Stability and dynamical property for two-species ultracold atoms in double wells

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We study undistinguishable two-species Bose-Einstein condensates (BECs) trapped in double wells. In the mean-field approximation we map the quantum system into a classical model and investigate the existence and stability of the fixed point solutions. By appropriately varying the amplitude of the high-frequency periodic modulation on the energy bias between the two wells, we tune the tunneling strength in the adiabatic Rosen-Zener type effectively. Starting from two typical initial states, we study the evolution of the system and find that the interspecies interaction can produce totally distinct results, which is expected to provide a route to manipulate the BEC distribution.

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I. INTRODUCTION

Bose-Einstein condensates (BECs) in double wells offer a powerful tool to study the quantum tunneling phenomena [1] since almost every parameter can be tuned experimentally, i.e., the interwell tunneling strength, on-site interaction strength, and also the energy bias between the two wells. There have been extensive studies on the single-species BEC in double wells in recent years. The most prominent feature of its tunneling is the nonlinear dynamics arising from the interaction between atoms. Several phenomena like self-trapping (ST) and Josephson oscillation (JO) [2–4] were observed in experiments recently [5]. Such a quantum system can be mapped into a classical Hamiltonian which gives rise to fixed-point solutions. The emergence of phenomena ST and JO are strongly related to these fixed point solutions. This is because there is a correspondence between the quantum eigenstates and the classical fixed points [6]. However, if the initial state of the system is merely a fixed point, it will keep “fixed” and never evolve [7]. If the tunneling strength is tuned adiabatically, the system will evolve due to the shift of fixed points. The complete population transfer between the two wells may occur for the adiabatic Rosen-Zener form of tunneling strength [9], which was originally proposed in Ref. [8] to study the spin-flip problem of two-level atoms. Compared with other forms of tunneling, such as the monotonic ones, this result may be natural. However, it is somewhat counterintuitive for Rosen-Zener form since the initial and final tunneling strengths are the same. There are several ways to tune the tunneling strength, e.g., by changing the distance between the two wells or the height of the potential barrier separating the two wells. Additionally, the tunneling strength can also be effectively tuned through periodically modulating the energy bias between the wells [10–12].

In comparison to the single-species case, the multispecies BECs exhibit richer physics due to the existence of the interspecies interaction. Previous studies focus on the equilibrium properties of the system, e.g., the component separation [13,14], the cancelation of the mean-field energy shift [15], etc. When trapped in double wells, the tunneling process owns more new features, e.g., quantum-correlated tunneling between the two species [16], the renewed macroscopic JO and ST effects [17,18], and the spin tunneling [19,20], etc. In this paper we consider an undistinguishable two-species BEC system confined in double wells. We study its adiabatic dynamical properties with the application of a high-frequency periodic modulation on energy bias between the wells which has never been studied before. In the next section, we employ the mean-field approximation to map the quantum model into a classical Hamiltonian and derive the corresponding dynamical equations. In Sec. III, we solve the fixed points and discuss their stabilities. In Sec. IV, we give the numerical results where two typical initial states are considered, respectively. Our results are briefly summarized and discussed in Sec. V.

II. MODELING THE SYSTEM

We consider a two-species Bose-Einstein condensate system trapped in double wells. The Hamiltonian of such a system is given by

\[ H = -\sum_{\sigma} (J_{\sigma} c_{\sigma\uparrow} c_{\sigma\downarrow} + H.c.) + \sum_{\sigma, \sigma', i} \frac{1}{2} \tilde{U}_{\sigma\sigma'} \hat{n}_{\sigma i} \hat{n}_{\sigma' i} - \sum_{\sigma, i} \frac{1}{2} \tilde{U}_{\sigma i} \hat{n}_{\sigma i}, \]

