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Rapidly rotating Bose-Einstein condensates in anharmonic confinement

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Abstract: We study a rapidly rotating Bose-Einstein condensate in anharmonic confinement and find that many properties, such as the critical rotating frequency and phase diagram, are different from those in a harmonic trap. We investigate the phase transitions between various vortex lattices and find that a hole emerges in the center of the cloud when the rotating frequency Ω reaches Ω_h but it becomes invisible when $\Omega > 1.0842\omega_\perp$.

Keywords: Rotating Bose-Einstein condensate, critical rotational frequency, vortex lattice, vortex hole
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1 Introduction

There have been many studies on Bose-Einstein Condensates (BECs) rotating with high angular momentum, of which many features resemble some of the novel solid-state systems such as type-II superconductors and quantum Hall liquids. In a number of earlier experiments [1–4] where the gases are confined in harmonic traps, the energetically favorite state takes the form of a triangular lattice of singly quantized vortices [5]. For a harmonic trapping potential, the centrifugal force will exactly cancel the trapping force and the system will collapse if Ω reaches the trap frequency ω_{xy}. Thus the centrifugal force prevents the rotation frequency Ω from being enhanced over ω_{xy}. This limit makes it impossible for the angular momentum to reach a higher magnitude, as expected. To resolve this problem, some theoretical studies suggest that Ω is no longer bounded by

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ω_{xy} once an anharmonic term is introduced into the trapping potential. Vortices are successfully created experimentally [6] by adding a quartic term in the trapping potential where phase transitions are investigated by varying Ω. Experimental images make the theoretical suggestion that the picture should become richer in an anharmonic trap more conceivable [7–14]. Indeed, for $\Omega > \Omega_c$ (Ω_c is the critical frequency), the gas can only form an array of regular vortices in a harmonic trap. However, in an anharmonic trap, it can also be a state of multiple quantization and a mixed state beyond the above state, The mixed state consists of a multiply quantized vortex (hole) at the center of the cloud and singly-quantized vortices around the hole.

Currently, people's interest extends to fast rotating BECs which exhibit richer pictures. The rotation of a Bose-Einstein condensate in a qudratic-plus-quartic trap was studied experimentally [6]. The experiment[6] presented puzzling pictures: the vortex hole can be generated when the rotation frequency Ω reaches some value Ω_h, the number of vortices which is visible becomes smaller and smaller with the increasing frequency Ω, and the vortex lattice becomes invisible when Ω reaches some value. In our paper, we give the theoretical explanation for the puzzling experiment. By means of a variational method we not only find the critical frequency Ω_h but also explain why the vortex lattice becomes invisible with increasing Ω. Our paper is organized into five sections. In section III, we determine the critical frequency Ω_c by means of a variational method which differs from that in a harmonic trap. We prove that the system in an anharmonic trap can indeed form a multiply-quantized state [15]. In section IV, we extend the approach proposed by Ho [5] to investigate the phase transitions between the mixed state and the state with a regular vortex lattice. Comparing energies for different l, we obtain Ω_h, above which a hole is generated at the center of the cloud. Increasing Ω will make the hole gradually absorb the singly-quantized vortices around it and finally form a giant vortex state.

2 The model

We consider a Bose-Einstein Condensate confined in an anharmonic trap whose potential is well approximated by a superposition of a quadratic and a quartic potential [16]:

$$v(\vec{r}) \simeq \frac{1}{2}M\omega_z^2 z^2 + \frac{1}{2}M\omega_{xy}^2 r^2 + \frac{M\omega_{xy}^2}{2a_\perp^2}\lambda r^4, \quad (\lambda > 0), \tag{1}$$

where M is the atomic mass; ω_z and ω_{xy} are the frequencies of the harmonic potentials along the z-axis and in the xy plane $\mathbf{r} \equiv (x, y)$ respectively; $a_\perp = [\hbar/(M\omega_{xy})]^{\frac{1}{2}}$ denotes the oscillator length and λ is a small dimensionless parameter characterizing the strength of the quartic potential which is $\lambda \approx 10^{-3}$ in the experiment [6]. The energy functional is given by

$$E[\Psi] = \int d^3r \Psi^*(h_z + h_\perp)\Psi + \frac{g}{2}\int d^3r|\Psi|^4, \tag{2}$$

where Ψ is the condensate wave function, h_z and $h_\perp$ refer to single-particle Hamiltonians along z-axis and in the xy plane, namely $h_z = -(\hbar^2/2M)\nabla_z^2 + (M\omega_z^2 z^2)/2$ and $h_\perp = -(\hbar^2/2M)\nabla_\perp^2 + (M\omega_{xy}^2 r^2)/2 + (M\omega_{xy}^2\lambda r^4)/(2a_\perp^2)$.

