x$-axis direction) when $\vec{N}_0$ is in the $x$-axis direction ($y$-axis direction), as shown in Fig. 2(d).

(b) We consider an inhomogeneous case $\alpha=ax$ which can be realized by tuning the applied gate voltage on the two-dimensional electron gas. Hence, $\vec{N}$ can be analytically solved in form of series expansion, namely, $N^x=N_0^x-\kappa_a\sum_{n=0}^{\infty}\frac{b_n}{n+2}x^{n+2}$, $N^y=N_0^y+\kappa_b\sum_{n=0}^{\infty}\frac{b_n}{n+1}x^{n+1}$, and $N^z=\sum_{n=0}^{\infty}b_nx^n$, with $\kappa_a=2ma/\eta$ and $\kappa_b=2m\beta/\eta$. The coefficients are given by $b_0=N_0^z$, $b_1=-\kappa_bN_0^y$, $b_2=\frac{1}{2}(\kappa_aN_0^x-\kappa_b^2N_0^z)$, $b_3=\kappa_b^3N_0^y/6$, and $b_{n+2}=-\kappa_b^2b_n/(n+2)(n+1)-\kappa_a^2b_{n-2}/(n+2)n$ for $n=2,3,\ldots$. When $\alpha$ is much smaller than $\beta$, the components of $\vec{N}$ can be approximatively written as

$$N^x=N_0^x+N_0^y\frac{\kappa_a}{\kappa_b\cos\theta'}\left[\frac{\sin(\kappa_bx+\theta')-\sin\theta'}{\kappa_b}-x\cos(\kappa_bx+\theta')\right],$$

$$N^y=\frac{N_0^y}{\cos\theta'}\cos(\kappa_bx+\theta'),$$

$$N^z=-\frac{N_0^y}{\cos\theta'}\sin(\kappa_bx+\theta'),\qquad(7)$$

with $\tan\theta'=-N_0^z/N_0^y$.

We plot these three components as functions of $x$ in Fig. 3 with $\beta=5\times10^{-12}$ eV m and $a=10^{-6}$ eV. If the initial direction is in the $y$-$z$ plane, $\vec{N}$ will almost precess in this plane for small $x$ since the vector potential in the $x$-axis direction $\mathcal{A}_x^x$ is much larger than that in the $y$-axis direction $\mathcal{A}_x^y$. As $x$ increases, the amplitude of the oscillation of $N^x$ becomes larger and $\vec{N}$ tilts out of the $y$-$z$ plane [see Fig. 3(a)]. If the initial direction is along the $x$-axis direction, $\vec{N}$ nearly keeps pointing in this direction for small $x$ and begins to precess as $x$ increases [see Fig. 3(b)].

We further consider sinusoid-type inhomogeneity, $\alpha=b\sin(qx)$. One can analytically solve $\vec{N}$ with the help of series expansion, in principle, but insufficient information can be obtained because the solution cannot be expressed either in a closed form or in terms of any known functions. Now, we numerically calculate it. The obtained results show that $\vec{N}$ behaves like a biperiodic function of $x$; one period is determined by the gauge potential $\vec{\mathcal{A}}_x$ and the other by $q$, as shown in Fig. 4 with $b=10^{-12}$ eV m and $q=10^7$ m$^{-1}$.

## IV. EQUATION OF MOTION FOR A SINGLE SPIN AND ITS APPLICATION

In this section, we focus on the motion of a single charged top $\vec{n}=(n^x,n^y,n^z)$ with constant module $n$ which is the classical analogy of an electron. The equations of motion for this top under both U(1) and SU(2) fields are obtained as

$$\frac{d\vec{n}(t)}{dt}=\eta(\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i)\times\vec{n}(t),$$

$$m\frac{dv_i}{dt}=\vec{\mathcal{E}}_i\cdot\vec{n}(t)+eE_i+\epsilon_{ijk}v_j[\vec{\mathcal{B}}_k\cdot\vec{n}(t)+eB_k].\qquad(8)$$

The first equation can be derived from Eq. (1) by adopting $\vec{N}(\mathbf{r},t)=\vec{n}(t)\delta[\mathbf{r}-\tilde{\mathbf{r}}(t)]$, with $\tilde{\mathbf{r}}(t)$ being the trajectory of a