where \( \hat{c}_{\sigma i} \) (\( \hat{c}_{\sigma i}^\dagger \)) creates (annihilates) a boson of species \( \sigma \) in the \( i \)th well, and \( \hat{n}_{\sigma i} = \hat{c}_{\sigma i}^\dagger \hat{c}_{\sigma i} \) denotes the particle number operator. Here the subscripts \( i \) (\( i=1,2 \)) specify the two wells while \( \sigma \) (\( \sigma = a, b \)) refer to two different species whose interwell tunneling strengths are denoted by \( J_a \) and \( J_b \), respectively; \( \tilde{U}_{aa} \) and \( \tilde{U}_{bb} \) are the intraspecies interaction strength while \( \tilde{U}_{ab} \) the interspecies interaction strength. Such a two-species BEC mixture may consist of different atoms, or different isotopes, or different hyperfine states of the same kind of atom.

In order to investigate the dynamical properties of the system, we need to work with the equations of motion. In the
mean-field approximation $(\hat{c}_n)=c_n\sigma$ with $c_n\sigma$ being $c$ numbers, the Heisenberg equations give rise to the following equations:

\[ i\dot{c}_n = -J_{\sigma\sigma}\dot{c}_n + U_{\sigma\sigma}|c_n|^2c_n + U_{\sigma\alpha}|c_n|^2c_n, \]

where $U_{\sigma\sigma}=U_{\sigma\sigma}N$ and $U_{\sigma\alpha}=-U_{\sigma\alpha}N$ with $N$ being the particle number of each species ($N_\sigma=N_\alpha=N$ for simplicity). Here the overhead dot denotes the time derivative and $\hat{c}$ is set to unit. Note that $\sigma$ and $\dot{\sigma}$ refer to different species while $i$ and $\dot{i}$ to different wells. Equation (2) is the well-known Gross-Pitaevski equation.

We further express $c_n\sigma$ as $\sqrt{\rho_{n\sigma}}e^{i\theta_n\sigma}$ and define $\phi_n\sigma=\theta_n\sigma-\theta_n$, $z_n\sigma=\rho_{n\sigma}-\rho_{\sigma\sigma}$ with $\rho_{n\sigma}$ and $\theta_n\sigma$ being the distribution probability and phase of species $n$ in the $i$th well, respectively. The conservation of particle numbers of each species requires $\rho_{n\sigma}+\rho_{\sigma\sigma}=1$ and $\rho_{n\sigma}+\rho_{\sigma\sigma}=1$. Since $\phi_n\sigma$ and $z_n\sigma$ are mutually canonical conjugations, the equations of motion can be rewritten as

\[ \dot{\phi}_a = J_a \cos \phi_a \sqrt{1-z_a^2} + U_{ia} z_a + U_{ia} \bar{a}, \]

\[ \dot{z}_a = -2J_a \sin \phi_a \sqrt{1-z_a^2}, \]

\[ \dot{\phi}_b = J_b \cos \phi_b \sqrt{1-z_b^2} + U_{ib} \bar{b} + U_{ib} z_b, \]

\[ \dot{z}_b = -2J_b \sin \phi_b \sqrt{1-z_b^2}, \]

and the corresponding classical Hamiltonian is given by

\[ H_{mf} = -2J_a \cos \phi_a \sqrt{1-z_a^2} + \frac{1}{2} U_{ia} z_a^2 - 2J_b \cos \phi_b \sqrt{1-z_b^2} + \frac{1}{2} U_{ib} \bar{b} + U_{ib} z_b. \]

In the following, we only consider the case $U_{\sigma\sigma}=U_{\sigma\alpha}=U$, $J_a=J_b=J$ for simplicity [21]. We also assume positive $U$, $U_{ia}$, and $J$ and take $U_{ia}=U$ so that the miscibility condition [13] is fulfilled.