To find the optimal form of Ψ for a rapidly rotating system, we ought to minimize the energy Eq.(2) subject to two constraints. Since both the total number of particles and angular momentum are constants, two Lagrange multipliers are introduced such that $\delta(E - \mu N - \Omega l_z) = 0$. Here the chemical potential μ and rotation frequency Ω of the trap can guarantee the particle number as well as angular momentum to be constants, respectively [17]. We therefore solve the condensate wave function by minimizing the following Gross-Pitaevskii functional:

$$K = \int d^3r\Psi^*[h_z + h_\perp - \Omega L_z - \mu]\Psi + \frac{g}{2}\int d^3r|\Psi|^4, \tag{3}$$

in which L_z is the angular momentum along the positive direction of the z-axis ; $g = (4\pi\hbar^2 a_{sc})/M$ is the strength of the effective two-body interaction with a_{sc} being the s-wave scatting length.

3 Critical rotational frequency

In reality, for a large condensate, the density profile along z is similar to the stationary case. Since the gas rotates around the z-axis and its features in the xy plane are more interesting, we simply assume that the particle density along z is a constant ρ [16] so that we have a two dimensional quantum system (e.g., $z = 0$ plane without loss of generality).

It is instructive to consider the limiting case $g = 0$ and $\lambda = 0$ in which the Hamiltonian in the xy plane is similar to that in the quantum Hall regime. In this case the eigenenergy gives rise to Landau Levels for which the wave function Φ_m for the lowest Landau Level (LLL) reads: $\Phi_m \propto r^m \exp(im\phi)\exp(-r^2/2a_\perp^2)$ [18]. In the experiment, the g and λ are nonzero but their magnitudes are relatively small, so we can solve the problem by means of a variational method. Let us write the variational wave function as $\Phi = Cr^m \exp(im\phi)\exp(-\beta^2 r^2/2a_\perp^2)$ with variational parameter β. Here $C = \beta^{m+1}/[a_\perp^m(\pi a_\perp{}^2 m!)^{1/2}]$ is the normalization constant. The energy for the state $\Psi = \sqrt{\rho}\Phi$ is given by (in units of $\rho\hbar\omega_{xy}$)

$$\begin{aligned} K &= (\frac{1}{2}(\beta^2 + \frac{1}{\beta^2}) - \frac{\Omega}{\omega_{xy}})\Gamma_1(m) + \frac{\lambda}{2\beta^4}\Gamma_2(m) \\ &\quad + \rho a_{sc}\beta^2\Gamma_3(m) + (\frac{\Omega}{\omega_{xy}} - \mu), \end{aligned} \tag{4}$$

where $\Gamma_1(m) = \beta^2\langle r^2\rangle/a_\perp^2 = m + 1$ refers to the width of the wave function in the xy plane in units of $\beta^2/a_\perp^2$, $\Gamma_2(m) = \beta^4\langle r^4\rangle/a_\perp^4 = (m+1)(m+2)$ refers to the expectation value of r^4, and $\Gamma_3(m) = 2\pi a_\perp^2/\beta^2 \int d^2r \mid \Phi \mid^4 = 2m!/(2^{2m}(m!)^2)$ refers to the interaction contribution. Minimizing the energy with respect to the variational parameter β, we obtain the following two equations in β for the cases of $m = 0$ (non-vortex configuration) and $m = 1$ (one vortex of unit strength), respectively:

$$\begin{aligned} \beta_0^6 - \beta_0^2 - 4\lambda + 2\rho a_{sc}\beta_0^6 &= 0, \\ 2\beta_1^6 - 2\beta_1^2 - 12\lambda + \rho a_{sc}\beta_1^6 &= 0. \end{aligned} \tag{5}$$