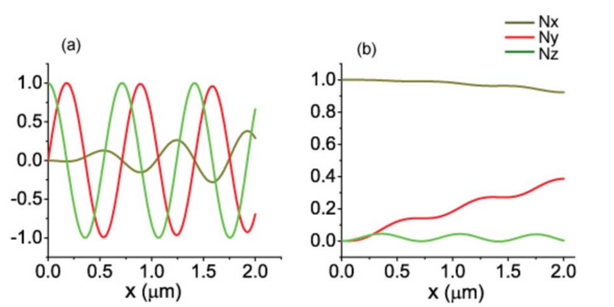


FIG. 3. (Color online) $N^x$, $N^y$, and $N^z$ are plotted in unit of $\hbar/2$ as functions of $x$ with $\beta=5\times10^{-12}$ eV m and $a=10^{-6}$ eV. The initial direction $\vec{N}_0$ is along the $z$ direction in panel (a) and along the $x$-axis direction in panel (b).

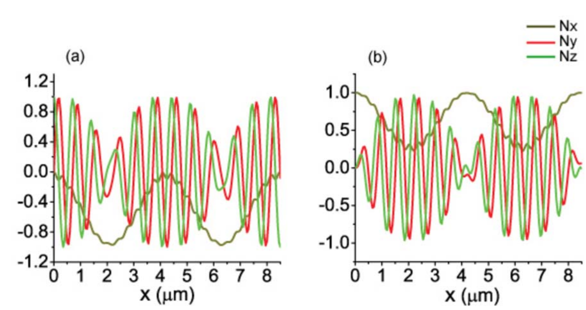


FIG. 4. (Color online) $N^x$, $N^y$, and $N^z$ are plotted in unit of $\hbar/2$ for $\alpha=b\sin(qx)$ with $q=10^7$ m$^{-1}$. The initial direction $\vec{N}_0$ is along the $z$-axis direction in panel (a) and along the $x$-axis direction in panel (b).

single top. The second equation is due to the fact that its translational motion is governed by both the Lorentz force and the spin-related force given in Eq. (5). Clearly, Eq. (8) gives rise to some immediate consequences:

*Case 1*. The first equation for the time rate of $\vec{n}$ clearly manifests that the vector $\vec{n}$ does not precess when it is parallel to $\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i$, or more specially, $\vec{\mathcal{A}}_0-v_i\vec{\mathcal{A}}_i=0$. In such cases, the spin orientation remains unchanged, which results in an infinite spin relaxation time. A typical example is that the infinite spin relaxation time occurs in the $\pm[1,\pm 1,0]$ direction when $\vec{\mathcal{A}}_0=0$ and $\alpha=\pm\beta$, as discussed in Ref. 19. This is analogous to the case in the classical electrodynamics where an electron moving in the uniform orthogonal electromagnetic fields with certain velocity, $\mathbf{v}=\mathbf{E}\times\mathbf{B}/B^2$, does not feel the Lorentz force.

*Case 2*. In virtue of the coupling between $\vec{n}$ and the SU(2) field in the second equation of Eq. (8), the time rate of $\vec{n}$ leads to the time-dependent effective fields even when the SU(2) fields are time independent.

As a concrete example, we consider a spin Hall system with constant Rashba and Dresselhaus SOCs. Thus, $\vec{\mathcal{A}}_0=0$, $\vec{\mathcal{E}}_i=0$, and $\vec{\mathcal{B}}_i=\frac{\eta}{2}\epsilon_{ijk}\vec{\mathcal{A}}_j\times\vec{\mathcal{A}}_k$, and we first neglect the electric field. The second equation of Eq. (8) reduces to

$$m\frac{dv_i}{dt}=\eta\vec{\mathcal{A}}_i\cdot(v_j\vec{\mathcal{A}}_j)\times\vec{n}, \tag{9}$$

which gives rise to a relation between $\vec{n}$ and $v_i$,

$$v_i(t)=-\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}(t)+C_i, \tag{10}$$

where $C_i=v_{0i}+\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}_0$ are determined by the initial values $v_i(0)=v_{0i}$ and $\vec{n}(0)=(n_0^x,n_0^y,n_0^z)$. Consequently, one only needs to solve one equation,

$$\frac{d\vec{n}}{dt}=\eta\left(\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}-C_i\right)(\vec{\mathcal{A}}_i\times\vec{n}), \tag{11}$$

which can be explicitly written as

$$\frac{dn^x}{dt}=-\frac{2m}{\eta}n^z\left[\alpha C_1-\beta C_2-\frac{4\alpha\beta n^x+2(\alpha^2+\beta^2)n^y}{\eta^2}\right],$$

