Atom-to-molecule conversion efficiency and adiabatic fidelity

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The efficiency of converting two-species fermionic atoms into bosonic molecules is investigated in terms of mean-field Lagrangian density. We find that the stimulated Raman adiabatic passage technique aided by Feshbach resonance is more effective than the bare Feshbach resonance for ^{6}Li atoms rather than ^{40}K atoms. We also generally consider the symmetry and its relevant conservation law, which enables us to introduce a natural definition of adiabatic fidelity for a coherent population trapping state. The calculated values of the fidelity are presented and then we provide an explanation of why the conversion efficiencies for ^{40}K and ^{6}Li are distinctly different.

I. INTRODUCTION

There has been much attention paid to the molecular Bose-Einstein condensate (BEC) which is versatile not only for cold atomic physics but also for other research areas, such as condensed matter theory and quantum field theory. It is no longer a pure Bose system as the molecular BEC can be a system of Bose-Fermi mixtures. This makes the theoretical structure rich and colorful. To realize molecular BEC, one must create stable molecules with long lifetimes. In recent experiments [1–3], the technique of Feshbach resonance plays an important role in the creation of molecules. Since the molecules created through such a technique usually suffer from fast decay due to the vibrational excitation, the stimulated Raman adiabatic passage (STIRAP) in photoassociation [4] has been regarded as an effective approach to create ground-state molecules. The success of the STIRAP technique requires the existence of a coherent population trapping (CPT) state which can be followed adiabatically [5]. Such a condition can be fulfilled for linear Λ systems by appropriately choosing laser frequencies. However, for systems with interparticle interactions, the two-photon resonance condition dynamically changes when population is transferred from atomic states to molecular states. This makes the CPT state more difficult to follow adiabatically. The adiabatic property was studied by means of adiabatic fidelity in a recent theoretical work [6] for a simplified model of monoatomic systems without interparticle interaction. It is worthwhile to appropriately define the adiabatic fidelity to study the adiabatic property for a more realistic system, such as the diatomic system which has been realized in several experiments [7–12].

In this paper, we consider systems consisting of fermionic atoms in different hyperfine states and their compounded molecules coupled through the STIRAP technique aided by Feshbach resonance. We discuss systems of ^{6}Li and ^{40}K as concrete examples. In Sec. II, we model the systems with interparticle interaction and derive dynamical equations for them. We make a general consideration on the symmetry and the relevant conservation law, and then introduce the definition of fidelity for our system. In Sec. III, we look for solutions of the CPT state. In Sec. IV, we calculate the conversion efficiency for ^{40}K atoms and ^{6}Li atoms, respectively,

and discuss the corresponding features. In Sec. V, we study the relation between the atom-to-molecule conversion efficiency and the adiabatic fidelity for the CPT state. We also study the effect of the decay of quasibound molecules and compare the difference between ^{40}K and ^{6}Li systems. Our results are briefly summarized in Sec. IV.

II. MODEL AND ITS GENERAL PROPERTIES

We consider that two species of fermionic atoms are converted into stable molecules via the STIRAP technique aided by Feshbach resonance. Here we use $|a\rangle$ and $|b\rangle$ for the states of the free atoms in the open channel and $|m\rangle$ and $|g\rangle$ for the quasibound and ground molecular states in the closed channel, respectively. A pair of fermionic atoms are coupled with the quasibound molecular state $|m\rangle$ through a Feshbach resonance, where the coupling strength is denoted by α' and the detuning by $\mathcal{E}'$. Additionally, the states $|m\rangle$ and $|g\rangle$ are coupled with each other through a laser field with the coupling strength Ω' and detuning Δ'. Since the system is a Fermi-Bose mixture whose Hilbert space has the representation of a graded unitary group SU(2|2), it will be arduous work to study the dynamics of the system by means of the Heisenberg equation of motion. Whereas, with the help of mean-field Lagrange density [13], one can investigate the dynamics conveniently,

$$
\begin{aligned}
L = \sum_i & \left[\frac{i\hbar}{2}\left(\psi_i^* \frac{\partial \psi_i}{\partial t} - \psi_i \frac{\partial \psi_i^*}{\partial t} \right) - T_i \right] - \mathcal{E}' \psi_m^* \psi_m - \alpha'(\psi_m^* \psi_a \psi_b \\
& + \text{H.c.}) + \Omega'(\psi_m^* \psi_g + \text{H.c.}) - \Delta' \psi_g^* \psi_g - \frac{1}{2}\sum_{i\neq j} \lambda_{ij}'|\psi_i|^2|\psi_j|^2 \\
& - \frac{1}{2}(\lambda_{mm}'|\psi_m|^4 + \lambda_{gg}'|\psi_g|^4) - \frac{3}{5}(A_a'|\psi_a|^{10/3} + A_b'|\psi_b|^{10/3}),
\end{aligned}
$$