Their corresponding energies are

$$\begin{aligned} K_0 &= (\frac{1}{2}(\beta_0^2 + \frac{1}{\beta_0^2}) - \frac{\Omega}{\omega_{xy}}) + \frac{\lambda}{2\beta_0^4} \times 2 + \rho a_{sc}\beta_0^2 \\ &+(\frac{\Omega}{\omega_{xy}} - \mu), \\ K_1 &= (\frac{1}{2}(\beta_1^2 + \frac{1}{\beta_1^2}) - \frac{\Omega}{\omega_{xy}}) \times 2 + \frac{\lambda}{2\beta_1^4} \times 6 + \rho a_{sc}\beta_1^2 \times \frac{1}{2} \\ &+ (\frac{\Omega}{\omega_{xy}} - \mu). \end{aligned} \tag{6}$$

The experimental data: $\lambda = 0.001$, $N = 3 \times 10^5$, $\omega_{xy} = 2\pi \times 65.6$Hz, $\omega_z = 2\pi \times 11.0$Hz, and $a_{sc} = 53$Å; give rise to $\rho a_{sc} \approx 20$ [16]. Thus we have approximately $\beta_0 = 0.397664$ and $\beta_1 = 0.551786$. For K_0=K_1, we obtain $\Omega_c/\omega_{xy} = 0.2223$ where Ω_c is the critical frequency. The state without a vortex is stable when $\Omega < \Omega_c$, but when $\Omega > \Omega_c$ the state with a vortex is stable, so that vortices can be generated only when the rotation frequency Ω is larger than Ω_c. For $\lambda = 0$, the critical frequency is $\Omega_c/\omega_{xy} = 0.136$ which is smaller than that for $\lambda \neq 0$. It has been shown [17] that a noninteracting gas in a harmonic trap does not generate vortices. The contributions of both interaction and rotation frequency Ω result in vortices in the condensate. The larger the value λ is, the smaller the contribution of the interaction term in the total energy will be. Thus, larger rotation frequency Ω is required for generating vortices. We examine further state configurations with larger units of vortex, viz., for $m = 2, 3\cdots$ [18]. The energy K_2 is minimized when $\beta_2 = 0.642021$. $K_1 = K_2$ gives $\Omega_2/\omega_{xy} = 0.7128$. This value agrees with some experimental and theoretical results[6, 19]. In the experiment[6], when $\Omega_{stir} \simeq \omega_\perp/\sqrt{2}$, the stirring laser resonantly excites the transverse quadrupole mode $m = 2$ of the condensate at rest. Clearly, when $\Omega_2/\omega_{xy} > 0.7128$ the K_1 becomes larger than K_2, which implies that the state with a doubly quantized vortex is more stable than the singly quantized vortex. Unlike the case in a harmonic trap where the state with singly quantized vortex (unit quantum number) is merely most favorable [18] when $\Omega > \Omega_c$, the stable state in a anharmonic trap favors a vortex with large quantum numbers (angular momentum). The faster (the larger Ω) the anharmonic trap rotates, the higher the quantum number of the vortex will be.

4 Vortex lattice

We consider the state with a large angular momentum by looking at configurations with a large number of vortices of unit strength centered around a giant vortex of larger strength. Since the dynamics along the z-axis is identical to the stationary case, the kinetic energy $\mid \nabla_z \Psi \mid^2$ in the z direction is negligible in the Thomas-Fermi approximation (TFA). Let us write $K = \int dz K(z)$:

$$K(z) = \int d^2 r \Psi^* \big[-\frac{\hbar^2}{2m}\nabla_\perp^2 + \frac{1}{2} + \frac{M\omega_{xy}^2}{2a_\perp^2}\lambda r^4 - \Omega L_z$$