III. FIXED POINTS AND THEIR STABILITY

The adiabatic evolution of quantum eigenstates corresponds to the movement of fixed points determined by the classical Hamiltonian. In order to get the fixed point solutions, we set $\phi_a=0$, $\phi_b=0$ in Eqs. (3) and obtain

\[ z_a = -\frac{2J}{U_{ia} \sqrt{1-z_b^2}} \xi_a + U, \]

\[ z_b = -\frac{2J}{U_{ib} \sqrt{1-z_a^2}} \xi_b + U, \]

where $\xi_a=\cos \phi_a$ and $\xi_b=\cos \phi_b$ where $\phi_a$ and $\phi_b$ are either 0 or $\pi$.

\[ \xi_a=\cos \phi_a \quad \text{and} \quad \xi_b=\cos \phi_b \]

where $\xi_a$ and $\xi_b$ are either 0 or $\pi$.

Equations (5) define two curves in the $(z_a, z_b)$ plane. We plot these curves for the case of $(\phi_a, \phi_b)=(\pi, \pi)$ in Fig. 1. The four panels correspond to different values of $U$, $U_{ia}$, and $J$ in the $(z_a, z_b)$ plane for $(\phi_a, \phi_b)=(\pi, \pi)$. The black curve and the gray curve correspond to the first and the second equation of Eq. (5), respectively. The intersections of the two curves stand for the fixed points solutions. The conditions for the existence of each panel are (a) $0<2J<(U+U_{ia})/(U+U_{ab})$, (b) $(U-U_{ab})/(U-U_{ia})<2J<(U+U_{ab})$, (c) $-U_{ab}<2J<(U+U_{ab})$, and (d) $2J>(U+U_{ab})$.

The four panels correspond to different values of $U$, $U_{ia}$, and $J$ in the $(z_a, z_b)$ plane for $(\phi_a, \phi_b)=(\pi, \pi)$. The black curve and the gray curve correspond to the first and the second equation of Eq. (5), respectively. The intersections of the two curves stand for the fixed points solutions. The conditions for the existence of each panel are (a) $0<2J<(U+U_{ia})/(U+U_{ab})$, (b) $(U-U_{ab})/(U-U_{ia})<2J<(U+U_{ab})$, (c) $-U_{ab}<2J<(U+U_{ab})$, and (d) $2J>(U+U_{ab})$.
The eigenvalues of $Q_b/(\mathcal{H}_9)$

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We substitute the aforementioned fixed point solutions into Eq. (7). In Table I we list the analytical expressions of eigenfrequencies and the corresponding eigenvectors for fixed points $S$, $A_n$, and $I$ (that for fixed points $A_s$ are unlisted because of the unavailable of their analytic solutions). There are two kinds of eigenvectors in the table: $(-1,1)$ and $(1,1)$. The former corresponds to the spin mode while the latter the density mode. The two modes can be understood as in- and out-of-phase oscillations of two coupled Josephson currents [18].

The expressions of eigenfrequencies can be simplified if we substitute the aforementioned fixed point solutions into Eq. (8).

The fixed points are dynamically stable as long as the corresponding frequencies of the system around each fixed point, which read

$$\omega_1^2 = 4J^2 + \frac{M_1^2 + M_2^2 - 2JU(Q_a + Q_b)}{2} \pm \sqrt{\left[(M_1^2 - M_2^2) - 2JU(Q_a - Q_b)\right]^2 + 16J^2 U_{ab}^2 Q_a Q_b},$$

where $M_1 = U_{a}^0 + U_{ab}^0$, $M_2 = U_{a}^0 + U_{ab}^0$, $Q_a = \sqrt{1 - (\zeta_b^0)^2}$, and $Q_b = \sqrt{1 - (\zeta_b^0)^2}$.

The fixed points are dynamically stable as long as the corresponding frequencies $\omega_+\,\omega_-$ are both real. The stability diagrams of fixed points $S$, $A_n$, $I$, and $A_s$ are plotted in the $(U_{ab},J)$ space for $(\phi_{A_s},\phi_{A_s})=(\pi,\pi)$ in Figs. 2(a)–2(d), respectively. In each panel the line labeled corresponds to the parameter space for the existence of a fixed point, and the grayed area to the stable region. As shown in Fig. 2(a), fixed points $S$ are always stable. For both fixed points $A_n$ and $I$, there exist a stable and an unstable region as represented in Figs. 2(b) and 2(c). Figure 2(d) shows that fixed points $A_s$ are always unstable. They are marked, respectively, by dots and circles in Fig. 1 so as to distinguish between stable and unstable fixed points.