(1)

where T_i denotes the kinetic energy term of the ith component and $i,j=a,b,m,g$. Here the coefficients $\lambda_{ij}'=\lambda_{ji}'=2\pi\hbar^2 a_{ij}/m_{ij}$ are the interaction strengths between particles with $m_{ij}=m_i m_j/(m_i+m_j)$ being the reduced mass and a_{ij} the s-wave scattering length.

A. The conservation law

The Lagrangian (1) is no longer invariant under a simultaneous global phase transformation due to the presence of the atom-to-molecule conversion term. As the Lagrangian does not include the term flipping the two fermionic components into each other, their corresponding phase parameters are not necessarily the same in general. One can find that the Lagrangian (1) is invariant under the phase transformation $(\psi_a, \psi_b, \psi_m, \psi_g) \mapsto (\psi_a, \psi_b, \psi_m, \psi_g) U(\vartheta_a, \vartheta_b)$ with

$$U(\vartheta_a, \vartheta_b) = \begin{pmatrix} e^{i\vartheta_a} & 0 & 0 & 0 \\ 0 & e^{i\vartheta_b} & 0 & 0 \\ 0 & 0 & e^{i(\vartheta_a+\vartheta_b)} & 0 \\ 0 & 0 & 0 & e^{i(\vartheta_a+\vartheta_b)} \end{pmatrix}. \quad (2)$$

Following the formulism of the Noether theorem, we evaluate the variation of the action caused by the infinitesimal phase transformation $U(\delta\vartheta_a, \delta\vartheta_b)$,

$$\delta I = \int \{L(\psi + \delta\psi, \psi^* + \delta\psi^*, \partial_\mu \psi + \delta\partial_\mu \psi, \partial_\mu \psi^* + \delta\partial_\mu \psi^*) - L(\psi, \psi^*, \partial_\mu \psi, \partial_\mu \psi^*)\} d(x).$$

For simplicity in the above expression, we omitted the subscripts of ψ labeling different components and abbreviated $dtdxdydz$ as $d(x)$ and $\partial/\partial x_\mu$ as ∂_μ with $\{x_\mu\}=\{t,x,y,z\}$. Because of, explicitly, $\delta\psi_a = i(\delta\vartheta_a)\psi_a$, $\delta\psi_b = i(\delta\vartheta_b)\psi_b$, $\delta\psi_{m(g)} = i(\delta\vartheta_a + \delta\vartheta_b)\psi_{m(g)}$, and their complex conjugations, we derive the following result:

$$\delta I = \int \{(\partial_\mu J_A^\mu)\delta\vartheta_a + (\partial_\mu J_B^\mu)\delta\vartheta_b\} d(x), \quad (3)$$

where $J_A^\mu = J_a^\mu + J_m^\mu + J_g^\mu$, $J_B^\mu = J_b^\mu + J_m^\mu + J_g^\mu$ with

$$J_i^\mu = \psi_i \frac{\delta L}{\delta(\partial_\mu \psi_i)} - \psi_i^* \frac{\delta L}{\delta(\partial_\mu \psi_i^*)}, \quad i = a, b, m, g. \quad (4)$$

The system is invariant under the transformation (2) such that $\delta I = 0$, which gives rise to two conserved currents

$$\partial_\mu J_A^\mu = 0, \quad \partial_\mu J_B^\mu = 0. \quad (5)$$

In the present paper, we focus on uniform system ($\nabla\psi_i \approx 0$) and hence neglect the kinetic and trapping potential terms. Then the conservation law (5) reads $d/dt(|\psi_a|^2 + |\psi_m|^2 + |\psi_g|^2) = 0$, $d/dt(|\psi_b|^2 + |\psi_m|^2 + |\psi_g|^2) = 0$, which implies that

$$|\psi_a|^2 + |\psi_m|^2 + |\psi_g|^2 = n_a,$$

$$|\psi_b|^2 + |\psi_m|^2 + |\psi_g|^2 = n_b, \quad (6)$$

where the constants n_a and n_b are determined by V, the volume of the system, together with $N_a(0)$ and $N_b(0)$, the initial numbers of species a and b, i.e., $n_a = N_a(0)/V$, $n_b = N_b(0)/V$. Here we assume that there are no molecules at the initial time in the system.

