Control of spiral breakup by an alternating advective field

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The control of spiral breakup due to Doppler instability is investigated. It is found that applying an alternating advective field with suitable amplitude and period can prevent the breakup of spiral waves. Further numerical simulations show that the growing meandering behavior of a spiral tip caused by decreasing the excitability of the medium can be efficiently suppressed by the alternating advective field, which inhibits the breakup of spiral waves eventually. © *2006 American Institute of Physics*. [DOI: 10.1063/1.2397075]

Spiral waves represent a typical example of self-organizing structures frequently encountered in various excitable media, such as the Belousov-Zhabotinsky (BZ) reaction,[1] the catalytic surface processes,[2] the cardiac tissue,[3] as well as the retinas.[4] The BZ reaction turned out to be the most suitable laboratory system to study the dynamics of spiral waves and to elaborate adequate means of control.

An understanding of the transition from spiral waves to spiral turbulence has been of great importance. Different spiral breakup scenarios are documented in experiments (mostly in chemical reaction-diffusion systems) and simulations, for example, the Eckhaus instability,[5] the line instability,[6] and the Doppler instability.[7,8] Up to now, various methods to control two- and three-dimensional spiral turbulences have been put forward.[9–13] However, the controlling approach to prevent spiral breakup is still poor and open. It is necessary to develop some efficient and practical control schemes to eliminate such events. Recently, Rappel *et al.*[14] prevented spiral breakup by applying a feedback current at a discrete set of control points. Xiao *et al.*[15] showed that the spiral breakup can also be controlled by injecting a time-delayed feedback signal into a local region around the spiral tip.

Advective fields are known to have pronounced effects on behaviors of spiral waves.[16–23] For instance, both constant and alternating advective fields can induce the spiral to drift: Steinbock *et al.*[16] and Agladze and Kepper[17] studied the drift of spiral waves induced by weak constant advective fields; Munuzuri *et al.*[18] investigated the resonance drift of spiral waves subjected to a weak alternating advective field (AAF) in the Belousov-Zhabotinsky reaction. In our recent work,[21] we derived an approximate formula of the advective-field-induced drift velocity. With this formula we are able to explain the main features appearing in the spiral drift problems. On the other hand, as the medium is superimposed by an AAF with an amplitude exceeding a critical value, spiral waves will break up and evolve into a disordered situation.[22,23] And the critical value of the amplitude depends on the frequency of the applied advective field.

In this paper, besides the above-mentioned spiral drift and spiral breakup induced by AAF, we will show a new effect on spiral waves: with suitable frequency and amplitude, AAF can suppress the Doppler instability of spiral waves effectively.

Let us consider the effect of an advective field on spiral dynamics using a two-variable reaction-diffusion model:

$$\frac{\partial u}{\partial t} = \frac{1}{\varepsilon} f(u,v) + D_u \Delta u + M_u E \frac{\partial u}{\partial x},$$
$$\frac{\partial v}{\partial t} = g(u,v) + D_v \Delta v + M_v E \frac{\partial v}{\partial x}, \qquad (1)$$

where the variables u and v can be viewed as the "fast" and "slow" variables, $1/\varepsilon$ is a parameter characterizing the excitability of the medium, E is the advective field aligned along the x axis, and the parameters M_u and M_v are the ion mobilities. In our numerical simulations, a modified FitzHugh-Nagumo model[8] is considered, where $D_u=1$, $M_u=1$ and $D_v=0$, $M_v=0$, $f(u,v)=-u(u-1)[u-(v+b)/a]$, and $g(u,v)=-v$, if $0 \le u < 1/3$; $g(u,v)=1-6.75u(u-1)^2-v$, if $1/3 \le u \le 1$; $g(u,v)=1-v$, if $u>1$. The simulation is performed on a sys-