Until now we have focused our attention on the case of $(\phi_{A_s},\phi_{A_s})=(\pi,\pi)$ which is relevant to our discussion in the next section. The other three cases can be derived by means of the similar strategy which we omit here for the sake of saving space.

TABLE I. Eigenfrequencies and the corresponding eigenvectors for fixed points $S$, $A_n$, and $I$.

<table>
<thead>
<tr>
<th>Fixed points</th>
<th>Eigenfrequencies</th>
<th>Eigenvectors</th>
</tr>
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<tbody>
<tr>
<td>$S$</td>
<td>$\omega_+ = \sqrt{(U + U_{ab})^2 - 4J^2 \frac{U - U_{ab}}{U + U_{ab}}}$</td>
<td>$(-1,1)$</td>
</tr>
<tr>
<td></td>
<td>$\omega_- = \sqrt{(U + U_{ab})^2 - 4J^2}$</td>
<td>$(1,1)$</td>
</tr>
<tr>
<td>$A_n$</td>
<td>$\omega_+ = \sqrt{(U - U_{ab})^2 - 4J^2 \frac{U + U_{ab}}{U - U_{ab}}}$</td>
<td>$(1,1)$</td>
</tr>
<tr>
<td></td>
<td>$\omega_- = \sqrt{(U - U_{ab})^2 - 4J^2}$</td>
<td>$(-1,1)$</td>
</tr>
<tr>
<td>$I$</td>
<td>$\omega_+ = 4J^2 - 2J(U - U_{ab})$</td>
<td>$(-1,1)$</td>
</tr>
<tr>
<td></td>
<td>$\omega_- = 4J^2 - 2J(U + U_{ab})$</td>
<td>$(1,1)$</td>
</tr>
</tbody>
</table>

IV. ADIABATIC EVOLUTION WITH HIGH-FREQUENCY PERIODIC MODULATION

The fixed points will shift accordingly in the phase space if the tunneling strength is tuned adiabatically. Instead of varying the value of $J$ directly, we apply a high-frequency periodic modulation of the energy bias between the two wells,

$$H' = A \sin(\omega_{hf}t) \sum_{\sigma} (\epsilon_{\sigma}^a \epsilon_{\sigma}^a - \epsilon_{\sigma}^b \epsilon_{\sigma}^b),$$

where $A$ and $\omega_{hf}$ stand for the amplitude and frequency of the modulation, respectively. After we take account of the influence of the above modulation, the equations of motion become

$$\dot{\phi}_{A_s} = J \cos(\phi_{A_s} - S) \frac{2 \sqrt{\zeta_a - 1 - \zeta_a}}{\sqrt{\zeta_a - 1 - \zeta_a} + U_{za} + U_{ab} \zeta_b},$$

FIG. 2. (Color online) The stability diagrams of fixed points in the $(U_{ab},J)$ space for $(\phi_{A_s},\phi_{A_s})=(\pi,\pi)$. Here $U_{ab}$ and $J$ are in units of $U$. Panels (a)–(d) correspond to fixed points $S$, $A_n$, $I$, and $A_s$, respectively. The lined areas refer to the parameter space for the existence of each kind of fixed points, and the grayed areas to the stable region for those fixed points accordingly.
\[ \dot{z}_a = -2J \sin(\phi_a - S) \sqrt{1 - z_a^2}, \]
\[ \dot{\phi}_b = J \cos(\phi_b - S) \frac{2z_b}{\sqrt{1 - z_b^2}} + U_z + U_{ab} \sin a, \]
\[ \dot{z}_b = -2J \sin(\phi_b - S) \sqrt{1 - z_b^2}, \]