To guarantee the compatibility with the constraints given by Eq. (6), we introduce two Lagrange multipliers μ_a and μ_b into the mean-field Lagrange density (1)

$$K = L + \mu_a|\psi_a|^2 + \mu_b|\psi_b|^2 + (\mu_a + \mu_b)(|\psi_m|^2 + |\psi_g|^2). \quad (7)$$

Here the real parameters μ_a and μ_b can be identified as the chemical potentials of the corresponding components. Owing to Eq. (6), we can conveniently introduce new notations $\phi_i = \psi_i/\sqrt{n_a + n_b}$, $\lambda_{ij} = \lambda_{ij}'(n_a + n_b)/\hbar$, $\alpha = \alpha'\sqrt{n_a + n_b}/\hbar$, $A_i = A_i'(n_a + n_b)^{2/3}/\hbar$, $\Omega = \Omega'/\hbar$, $\mathcal{E} = \mathcal{E}'/\hbar$, and $\Delta = \Delta'/\hbar$. Reexpressing Eq. (7) in terms of ϕ's and substituting it into the Euler-Lagrange equation, we obtain a set of equations

$$i\frac{\partial\phi_a}{\partial t} = \sum_{i\neq a} \lambda_{ai}|\phi_i|^2\phi_a + A_a|\phi_a|^{4/3}\phi_a + \alpha\phi_b^*\phi_m - \mu_a\phi_a,$$

$$i\frac{\partial\phi_b}{\partial t} = \sum_{i\neq b} \lambda_{bi}|\phi_i|^2\phi_b + A_b|\phi_b|^{4/3}\phi_b + \alpha\phi_a^*\phi_m - \mu_b\phi_b,$$

$$i\frac{\partial\phi_m}{\partial t} = \sum_i \lambda_{mi}|\phi_i|^2\phi_m + \alpha\phi_a\phi_b - \Omega\phi_g + \mathcal{E}\phi_m - i\gamma\phi_m - (\mu_a + \mu_b)\phi_m,$$

$$i\frac{\partial\phi_g}{\partial t} = \sum_i \lambda_{gi}|\phi_i|^2\phi_g - \Omega\phi_m + \Delta\phi_g - (\mu_a + \mu_b)\phi_g, \quad (8)$$

where a phenomenological parameter γ is introduced to characterize the decay of quasibound molecules. In terms of ϕ_i's, the conservation relations (6) turn out to be

$$|\phi_a|^2 + |\phi_m|^2 + |\phi_g|^2 = (1 + \delta)/2,$$

$$|\phi_b|^2 + |\phi_m|^2 + |\phi_g|^2 = (1 - \delta)/2, \quad (9)$$

with $\delta = (n_a - n_b)/(n_a + n_b)$ characterizing the population imbalance between fermionic atoms in different states.

B. The definition of fidelity

Now we are in the position to introduce a proper definition of fidelity for our system. As our system is a four-component system of Bose-Fermi mixture which is related to a graded unitary group SU(2|2), we need to define the fidelity carefully as it must obey several basic properties [16]. Equation (9) provides us the normalization condition

$$|\phi_a|^2 + |\phi_b|^2 + 2|\phi_m|^2 + 2|\phi_g|^2 = 1,$$

which can be expressed as

$$\langle\phi|F^*(\phi)F(\phi)|\phi\rangle = 1, \quad (10)$$

where $\langle\phi|$ denotes $(\phi_a^*, \phi_b^*, \phi_m^*, \phi_g^*)$. One might consider a naive expression for the F matrix $F = \text{diag}(1, 1, \sqrt{2}, \sqrt{2})$. However, because the relation of Eq. (10) should be invariant under the transformation given in Eq. (2), just as the conventional inner product in quantum mechanics is invariant under the U(1) transformation, the simplest correct expression of the F matrix ought to be

$$F(\phi) = \begin{pmatrix} \dfrac{\phi_b}{|\phi_b|} & 0 & 0 & 0 \\[2mm] 0 & \dfrac{\phi_a}{|\phi_a|} & 0 & 0 \\[2mm] 0 & 0 & \sqrt{2} & 0 \\[2mm] 0 & 0 & 0 & \sqrt{2} \end{pmatrix}. \tag{11}$$