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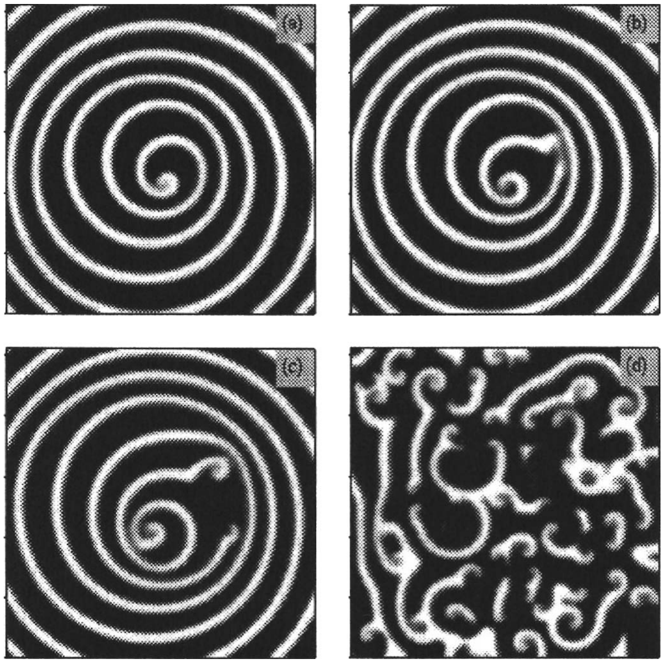

FIG. 1. The evolution of spiral turbulence undergoing a Doppler instability. The excitability parameter $\varepsilon=0.072>\varepsilon_{C1}=0.0715$: (a) $t=0$ (a fully developed spiral at $\varepsilon=0.071$ just below the critical value ε_{C1} as the initial pattern), (b) $t=42$, (c) $t=50$, and (d) $t=140$.

tem of 100×100 size with no-flux boundary conditions. Discretizations with $\Delta x=\Delta y=0.39$ and $\Delta t=0.02$ have been used in an Euler scheme.

The spiral dynamics in the modified FitzHugh-Nagumo model[8] is characterized by the excitability that is defined by the inverse of ε and quantifies the relative excitation and recovery rates. The spatiotemporal dynamics ($E=0$) is investigated by varying ε and fixing $a=0.84$ and $b=0.07$. In the range $0.01<\varepsilon<0.06$, suitable initial conditions lead to steadily rotating spiral waves. At $\varepsilon=0.06$, the spiral undergoes the supercritical Hopf bifurcation and indicates quasiperiodic meandering. For $\varepsilon>\varepsilon_{C1}=0.0715$, growing Doppler effect destabilizes the spiral, which leads to the spiral breakup and makes it evolve into a turbulence state. Figure 1 shows the process of spiral breakup resulting from Doppler instability: With the evolution of time, the compressed waves approach each other; overwhelmingly, they collide with each other in the region near the tip [see Fig. 1(b)]. The spiral starts to breakup and generate a pair of defects, as shown in Fig. 1(c); the newly generated defects drift apart and self-organize into new spirals; similarly, the newly generated spirals also break up and form defects. At last, a spatiotemporal chaos is observed in Fig. 1(d).

The AAF is added in the following procedure: At first, we get a fully developed spiral at $\varepsilon=0.071$ just below the critical value ε_{C1} as the initial pattern; then, we increase ε larger than ε_{C1} in a jump and apply an AAF at the same time. Without control, the spiral will break up and evolve into a chaotic state after ε is changed from 0.071 to 0.073. When an AAF $[E=A\cos(2\pi t/T_E)]$ is applied to the system, the phenomenon is dramatically changed. In Fig. 2, the value of ε is changed directly from 0.071 to 0.073 and an AAF with suitable frequency and amplitude is added at the same time.