$$\frac{dn^y}{dt}=\frac{2m}{\eta}n^z\left[\beta C_1-\alpha C_2-\frac{4\alpha\beta n^y+2(\alpha^2+\beta^2)n^x}{\eta^2}\right],$$

$$\frac{dn^z}{dt}=\frac{2m}{\eta}n^x\left[\alpha C_1-\beta C_2-\frac{4\alpha\beta n^x}{\eta^2}\right]-\frac{2m}{\eta}n^y\left[C_1\beta-C_2\alpha-\frac{4\alpha\beta n^y}{\eta^2}\right]. \tag{12}$$

For $\alpha=\beta$ which is called ReD field in Ref. 19, we can analytically solve these equations,

$$n^x=-\frac{n_0^z}{\sqrt{2}\cos\varphi}\sin(\omega t+\varphi)+\frac{1}{2}(n_0^x+n_0^y),$$

$$n^y=\frac{n_0^z}{\sqrt{2}\cos\varphi}\sin(\omega t+\varphi)+\frac{1}{2}(n_0^x+n_0^y),$$

$$n^z=\frac{n_0^z}{\cos\varphi}\cos(\omega t+\varphi), \tag{13}$$

where $\tan\varphi=(n_0^y-n_0^x)/(\sqrt{2}n_0^z)$ is determined by the initial conditions. It is clear that the tip of $\vec{n}$ experiences a cyclotron rotation with frequency $\omega=2\sqrt{2}m\alpha(v_{0x}-v_{0y})/\eta$. The instantaneous velocity solved from Eq. (10) is just its initial value $v_x=v_{0x}$ and $v_y=v_{0y}$, i.e., the electron undergoes a motion with uniform velocity. This is due to that the time-dependent parts of $n^x$ and $n^y$ only differ from each other by a minus sign. Specially, when $v_x=v_y$, the spin vector $\vec{n}$ does not precess since $\omega=0$, which recovers the result in Ref. 19.

When an external electric field $\vec{E}=(E_x,E_y)$ is applied, which mimics to the usual spin Hall effect in current literature, we obtain $v_i(t)=-\frac{1}{m}\vec{\mathcal{A}}_i\cdot\vec{n}(t)+C_i+\frac{e}{m}E_it$. The equation of motion for $\vec{n}$ is almost the same as Eq. (11) if $C_i$ is replaced by $\tilde{C}_i(t)=C_i+\frac{e}{m}E_it$. For the electric field that is sufficiently weak, the perturbation theory is applicable and we can expand $\vec{n}$ in power series of the electric field, $\vec{n}=\vec{n}^{(0)}+\vec{n}^{(1)}+\cdots$. The zeroth-order results take the form of Eq. (13), while the first order corrections are

$$n^{x(1)}=-\lambda(E)t^2\cos(\omega t+\varphi),$$

$$n^{y(1)}=\lambda(E)t^2\cos(\omega t+\varphi),$$

$$n^{z(1)}=-\sqrt{2}\lambda(E)t^2\sin(\omega t+\varphi), \tag{14}$$

with $\lambda(E)=e\alpha n_0^z(E_x-E_y)/(\eta\cos\varphi)$. The form of the entire motion for $\vec{n}$ is similar to the case without external electric fields, but with a time-dependent amplitude $\tilde{a}(t)=[(n_0^z)^2/(2\cos^2\varphi)+\lambda^2t^4]^{1/2}$ and a phase $\tilde{\varphi}(t)=\varphi+\tan^{-1}(\sqrt{2}\lambda t^2\cos\varphi/n_0^z)$.

## V. DIFFUSION EQUATIONS AND *ZITTERBEWEGUNG* EFFECTS

In the presence of U(1) and SU(2) fields, $mv_i=p_i(\mathbf{r},t)-eA_i-\eta\vec{\mathcal{A}}_i\cdot\vec{N}$, we can write out the charge current $e\rho v_i$ and spin current $\rho v_i\vec{N}$ as follows:

$$ej_i=\frac{e\rho}{m}(p_i-eA_i)-\frac{e\eta}{m}\vec{\mathcal{A}}_i\cdot\vec{\sigma},$$

$$\vec{J}_i=\frac{\vec{\sigma}}{m}(p_i-eA_i)-\frac{\eta}{m}\vec{N}(\vec{\mathcal{A}}_i\cdot\vec{\sigma}). \tag{15}$$