As a result, a nature definition of fidelity of a state labeled by ϕ with that labeled by ϕ' is given by

$$f(\phi, \phi') = |\langle \phi | F^*(\phi) F(\phi') | \phi' \rangle|, \tag{12}$$

where the F matrix was given in Eq. (11). Clearly, such a definition fulfills $f(\phi, U(\vartheta_a, \vartheta_b)\phi') = f(U(\vartheta_a, \vartheta_b)\phi, \phi') = f(\phi, \phi')$, which means the phase transformation given in Eq. (2) does not vary the magnitude of fidelity; and the fidelity of a state with itself is always the unit $f(\phi, \phi) = 1$ which is the normalization condition.

III. COHERENT POPULATION TRAPPING STATES

Now we consider stationary states where we neglect the decay of quasibound molecules (i.e., $\gamma = 0$). We know that the existence of stationary solutions of Eq. (8) requires that the system satisfies the adiabatic approximation condition. Once the adiabatic approximation is valid, i.e., $\partial \phi_i / \partial t \approx 0$, the time evolution (8) becomes algebraic for ϕ's, namely,

$$\mu_a \phi_a = \sum_{i \neq a} \lambda_{ai} |\phi_i|^2 \phi_a + A_a |\phi_a|^{4/3} \phi_a + \alpha \phi_b^* \phi_m,$$

$$\mu_b \phi_b = \sum_{i \neq b} \lambda_{bi} |\phi_i|^2 \phi_b + A_b |\phi_b|^{4/3} \phi_b + \alpha \phi_a^* \phi_m,$$

$$(\mu_a + \mu_b)\phi_m = \sum_i \lambda_{mi} |\phi_i|^2 \phi_m + \alpha \phi_a \phi_b - \Omega \phi_g + \mathcal{E} \phi_m,$$

$$(\mu_a + \mu_b)\phi_g = \sum_i \lambda_{gi} |\phi_i|^2 \phi_g - \Omega \phi_m + \Delta \phi_g. \tag{13}$$

Although it is difficult to find the exact solutions of the above equations, one can easily obtain a set of steady-state solutions for Eq. (13) by taking $\phi_m = 0$. Such a state is called the coherent population trapping state which yields

$$|\phi_a^0|^2 = \frac{(\delta - \tilde{\Omega}^2) + \sqrt{(\delta - \tilde{\Omega}^2)^2 + 2\tilde{\Omega}^2(1 + \delta)}}{2},$$

$$|\phi_b^0|^2 = |\phi_a^0|^2 - \delta,$$

$$|\phi_g^0|^2 = \frac{1 + \delta}{2} - |\phi_a^0|^2, \quad \mu_a = \lambda_{ab} |\phi_b^0|^2 + \lambda_{ag} |\phi_g^0|^2 + A_a |\phi_a^0|^{4/3},$$

$$\mu_b = \lambda_{ab} |\phi_a^0|^2 + \lambda_{bg} |\phi_g^0|^2 + A_b |\phi_b^0|^{4/3}, \tag{14}$$

where $\tilde{\Omega} = \Omega / \alpha$. The resonance condition corresponding to this solution is

$$\Delta = (\lambda_{ab} - \lambda_{ag})|\phi_a^0|^2 + (\lambda_{ab} - \lambda_{bg})|\phi_b^0|^2 + (\lambda_{ag} + \lambda_{bg} - \lambda_{gg})$$
$$\times |\phi_g^0|^2 + A_a |\phi_a^0|^{4/3} + A_b |\phi_b^0|^{4/3}. \tag{15}$$