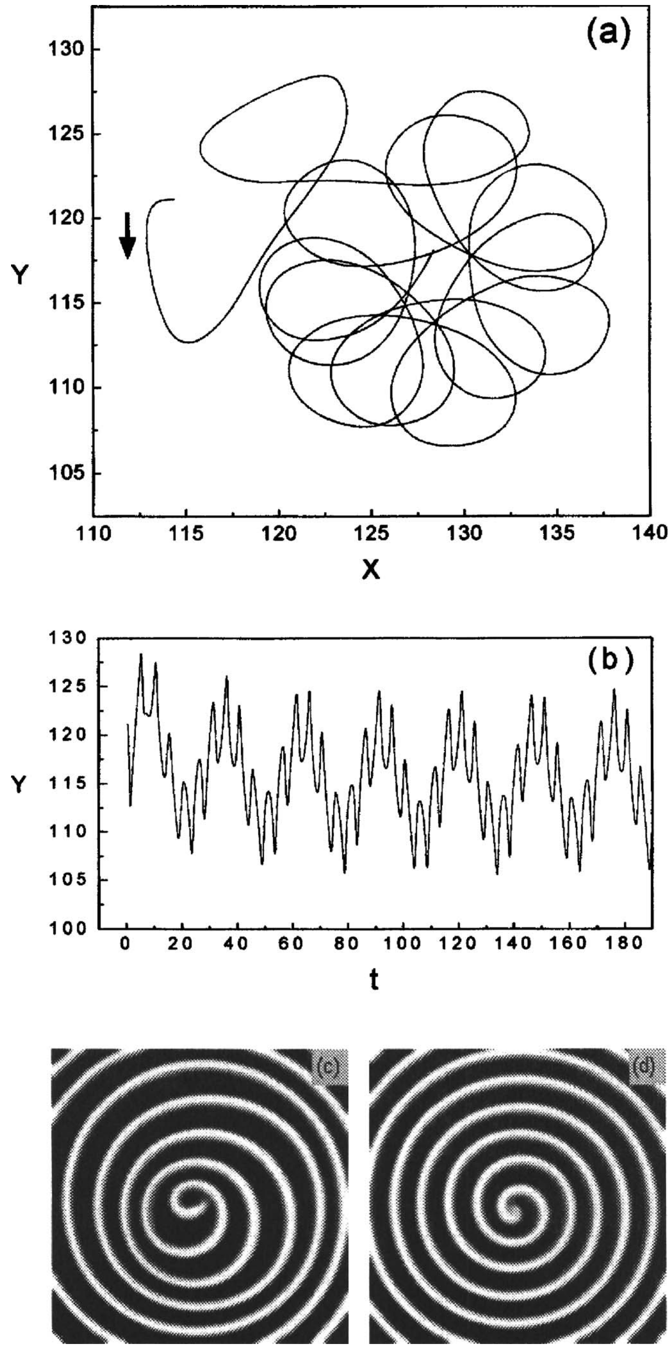

FIG. 2. Control of spiral breakup ($\varepsilon=0.073$) by an AAF $[E=A\cos(2\pi t/T_E)]$ with $A=0.5$ and $T_E=5.0$. (a) Orbit of spiral tip from $t=0$ to $t=60$. (b) The evolution of the y coordinate of the spiral tip from $t=0$ to $t=190$. Spiral waves at times (c) $t=0$ (a developed spiral at $\varepsilon=0.071$) and (d) $t=990$.

Strikingly, the effort to prevent spiral breakup succeeds. Figures 2(a) and 2(b) show the corresponding orbit of the spiral tip after AAF is applied. The changed flower petals indicate that the spiral is stabilized and still meanders. Figure 2(d) shows that the spiral wave is stable at low excitability after a long time evolution. Comparing the initial spiral [Fig. 2(c)] with that in Fig. 2(d), one distinctive phenomenon is that the meandering behavior is modulated. The dilated and compressed wave fronts are moderated.

The relation between the period of external force and that of the spiral is vital to the dynamics behavior of the