$$- \mu(z)\Big]\Psi + \frac{1}{2} f \int d^2 r \mid \Psi \mid^4, \tag{7}$$

where $\mu(z) = \mu - M\omega_z^2 z^2/2$. The case in the absence of the quartic term was considered in [20], where the authors obtained precise positions for 52 and 70 vortices respectively. Now we calculate the $K(z)$ in Eq. (7). The condensate wave function in the xy plane is expressed as $\Phi = \sum_{m=0}^{q} C_m \Phi_m \propto \exp(-r^2/2a_\perp^2)F(u)$ with $F(u) = \Pi_{\alpha=1}^{q}[u - b_\alpha]$, the vortex function of u. Here $u \equiv (x + iy)/a_\perp$ and b_α refers to the position of the vortex [5]. A hole in the center of the gas was shown to be generated by enhancing Ω in the weakly repulsive interaction regime [7], which was also noted in a numerical simulation [8]. Hence we simply assume that $\{b_\alpha\}$ form a regular lattice in which the vortex at the center of the site carries l units of angular momentum while the vortices at the other sites merely carry a unit of angular momentum. Then $F(u)$ can be written as

$$F(u) = u^{l-1} \prod_{\alpha=1}^{q} (u - b_\alpha), \tag{8}$$

where $b_0 = 0$ is implied. The general form of Ψ is likely

$$\Psi = f(z)\Phi, \quad \int d^2 r \mid \Phi \mid^2 = 1, \tag{9}$$

where $f(z)$ describes the density profile of the gas along the z-axis and $\Phi = \bar{\Phi}/D(z)$ with

$$\bar{\Phi} = \exp(-r^2/2a_\perp^2) u^{l-1} \prod_{l=1}^{q} (u - b_\alpha), \tag{10}$$

and $D^2 = \int d^2 r \mid \bar{\Phi} \mid^2$, the normalization constant for Φ. Now the number constraint $\int d^2 r \mid \Psi \mid^2$ becomes $\int dz \mid f \mid^2 = N$. Noting that in complex coordinates

$$L_z = \hbar(u\partial u - u^* \partial u^*),$$

we can obtain the expectation value of L_z,

$$\int \Psi^* L_z \Psi = \hbar \int [(r/a_\perp)^2 - 1] \mid \Psi \mid^2 . \tag{11}$$

Then Eq.(7) becomes

$$K(z) = \Big[-\tilde{\mu}(z) + \frac{\hbar(\omega_{xy} - \Omega)\langle r^2 \rangle_\Phi}{a_\perp^2}\Big] f^2 + \frac{M\omega_{xy}^2}{2a_\perp^2} \lambda f^2 \langle r^4 \rangle_\Phi + \frac{1}{2} g I_\Phi f^4, \tag{12}$$

where $\tilde{\mu}(z) = \mu - \hbar\Omega - (M\omega_z^2 z^2/2)$, $\langle r^2 \rangle_\Phi = \int d^2 r r^2 \mid \Phi \mid^2$, $\langle r^4 \rangle_\Phi = \int d^2 r r^4 \mid \Phi \mid^2$ and $I_\Phi = \int d^2 r \mid \Phi \mid^4$. The minimization of the energy is performed by the variation of the parameter $\{b_\alpha\}$.

To evaluate $\langle r^2 \rangle_\Phi$, $\langle r^4 \rangle_\Phi$ and I_Φ, we note that

$$\mid \bar{\Phi} \mid^2 = e^{-H}, \quad H = H_1 + H_2, \tag{13}$$

where $H_1 \equiv r^2/a_\perp^2 - 2\sum_\alpha \ln \mid \overrightarrow{r} - \overrightarrow{b}_\alpha \mid$ and $H_2 \equiv -2(l-1)\ln \mid \overrightarrow{r} \mid$. Here we assume that $\{\overrightarrow{b_\alpha}\}$ forms an infinite regular lattice, $\overrightarrow{b}_{n_1,n_2} = n_1\overrightarrow{c_1} + n_2\overrightarrow{c_2}$ (n_1, n_2 integers); then $\sum_\alpha \delta(\overrightarrow{r} - \overrightarrow{b_\alpha}) = v^{-1}\sum_{\overrightarrow{K}} e^{i\overrightarrow{K}.\overrightarrow{r}}$, where $\overrightarrow{c_1}, \overrightarrow{c_2}$ are the basis vectors and $v =\mid \overrightarrow{c_1} \times \overrightarrow{c_2} \mid$ is the size of the unit cell of the vortex lattice, and $\overrightarrow{K} = l_1\overrightarrow{K_1} + l_2\overrightarrow{K_2}$ (l_1, l_2 integers) are the reciprocal lattice vectors $[\overrightarrow{K_1} = (2\pi/v)\overrightarrow{c_2} \times \hat{z}, \overrightarrow{K_2} = (2\pi/v)\hat{z} \times \overrightarrow{c_1}]$ [5]. This is a Fourier transform, in fact; by the transform one obtains