where \( S = \frac{24}{\omega_0} \cos(\omega_0 t) \). In the high-frequency approximation, \( \omega_{hf} \) being much larger than the intrinsic frequencies \( \omega_a \) and \( \omega_b \), the above equations reduce to a set of equations that are formally the same as Eqs. (3) with \( J \) being replaced by the effective tunneling term \( J_{hf} = J_{0} \frac{24}{\omega_0} \). Here \( J_0 \) denotes the zeroth Bessel function of the first kind. Clearly, the value of \( J_{eff} \) can be tuned by varying the amplitude \( A \).

It is interesting to consider the Rosen-Zener form of \( J_{eff} \), i.e., \( J_{eff} \) increases from zero to its maximum value \( J \) and then decreases to zero again in the end. For pulse-shape \( J_{eff} \), the amplitude \( A \) varies with respect to time in the following form:

\[ A(t) = \begin{cases} 
2.405 \omega_{hf} \left( \frac{1}{2} - \frac{t}{T} \right), & \text{if } 0 \leq t < \frac{T}{2}, \\
2.405 \omega_{hf} \left( \frac{t}{2} - \frac{1}{2} \right), & \text{if } \frac{T}{2} \leq t \leq T,
\end{cases} \]

where the coefficient 2.405 is given by the first zero point of \( J_0 \) and \( T \) is the modulation interval. The adiabatic condition is satisfied when \( T > \max \{ 1/\omega_a, 1/\omega_b \} \). Note that the precise form of \( A \) is not important as long as it satisfies the adiabatic condition; and the present form is chosen just for its simplicity. With this form of \( A \) we plot the time dependence of \( J_{eff} \) in Fig. 3.

The four panels of Fig. 2 are projected into one shown in Fig. 4. For given \( U \) and \( U_{ab} \), the three critical tunneling strengths which divide the parameter space \((U_{ab}, J)\) into four different regions are \( J_{c1} = (U-U_{ab})/(U-U_{ab})/(U+U_{ab})/2 \), \( J_{c2} = (U-U_{ab})/2 \), and \( J_{c3} = (U+U_{ab})/2 \). In the following numerical simulation, we consider two cases for the initial states, namely, symmetrical \((z_a, z_b) = (1,1)\) and antisymmetrical \((z_a, z_b) = (-1,1)\) cases. The former state corresponds to one of fixed points \( S \), while the latter to another fixed points \( A \) at the beginning of the evolution. The initial values of \( \phi_a \) and \( \phi_b \) are the well-defined for these two cases. However, as \( J_{eff} \) increases above zero adiabatically, the symmetry will be broken and the system will collapse into the \((\phi_a, \phi_b) = (\pi, \pi)\) case smoothly just like the single-species case [9]; and, if the adiabatic condition is always satisfied during the evolution, the system will remain in the \((\phi_a, \phi_b) = (\pi, \pi)\) case. That is why we only focused our attention on the \((\phi_a, \phi_b) = (\pi, \pi)\) case in the previous sections.

A. Symmetrical initial state

There exist stable fixed points \( S \) in the three regions I, II, and III shown in Fig. 4. Consequently, among the three critical tunneling strengths, only \( J_{c3} \) is related to the adiabatic evolution of fixed points \( S \) when the values of \( U \) and \( U_{ab} \) are fixed. As shown in Fig. 5, the adiabatic evolutions for \( J < J_{c3} \) or \( J > J_{c3} \) are quite different.