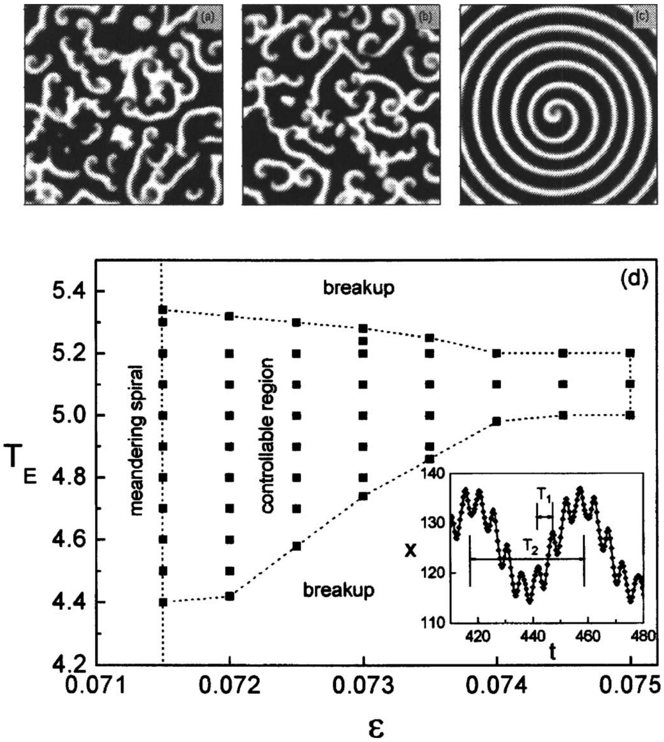

FIG. 3. Pattern at $\varepsilon=0.073$ under the influence of an AAF with $A=0.5$ and different periods (T_E) at $t=1000$: (a) $T_E=5.8$, (b) $T_E=4.4$, and (c) $T_E=5.0$. (d) The controllable phase diagram in the ε-T_E plane. Inset: the orbit of the spiral tip in the x direction without control ($\varepsilon=0.071$). Numerical simulations show that $T_1\approx5.0$ and $T_2\approx40.0$.

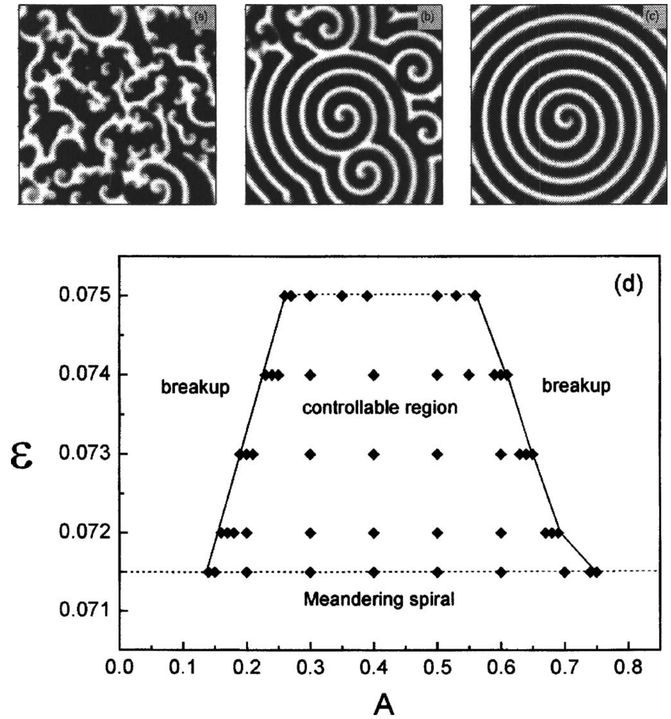

FIG. 4. Pattern controlled by AAF with $T_E=5.0$ and different amplitudes at $t=1000$ ($\varepsilon=0.074$): (a) $A=0.2$, (b) $A=0.7$, and (c) $A=0.4$. (d) A-ε phase diagram.

system and has been studied widely, such as resonant pattern formation in chemical system,[24,25] resonant drift behavior of spiral waves,[18–21] etc. In order to stabilize the spiral waves in low excitability, we must select suitable periods T_E of AAF. Too large T_E is just similar to the constant advective field and cannot attain the goal to prevent the spiral breakup, which is demonstrated in Fig. 3(a). Also, the spiral cannot be stabilized with small T_E [see Fig. 3(b), the final turbulence state is shown]. In the inset of Fig. 3(d), we show the orbit of the spiral tip in the x direction without control at $\varepsilon=0.071$. The meandering orbit shows two periods: the first period $T_1\approx5.0$ and the second period $T_2\approx40.0$ that is induced by Hopf bifurcation. Numerical simulations indicate that when T_E is in the vicinity of the first period T_1 of the initial spiral waves, the control of spiral breakup may be achieved.

The dependence of T_E on the excitability is presented in Fig. 3(d). For a given ε, there exist two critical periods of AAF. For example, at $\varepsilon=0.073$, the high critical value $T_{C\uparrow}$ is 5.28 and the low critical value $T_{C\downarrow}$ is 4.74. One can see that the controllable region of T_E becomes small as the value of ε is increased. Once ε is increased to $\varepsilon_{C2}=0.076$, the spiral waves can no longer be stabilized by AAF.