$$\begin{aligned}
\nabla_\perp^2 H_1 &= \frac{1}{\sigma^2} - \frac{4\pi}{v}\sum_{k\neq 0}\cos\vec{k}\cdot\vec{r}, \\
\mid \bar{\Phi} \mid^2 &= r^{2(l-1)}e^{-(r^2/\sigma^2)}\prod_{k\neq 0}\exp(\zeta_k \cos\vec{k}.\vec{r}), \\
\frac{1}{\sigma^2} &= \frac{1}{a_\perp^2} - \frac{\pi}{v}, \qquad \zeta_k = \frac{4\pi}{v \mid \vec{k} \mid}.
\end{aligned} \tag{14}$$

In the average-vortex approximation [5], the $|\bar{\Phi}|^2$ becomes $|\bar{\Phi}|^2 = r^{2(l-1)}\exp-(r^2/\sigma^2) = h(r)\exp-(r^2/\sigma^2)$; here $h(r) \equiv r^{2(l-1)}$ is a slowly varying envelope function, which gives rise to the components in higher Landau Levels [21]; the Φ then becomes $\mid \bar{\Phi} \mid^2 = (r^{2(l-1)}\exp(-r^2/\sigma^2))/(\pi\Gamma(l)\sigma^{2l})$. Using the above result, we can easily evaluate the values: $\langle r^2\rangle_\Phi = l\sigma^2$, $\langle r^4\rangle_\Phi = l(l+1)\sigma^4$, $I_\Phi = \xi/(2\pi\sigma^2)$, and $\xi \equiv (2l-1)!/(2^{2l-2}[(l-1)!]^2)$. Substituting these results into Eq(12), we have

$$\begin{aligned}
K(z) &= \Big[-\tilde{\mu}(z) + \frac{l\hbar(\omega_{xy} - \Omega)\sigma^2}{a_\perp^2}\Big]f^2 \\
&+ \frac{\hbar\omega_{xy}\lambda}{2a_\perp^4}l(l+1)\sigma^4 f^2 + \frac{\hbar\omega_{xy}a_{sc}a_\perp^2}{\sigma^2}f^4\xi.
\end{aligned} \tag{15}$$

Minimizing the energy, we obtain two equations:

$$\begin{aligned}
\frac{\partial K(z)}{\partial f} &= 2\Big[-\tilde{\mu}(z) + \frac{l\hbar(\omega_{xy} - \Omega)\sigma^2}{a_\perp}\Big]f + 4\frac{\hbar\omega_{xy}a_{sc}a_\perp^2}{\sigma^2}\xi f^3 \\
&+ \frac{\hbar\omega_{xy}\lambda}{a_\perp^4}l(l+1)\sigma^4 f = 0, \\
\frac{\partial K(z)}{\partial \sigma} &= \frac{2l\hbar(\omega_{xy} - \Omega)\sigma}{a_\perp^2}f^2 + \frac{2\hbar\omega_{xy}\lambda}{a_\perp^4}l(l+1)\sigma^3 f^2 \\
&- \frac{2\hbar\omega_{xy}a_{sc}a_\perp^2}{\sigma^3}f^4\xi = 0.
\end{aligned} \tag{16}$$

In the light of the fact that the above nonlinear equations are difficult to solve analytically, we study those equations for the cases of $\lambda = 0$ and $\lambda \neq 0$ so as to capture some physical implications.