We consider a laser pulse with $\tilde{\Omega} \to \infty$ for $t \to 0$ and $\tilde{\Omega} \to 0$ for $t \to \infty$. For such a laser field, at the initial time ($t = 0$), we have $|\phi_a^0|^2 = (1 + \delta)/2$, $|\phi_b^0|^2 = 1 - \delta/2$ and $|\phi_g^0|^2 = 0$, which implies that there are no molecules in the system at the initial time. At the final time ($t \to \infty$), $|\phi_a^0|^2 \to \delta$, $|\phi_b|^2 \to 0$, and $|\phi_g^0|^2 \to (1 - \delta)/2$ for $\delta > 0$; whereas $|\phi_a^0|^2 \to 0$, $|\phi_b|^2 \to -\delta$, and $|\phi_g^0|^2 \to (1 + \delta)/2$ for $\delta < 0$. With the help of initial values of $|\phi_i^0|^2$ and their asymptotic values at final time, it is easy to find that those fermionic atoms, in the presence of their counterparts, can be converted into molecules if the CPT states can be followed adiabatically. The residual atoms cannot be converted into molecules due to the lack of counterpart atoms. After the numerical calculation in the next section, we will go back to study whether the CPT state can be followed adiabatically with the help of the useful concept of adiabatic fidelity.

IV. CONVERSION EFFICIENCIES

We know that there are two sorts of fermionic atoms ^{6}Li and ^{40}K in the group of alkali-metal atoms. They have been converted into molecules in experiments successfully through the bare Feshbach resonance [7–10]. In those experiments, 60 to 80 % of ^{40}K atoms and no more than 85% of ^{6}Li atoms can be converted into molecules. For ^{6}Li atoms, the atom-to-molecule conversion efficiency via bare Feshbach resonance is lower than that via the STIRAP technique aided by Feshbach resonance, which is in contrast to the case for ^{40}K atoms. Due to the difference in atomic properties, the atom-to-molecule conversion efficiency differs for different atoms even if the same technique is applied.

Now we evaluate the atom-to-molecule conversion efficiency, respectively, for ^{6}Li and ^{40}K atoms with concrete magnetic and laser fields. For these two sorts of atoms, we adopt the same time-dependent Rabi frequency

$$\Omega(t) = \Omega_{\max}\left[1 - \tanh\left(\frac{t - t_0}{\tau}\right)\right], \tag{16}$$

where the parameters $\Omega_{\max}$, t_0, and τ are determined by the applied laser field that couples the two molecular states. In the numerical calculation, the detuning strength Δ is given by Eq. (15). We assume there are no molecules in the system at the initial time, i.e., $\phi_{m,g} = 0$ at $t = 0$.

For ^{40}K atoms, we know that the Feshbach resonance occurs at a magnetic field strength of 202.1 G and the resonance width is about 7.8 G [14]. Then we can get the atom-to-molecule coupling strength $\alpha' = 16.6 \times 10^{-39}$ J according to Ref. [13]. Here we choose that the magnetic field is 201.7 G and particle density $n_a + n_b$ is about 10^{20} m^{-3}. It is easy to obtain $A_a = 0.16\alpha$, $\lambda_{ab} = 0.24\alpha$, and $\mathcal{E} = -4.4\alpha$. The time evolution of the corresponding population can be obtained by solving Eq. (8). We plot the numerical results in Fig. 1. From this figure, we can see that the conversion efficiency $2|\phi_g(t = \infty)|^2$ for ^{40}K atoms is less than 60%, lower than that via the technique of bare Feshbach resonance [10]. The low conver-
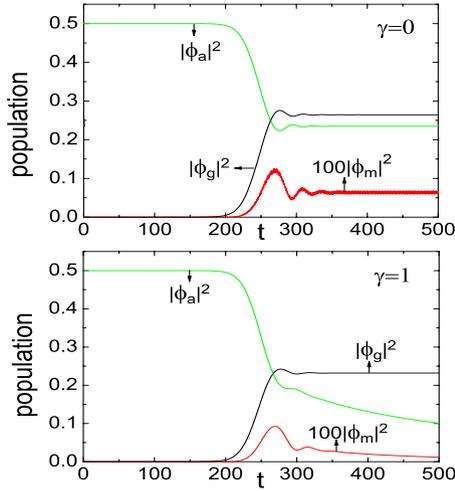
FIG. 1. (Color online) The time dependence of the population of particles for ^{40}K system for different γ. The parameters are $\delta=0$, $\lambda_{ab}=0.24$, $A_a=A_b=0.16$, $\mathcal{E}=-4.4$, $\Omega_{\max}=200$, $t_0=120$, $\tau=40$, and the other parameters are zero. Time is in unit of $1/\alpha$ and all other coefficients are in units of α, where $\alpha=16.6\times10^{-29}$ J.

sion efficiency implies that the CPT state cannot be followed adiabatically, which will be confirmed confidently by evaluating the adiabatic fidelity in the next section. Comparing the two panels in Fig. 1, we can know that the influence of γ on the atom-to-molecule conversion efficiency is very small. This is due to the fact that $|\phi_m|^2$ is close to zero at any time in contrast to the case for ^{6}Li atoms.