Not all the values of the amplitude of AAF are valid to keep the spiral stable in low excitability. It is known that too strong AAF can cause a stable spiral to break up.[23] On the other hand, weak AAF has less effect on spiral waves and does not work. To verify this point, a weak AAF (amplitude A is 0.2, below the low critical value $A_{C\downarrow}=0.23$) is added when ε is changed to 0.074 in Fig. 4(a). The small AAF fails to suppress the spiral breakup and the system evolves into a disorder state at last. As the amplitude ($A=0.7$) exceeds the high critical value ($A_{C\uparrow}=0.61$), the effort to prevent the spiral breakup cannot be obtained [see Fig. 4(b)]. Only with suitable amplitude can the spiral breakup be suppressed and then the spiral can be stable finally [Fig. 4(c)]. In Fig. 4(d), we present the dependence of the amplitudes of AAF on ε. One can see that the controllable region between $A_{C\downarrow}$ and $A_{C\uparrow}$ is decreased as the value of ε is increased. When ε is increased to $\varepsilon_{C2}=0.076$, AAF cannot stabilize the spiral waves any more.

In order to give insight into the mechanism underlying the suppression of spiral breakup via AAF, we present the orbit of the spiral tip in different excitabilities without control and that with AAF in Fig. 5. One can see that the tip follows a hypocycloid trace and the region of meandering motion increases as the excitability is decreased [Figs. 5(a) and 5(b)]. After a suitable AAF is switched on, the tip also follows a hypocycloid trace finally, but the region of meandering motion considerably decreases [see Figs. 5(c) and 5(d)].

In Fig. 6, we plot the radius of the region of meandering motion to make a quantitative analysis. We find a sudden decrease of the radius after the system is subjected to a suitable AAF, showing a large gap. The gap ΔR is about 1/3 of the original radius value at $\varepsilon=0.071$ without control. We conclude that it is the efficient decreasing of the highly meandering motion of the spiral tip by AAF that stabilizes the spiral waves. Then, the parameter region of low excitability that supports the stable spiral is extended.

In all the above discussions, we choose a fully developed spiral at $\varepsilon=0.071$ as the initial condition; then, we increase ε in a jump and apply AAF at the same time. In the following

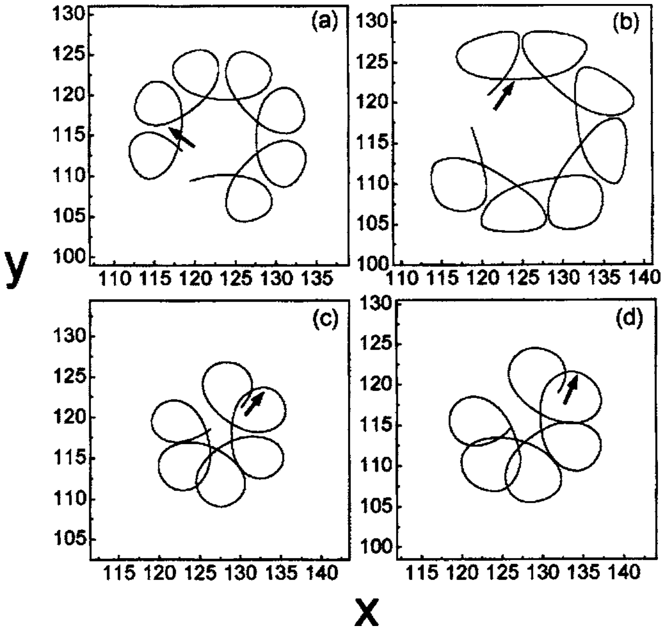

FIG. 5. Orbits of the spiral tip without control: (a) $\varepsilon=0.068$ and (b) $\varepsilon=0.071$. Orbits controlled by an AAF with $A=0.3$ and $T_E=5.0$: (c) $\varepsilon=0.071$ and (d) $\varepsilon=0.073$.