As \( J_{eff} \) increases from zero, the system evolves towards the region IV given in Fig. 4, which corresponds to the approach of fixed points \( S \) to \( I \) in Fig. 1. However, when \( J < J_{c3} \), no merger between fixed points \( S \) and \( I \) occurs. When \( J_{eff} \) decreases to zero again, the system can safely remain on the fixed point and come back to the initial state smoothly. Thus there will be no population transfer as shown in Figs. 5(a) and 5(b). For \( J > J_{c3} \), when \( J_{eff} \) increases above \( J_{c3} \), the fixed points \( S \) and \( I \) merge together and the state becomes fixed point \( I \) which is stable in region IV given in Fig. 4. Additionally, when \( J_{eff} \) decreases below \( J_{c3} \), there are some probabilities for the system to end up in \((z_a, z_b) = (-1,-1)\) which is the other \( S \) fixed point. However, as shown in Figs. 5(c) and 5(d), which symmetrical final state the system will end up depends on the parameters of the system \( U \), \( U_{ab} \) and \( J \) as well as the external periodic modulation. Note that for the symmetrical initial state the whole evolution is just like
If the value of $J$ enters the unstable region as $J_c$ region, the system will enter one of the stable fixed points $S$. As we know that the fixed point $I$ is also unstable in this limit, i.e., $J_{eff}$ increases from zero. However, it will not enter that region when $J < J_c$. When $J_{eff}$ decreases, the system returns to its initial state as shown in Fig. 6(b). When $J > J_c$, the system enters the unstable region as $J_{eff}$ increases above $J_c$. The value of $U_{ab}$ is comparable with $U$, such an instability can drive the evolution to be chaotic as shown in Fig. 6(a) no matter how slow the value of $A$ is varied. For this reason, the final state is beyond the fixed point solutions and becomes totally unpredictable. The adiabatic evolution of fixed points seemingly fails for $J > J_c$ since the unstable region is inevitable if $U_{ab} \neq 0$. However, when the value of $U_{ab}$ is in the weak limit, i.e., $U_{ab} \ll U$, it can produce totally different results.

When $U_{ab} \ll U$ and $J_{cl} < J < J_c$, as $J_{eff}$ increases above $J_{cl}$, the tiny instability can drive the system out of its unstable state, and there are three states (two degenerate symmetrical ones $S$ and the isotropic one $I$) for it to jump into. As we know that the fixed point $I$ is also unstable in this region, the system will enter one of the stable fixed points $S$ indicated by the arrows in Fig. 1(b). When $J_{eff}$ decreases, the system remains in this fixed point and the two BECs will be transferred into one well at the end of the modulation. However, which well it will choose sensitively depends on the concrete parameters of which a slight difference can yield the opposite result as shown in Figs. 6(c) and 6(d). The critical value $J_c$ plays no role here. In the evolution of the particle distribution, as $J_{eff}$ increases above $J_{cl}$, the system has already transitioned to one of the symmetrical fixed points and stays there.

However, if $J > J_3$, all the aforementioned fixed points merge into the isotropic one $I$ which is stable when $J_{eff} > J_3$. Then when $J_{eff}$ decreases below $J_3$ again, the system can enter either the symmetrical fixed points or antisymmetrical ones, which sensitively depends on the relevant parameters as illustrated in Figs. 6(e)–6(h).

**V. SUMMARY AND DISCUSSION**

We studied undistinguishable two-species BECs trapped in double wells in the mean-field approximation and obtained that there exist four types of fixed point solutions rather than two types in the single-species case. The stability of each kind of fixed points was analyzed. The effective tunneling strengths were tuned in the Rosen-Zener type with the help of high-frequency periodic modulation on the energy bias between the two wells. Furthermore, we studied the adiabatic evolution of the particle distribution difference starting from two typical initial states, symmetrical $(z_a,z_b)=(1,1)$ and antisymmetrical $(z_a,z_b)=(1,-1)$. We found that the whole adiabatic evolution is similar to the single-species case [9] for the symmetrical initial state where the role of $U$ is replaced by $U+U_{ab}$ and stays there.

The single-species case [9] while the role of $U$ is replaced by $U+U_{ab}$.