For ^{6}Li atoms, the Feshbach resonance occurs at two distinct strengths of magnetic field (the so-called narrow and broad Feshbach resonances, respectively). In our calculation, we focus on the narrow Feshbach resonance for which the atom-to-molecule coupling strength α' is about 3.29×10^{-37} J. If the particle density n_a+n_b is about 10^{20} m^{-3} and the magnetic field is about 543.6 G, one can get $A_a =0.055\alpha$, $\lambda'_{ab}=0.0027\alpha$, and $\mathcal{E}=-0.1125\alpha$. Figure 2 shows the time evolution of particle populations for the conversion of ^{6}Li atoms into molecules. From the top panel, we find that almost all of ^{6}Li atoms can be converted into molecules. For ^{6}Li atoms, the atom-to-molecule conversion efficiency via the STIRAP technique aided by Feshbach resonance is higher than that via bare Feshbach resonance [8,9], which is in contrast to the case for ^{40}K atoms. From the bottom panel, we can see that the atom-to-molecule conversion efficiency decreases distinctly due to the existence of the quasibound molecular decay.

The dependence of the conversion efficiency on the decay rate γ of quasibound molecules for both ^{40}K and ^{6}Li is plotted in Fig. 3. Clearly, the influence of the decay on the conversion efficiency for ^{6}Li atoms is more distinct than that for ^{40}K atoms, which confirms the previous interpretation.

V. ADIABATIC FIDELITY FOR CPT STATES

From the above numerical results, we can find that the atom-to-molecule conversion efficiencies of the STIRAP technique aided by Feshbach resonance for ^{40}K and ^{6}Li are
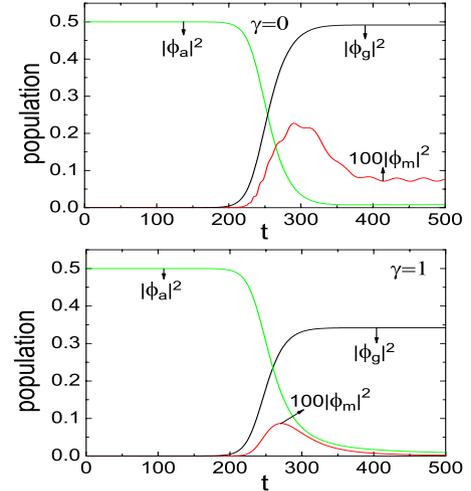

FIG. 2. (Color online) The time dependence of the population of particles for the ^{6}Li system for different γ. The parameters are $\delta =0$, $\lambda_{ab}=0.0027$, $A_a=A_b=0.055$, $\mathcal{E}=-0.1125$, $\Omega_{\max}=200$, $t_0=120$, $\tau=40$, and the other parameters are zero. Time is in units of $1/\alpha$ and all other coefficients are in units of α, where $\alpha=3.29 \times 10^{-27}$ J.

distinctly different although the CPT states are assumed to exist for the two systems. As we know, most of the atoms can be converted into molecules only when the CPT state is followed adiabatically and the existence of the CPT state does not guarantee that the state can be followed adiabatically [15]. The adiabatic properties for the atom-to-molecule conversion have been studied in Ref. [4]. Whereas we cannot apply this method to our system since all the possible solutions of the stationary Eq. (8) must be known in the approach [4] and it is difficult to obtain the other solutions beyond the CPT-state solution in our system.