The first terms on the right-hand sides of Eq. (15) correspond to the conventional nonrelativistic currents. It is worthwhile to pay attention to the second terms which are related to the *Zitterbewegung*. It was ever believed that the *Zitterbewegung*[20] as a relativistic effect cannot be directly observed due to the high frequency (of order $10^{20}$ Hz) and short length scale (of order 1 pm).[21] Recently, Schliemann *et al.* suggested to detect the *Zitterbewegung* in III-V semiconductor quantum wells[22] where the energy and length scale become available in experiments. We will show that the existence of *Zitterbewegung*-related phenomena in the spin transport makes the diffusion equations for the spin and charge densities couple to each other.

In comparison to the expression of the conventional nonrelativistic current density, there is an extra term $j_i'=[\frac{e}{m}\nabla\times\vec{\sigma}]_i$ in the expression derived from the Dirac equation in the nonrelativistic limit. This term $j_i'$ can be regarded as a result of the *Zitterbewegung*.[21,23] As long as we distinguish between the spin space and the conventional spatial space, we have

$$j_i' = \frac{e\eta}{m}\vec{\mathcal{A}}_i\cdot\vec{\sigma} + \frac{e}{m}[\nabla\rho\times\vec{N}]_i. \tag{16}$$

In the above derivation, we have adopted Eq. (1) and the fact that the matrix $(\mathbb{A})_{ij}\equiv\mathcal{A}_i^j$ is traceless. Hence, we can consider $\frac{e\eta}{m}\vec{\mathcal{A}}_i\cdot\vec{\sigma}$ arising from the *Zitterbewegung* for either uniform $\rho$ or $\nabla\rho\propto\vec{N}$. We will see in the following that $\frac{e\eta}{m}\vec{\mathcal{A}}_i\cdot\vec{\sigma}$ enters into the diffusion equations for both charge and spin densities which couple to each other.

The coupled charge-spin diffusion equations in a spin Hall system have been investigated by several groups in the view of quantum kinetic theory.[24] Starting from the Boltzmann equation as well as the above results, we can derive the semiclassical diffusion equations for both charge and spin densities. The introduced distribution function $f(\mathbf{r},\mathbf{p},t)$ obeys the Boltzmann equation,

$$\frac{\partial f}{\partial t} + v_i\frac{\partial f}{\partial x_i} + F_i\frac{\partial f}{\partial p_i} = \left.\frac{\partial f}{\partial t}\right|_{\text{coll}}. \tag{17}$$

For charged top, $F_i$ should contain both the Lorentz force caused by the U(1) fields $E_i$ and $B_i$ as well as the spin-related force caused by the SU(2) fields $\vec{\mathcal{E}}_i$ and $\vec{\mathcal{B}}_i$. The right-hand side of Eq. (17) represents the collision term contributed by the elastic scattering between electrons. For the system not driven far away from the equilibrium, the relaxation-time approximation is applicable and the distribution function $f$ can be decomposed into the equilibrium distribution $f_0$ and a small deviation $f'$, in which $f_0$ is independent of the direction of $\mathbf{p}$. For the spin Hall system with constant Rashba and Dresselhaus couplings, the magnetic field and the SU(2) electric field vanish and $\epsilon_{ijk}v_j\vec{\mathcal{B}}_k\cdot\vec{N}\frac{\partial f'}{\partial p_i}$ is of the second order and can be neglected. Hence, Eq. (17) is written as

$$\frac{\partial f}{\partial t} + \tilde{\nabla}_i\left(\frac{p_i}{m}f\right) - \frac{\eta}{m}\vec{\mathcal{A}}_i\cdot\frac{\partial\vec{g}}{\partial x_i} = -\frac{f-f_0}{\tau}, \tag{18}$$

where $\tilde{\nabla}_i=\nabla_i+eE_i(\partial/\partial\varepsilon)$, with $\varepsilon=p^2/2m$ and $\vec{g}=f\vec{N}$ is the distribution function for the spin density.