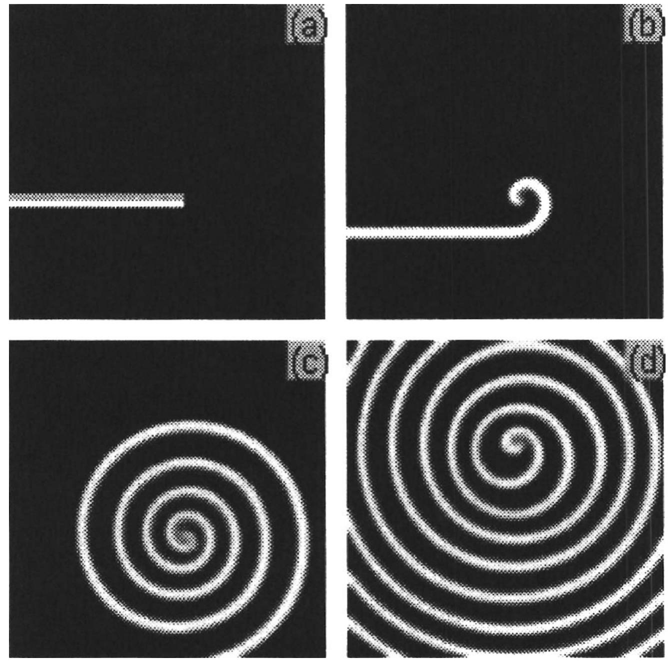

FIG. 7. Development of a spiral ($\varepsilon=0.074$) under the influence of an AAF with $A=0.3$ and $T_E=5.0$: (a) $t=0$, (b) $t=5$, (c) $t=20$, and (d) $t=1000$.

discussion, we choose a locally perturbed rest state [Fig. 7(a)] as the initial condition and study the effect of AAF on the stability of the system. Without control, a spiral will be developed from this initial condition for $\varepsilon<\varepsilon_{C1}$, while the system will evolve into spiral turbulence for $\varepsilon>\varepsilon_{C1}$.[8] In Fig. 7, starting the system from this initial condition, taking a value for the excitability parameter $\varepsilon=0.074$ (larger than $\varepsilon_{C1}=0.0715$), and applying an AAF with $A=0.3$ and $T_E=5.0$ (in the controllable region of Fig. 3) from $t=0$, one can see that the system is stable and a spiral is fully developed at last.

In Ref. 10, the authors showed that spiral turbulence can be eliminated by applying periodic force uniformly. We also do some numerical simulations to test whether AAF can

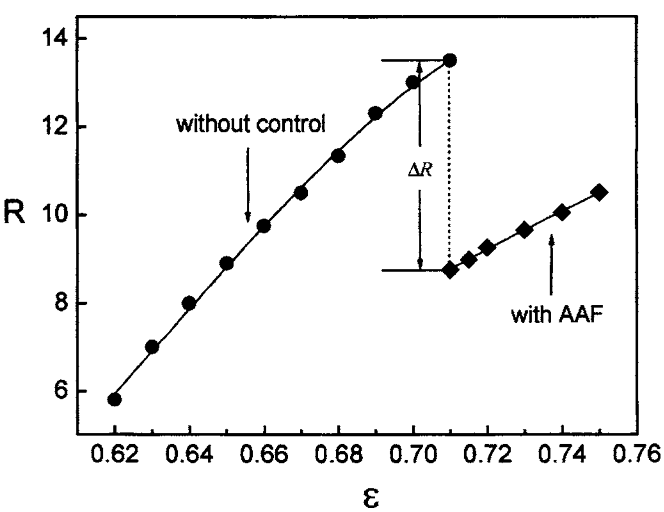

FIG. 6. The radius of the region of meandering motion of the spiral tip at different excitabilities. The circular dots show the radius without control and the diamond dots the radius under the control of an AAF with $A=0.3$ and $T_E=5.0$. The radius gap $\Delta R=4.8$ at $\varepsilon=0.071$.

eliminate turbulence in the Bar model (1). In a large range of the amplitude and the frequency of the AAF, we do not find that AAF can eliminate spiral turbulence. So, unlike the uniform periodic force,[10] AAF is able to avoid spiral breakup, but it is not able to eliminate spiral turbulence.

In summary, we have proposed a simple method to prevent spiral breakup induced by Doppler effects. As the amplitude of AAF is suitable and the period is in the vicinity of the first period T_1 of the initial spiral waves, we can stabilize the unstable spiral. After the AAF is applied, the changed flower petals of tip motion indicate that preventing spiral breakup is a result of the decrease of meandering motion of the spiral tip. Finally, we hope that our results can be observed in experiments, such as in the BZ reaction.[7]

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