Recently, the fidelity was employed [6] to characterize the adiabatic condition for systems with atom-to-molecule conversion. The key point is that the value of the fidelity should be close to unity if the system can adiabatically evolve in the CPT state. Now we evaluate the adiabatic fidelity $f(\phi^0(t),\phi(t))$ for our model, in which $\phi(t)$ is the wave function obtained by solving the dynamical Eq. (8) with $\gamma=0$, and $\phi^0(t)$ is the wave function corresponding to the CPT state which is given in Eq. (14). The time evolution of the adiabatic fidelity for CPT state is plotted in Fig. 4(a) for ^{40}K
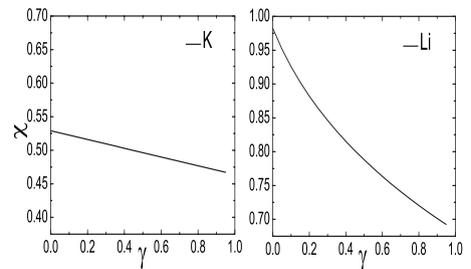

FIG. 3. The atom-to-molecule conversion efficiency versus the decay rate of the quasibound molecules for the ^{40}K (left panel) and ^{6}Li (right panel) systems. The parameter choice for the left (right) panel is the same as in Fig. 1 (Fig. 2).
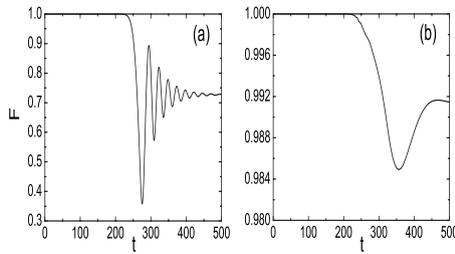
FIG. 4. The time dependence of adiabatic fidelity for the CPT state: (a) for ^{40}K, the same parameter choice as in Fig. 1; (b) for ^{6}Li, the same parameter choice as in Fig. 2.

system. One can see that the magnitude of fidelity is about 1 at the initial time, which implies the system adiabatically evolves along CPT state. At the time $275/\alpha$, the fidelity diminishes to the minimal value 0.35, which implies that the system deviates away from the CPT state distinctly at that time. Although the fidelity begins to fluctuate later on, its final value is still no more than 0.75. The above analysis implies that the CPT state cannot be followed adiabatically, so the atom-to-molecule conversion efficiency is not high for ^{40}K atoms. Figure 4(b) shows the time evolution of the adiabatic fidelity for the CPT state corresponding to the conversion of ^{6}Li atoms into molecules. Obviously, the fidelity for this system is very close to 1 at all times, which implies that the system adiabatically evolves along the CPT state. This result is consistent with the fact that almost all of the ^{6}Li atoms can be converted into molecules. Since the fidelity for the CPT state can reflect the atom-to-molecule conversion efficiency well, one can improve the conversion efficiency by optimizing the parameters of the system to achieve a higher adiabatic fidelity.

VI. SUMMARY

With the help of mean-field Lagrangian density, we studied the conversion of two-species fermionic atoms into bosonic molecules via the STIRAP technique aided by Feshbach resonance. We calculated conversion efficiencies for ^{40}K and ^{6}Li systems and found that almost all of the ^{6}Li atoms can be converted into molecules, which implies that the STIRAP technique aided by Feshbach resonance is more effective than the bare Feshbach resonance for ^{6}Li atoms rather than ^{40}K atoms. We also compared the influence of the decay rate of quasibound molecules on the conversion efficiency for ^{40}K and ^{6}Li systems, respectively, and found that there is a big difference between them. The success of the STIRAP technique not only requires the existence of the CPT state but also requires that the system can adiabatically evolve within such a state.

The adiabatic fidelity was recently recognized to be a useful measurement for characterizing the adiabatic properties. Our analysis of the symmetry and the corresponding conservation law for the systems under consideration helped us to introduce an appropriate definition of adiabatic fidelity for the CPT state. For the ^{40}K system, the CPT state cannot be adiabatically followed since the fidelity was found to be less than 0.75 at final time, which is well consistent with the corresponding low conversion efficiency. In order to improve the conversion efficiency in the ^{40}K system, one should achieve a higher fidelity through optimizing the parameters of the system. Whereas, for ^{6}Li system, the fidelity is very close to 1, hence the CPT state can be followed adiabatically, which is the reason for a high conversion efficiency. Our evaluation of the adiabatic fidelity enables us to understand why the conversion efficiencies for ^{40}K and ^{6}Li are distinctly different.

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