Integrating Eq. (18) over $\mathbf{p}$, we have the diffusion equations for the charge density $\rho=e\int f d\mathbf{p}$. In the calculation, we encounter $\int(\frac{p_i}{m}f)d\mathbf{p}=\int(\frac{p_i}{m}f')d\mathbf{p}$. For a steady state, $f'$ can be solved from Eq. (18),

$$f' = -\tau\left(\frac{p_j}{m}\tilde{\nabla}_j f - \frac{\eta}{m}\vec{\mathcal{A}}_j\cdot\vec{N}\frac{\partial f}{\partial x_j}\right). \tag{19}$$

Neglecting higher orders of $f'$, we have

$$\int d\mathbf{p}\left(\frac{p_i}{m}f'\right) \simeq -\tau\delta_{ij}\frac{\varepsilon}{m}\tilde{\nabla}_j\int d\mathbf{p}f_0 = -D\tilde{\nabla}_i\rho. \tag{20}$$

Here, $D=\varepsilon\tau$ is the diffusion coefficient which turns to be $v_F^2\tau/2m$ in the quantum kinetic equation, with $v_F$ being the Fermi velocity. As a result, we have

$$\frac{\partial\rho}{\partial t} - D\tilde{\nabla}^2\rho - \frac{\eta}{m}\vec{\mathcal{A}}_i\cdot\frac{\partial\vec{\sigma}}{\partial x_i} = -\frac{\rho-\rho_0}{\tau}, \tag{21}$$

and the charge current is given by

$$J_i = \int d\mathbf{p}v_i f' = -D\tilde{\nabla}_i\rho - \frac{\eta}{m}\vec{\mathcal{A}}_i\cdot\vec{\sigma}. \tag{22}$$

We derive the diffusion equation for the spin density in an analogous procedure,

$$\left(\frac{\partial}{\partial t} - \eta\vec{\mathcal{A}}_0\times\right)\vec{\sigma} - D(\tilde{\nabla}_i + \eta\vec{\mathcal{A}}_i\times)(\tilde{\nabla}_i + \eta\vec{\mathcal{A}}_i\times)\vec{\sigma} - \frac{\eta}{m}(\vec{\mathcal{A}}_i\cdot\vec{N})\vec{N}\frac{\partial\rho}{\partial x_i} = -\frac{\vec{\sigma}-\vec{\sigma}_0}{\tau}, \tag{23}$$

with the spin current being

$$\vec{J}_i = -D(\tilde{\nabla}_i + \eta\vec{\mathcal{A}}_i\times)\vec{\sigma} - \frac{\eta}{m}(\vec{\mathcal{A}}_i\cdot\vec{N})\vec{\sigma}. \tag{24}$$

Equations (21) and (23) are classical counterparts of the coupled spin-charge diffusion equations.[24] They are valid for a large time scale since our starting point is the semiclassical Boltzmann equation which implies the assumption of the energy conservation. For a short time scale $\delta t$, $\delta\varepsilon$ is large and the quantum kinetic theorem should be employed.

The last terms on the left hand sides of both Eqs. (21) and (23) are the contributions from the *Zitterbewegung*, which make these diffusion equations coupled to each other. To illustrate the effect of the *Zitterbewegung*-related contributions, we consider the one-dimensional quantum wire with constant Rashba and Dresselhaus SOCs. The charge density exponentially decays in the wire, namely, $\rho(x)\propto e^{-x/L_s}$ where the decay length is given by $L_s=[\sqrt{\xi^2+4D/\tau}+\xi/(2D)]^{-1}$, with $\xi=2(\alpha^2-\beta^2)(\alpha N_0^y+\beta N_0^x)/\eta(\alpha^2+\beta^2)$, shorter than that without the *Zitterbewegung*-related contributions $L_s=(\sqrt{D\tau})^{-1}$.

## VI. SUMMARY

Depicting moving electrons as a continuum constituted by charged tops, we investigated the motion of electrons in both U(1) and SU(2) fields and discussed the spin transport. We derived the classical counterpart of the continuitylike equations for the spin current, which takes the same form as that ever quantum-mechanically proposed in a previous paper.[12] This provided us an intuitive picture for elucidating the nonconservation of the spin current in the presence of the SOC.

We discussed the precession of the spin orientation in a ballistic onedimensional quantum wire with both Rashba and Dresselhaus SOCs and found that the initial direction can greatly affect the spin precession. We also formulated the equations of motion for a single spin in the presence of both U(1) and SU(2) fields. As a direct consequence, we obtained the condition for the emergence of infinite spin relaxation time. A special situation of our conclusion recovers the previous result discussed by other authors. Furthermore, we derived the semiclassical diffusion equations for both the charge and spin densities. We found that the *Zitterbewegung* causes these equations coupled each other and makes the decay length of the charge density much shorter in a one-dimensional quantum wire.

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