**B. Antisymmetrical initial state**

In contrast to the above symmetrical case, the adiabatic evolution of the particle distribution for the antisymmetrical initial state $(z_a,z_b)=(1,-1)$ is quite different. As shown in Fig. 4, the fixed point $A$ approaches the unstable region II as $J_{eff}$ increases from zero. However, it will not enter that region when $J < J_c$. When $J_{eff}$ decreases, the system returns to its initial state as shown in Fig. 6(b). When $J > J_c$, the system enters the unstable region as $J_{eff}$ increases above $J_c$. If the value of $U_{ab}$ is comparable with $U$, such an instability can drive the evolution to be chaotic as shown in Fig. 6(a) no matter how slow the value of $A$ is varied. For this reason, the final state is beyond the fixed point solutions and becomes totally unpredictable. The adiabatic evolution of fixed points seemingly fails for $J > J_c$ since the unstable region is inevitable if $U_{ab} \neq 0$. However, when the value of $U_{ab}$ is in the weak limit, i.e., $U_{ab} \ll U$, it can produce totally different results.

When $U_{ab} \ll U$ and $J_{cl} < J < J_c$, as $J_{eff}$ increases above $J_{cl}$, the tiny instability can drive the system out of its unstable state, and there are three states (two degenerate symmetrical ones $S$ and the isotropic one $I$) for it to jump into. As we know that the fixed point $I$ is also unstable in this region, the system will enter one of the stable fixed points $S$ indicated by the arrows in Fig. 1(b). When $J_{eff}$ decreases, the system remains in this fixed point and the two BECs will be transferred into one well at the end of the modulation. However, which well it will choose sensitively depends on the concrete parameters of which a slight difference can yield the opposite result as shown in Figs. 6(c) and 6(d). The critical value $J_3$ plays no role here. In the evolution of the particle distribution, as $J_{eff}$ increases above $J_{cl}$, the system has already transitioned to one of the symmetrical fixed points and stays there.

However, if $J > J_3$, all the aforementioned fixed points merge into the isotropic one $I$ which is stable when $J_{eff} > J_3$. Then when $J_{eff}$ decreases below $J_3$ again, the system can enter either the symmetrical fixed points or antisymmetrical ones, which sensitively depends on the relevant parameters as illustrated in Figs. 6(e)–6(h).

**FIG. 5. Time dependence of the particle distribution difference (PDD) for each species between the two wells for the symmetrical initial state $(z_a,z_b)=(1,1)$. The parameters are $U=2$, $U_{ab}=0.4U$, $\omega_0=25U$, and $T=6000/U$. In this case $J_3=0.7U$. (a) $J=0.4U$, (b) $J=0.6U$, (c) $J=0.75U$, and (d) $J=0.8U$.**
BECs that are initially prepared in different wells can be transferred into the same well. However, the sensitivity of the transfer result to the precise parameters of the system should be noticed. We have numerically simulated the evolution process for a wide range of parameters. For the transition shown in Fig. 5, we found that a tiny variation of $J$, i.e., $\Delta J = 0.0003 U$, can produce the opposite transfer result if all other parameters are fixed. This is seemingly a challenge for experiments. We have set the value of $T$ fixed in previous sections, whereas, if we fix the values of all other parameters but to tune the value of $T$, the variation of $T$, i.e., $\Delta T = 24/U$, can also produce the opposite transfer result. For the transitions shown in Figs. 6(c)–6(h), $\Delta J$ and $\Delta T$ are almost of the same order. In the typical BEC experiment with sodium [22], its lifetime is about 1 s, and $1/U = 3.92 \times 10^{-2}$ ms; and in the typical BEC experiment with rubidium [23], its lifetime is about 15 s while $1/U = 9$ ms. For both experiments, the values of $T$ can be both less than the lifetimes, which means the experimental realization of our proposal is possible in both cases.

**ACKNOWLEDGMENTS**

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[21] The same strategy can be generalized to more complicated cases where the interaction strengths and tunneling of the two species are different. More richer situations are expected to appear then.