For $\lambda = 0$, viz., the gas is confined in a harmonic trap,

$$\begin{aligned}
\Big(\frac{\sigma^2}{a_\perp^2}\Big)_0 &= \frac{\tilde{\mu}(z)^2}{3l\hbar(\omega_{xy} - \Omega)}, \\
(a_{sc}f^2)_0 &= \frac{[\tilde{\mu}(z)]^2}{9l^2\hbar^2\omega_{xy}(\omega_{xy} - \Omega)\xi}.
\end{aligned} \tag{17}$$

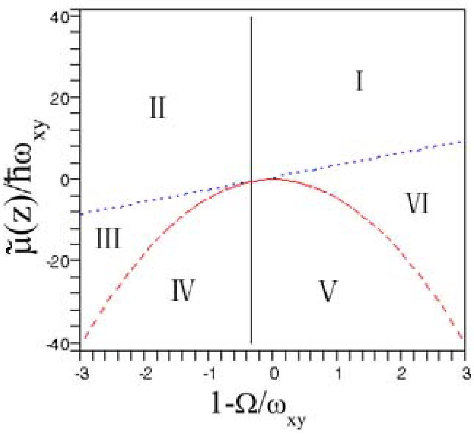

Fig. 1 (color online) The phase diagram in the plane of $\tilde{\mu}(z)$ versus $(1-\Omega/\omega_{xy})$ which is divided by the parabolic curve $\tilde{\mu}(z) = -9\hbar\omega_{xy}l(1-\Omega/\omega_{xy})^2/10(l+1)\lambda$, the oblique line $\tilde{\mu}(z) = \hbar\omega_{xy}[2.5l(l+1)\lambda+3l(1-\Omega/\omega_{xy})]$ and the vertical line $1-\Omega/\omega_{xy} = -5\hbar\omega_{xy}(l+1)\lambda/3$.

Substituting Eq.(17) into Eq.(15), we obtain

$$[K(z)]_0 = \frac{[\tilde{\mu}(z)]^3}{27\hbar^2\omega_{xy}(\omega_{xy}-\Omega)a_{sc}}\Big(-\frac{2}{l^2\xi}+\frac{1}{l^3\xi}\Big) \propto \alpha, \tag{18}$$

in which $\alpha \equiv (-\frac{1}{l^2\xi}+\frac{1}{l^3\xi})$ [22], for example, $[\alpha]_{l=1} = -1$, $[\alpha]_{l=2} = -3/4$, $[\alpha]_{l=3} = -40/81$ etc. This implies that the energy of the system becomes larger with increasing l. Thus the hole in the center of the system can not be formed in a harmonic trap, which also is confirmed by variational wave function $\Phi = Cr^m\exp(im\phi)\exp(\beta^2r^2/2a_\perp^2)$ by regarding β as the variation parameter.

For $\lambda \neq 0$, viz., the gas is confined in an anharmonic trap,

$$\begin{aligned}\Big(\frac{\sigma^2}{a_\perp^2}\Big)_\pm &= -\frac{3(1-\Omega/\omega_{xy})}{5(l+1)\lambda} \\ &\pm \frac{\sqrt{9(1-\Omega/\omega_{xy})^2+[10(l+1)\lambda\tilde{\mu}(z)]/(l\hbar\omega_{xy})}}{5(l+1)\lambda}, \\ \xi a_{sc}f^2 &= \frac{l(1-\Omega/\omega_{xy})\sigma^4}{a_\perp^4}+\frac{\lambda l(l+1)\sigma^6}{a_\perp^6}.\end{aligned} \tag{19}$$

Since $\sigma \geq a_\perp$, the condition for the existence of a solution of the above equations is plotted in Fig.1. In regimes I and II, only the $(\sigma^2/a_\perp^2)_+$ root satisfies the condition $\sigma^2 \geq a_\perp^2$. In regime III, both $(\sigma^2/a_\perp^2)_\pm$ satisfy that condition, which implies the system has double stable states and two possible configurations of vortex lattice can be formed. The vortices in this regime become invisible because it is easy to vary between the two configurations due to fluctuations. In the other regimes, $(\sigma^2/a_\perp^2)_\pm$ do not satisfy that condition. In the Thomas-Fermi approximation, $\mu = MR_i^2\omega_i^2/2$, R_i stand for the maximum extents of the condensate and ω_i for the frequencies in the three directions [17]. According to the experimental data [1, 2], the straight line $\tilde{\mu}(z) = \mu - \hbar\Omega - M\omega_z^2z^2/2$ must cross the

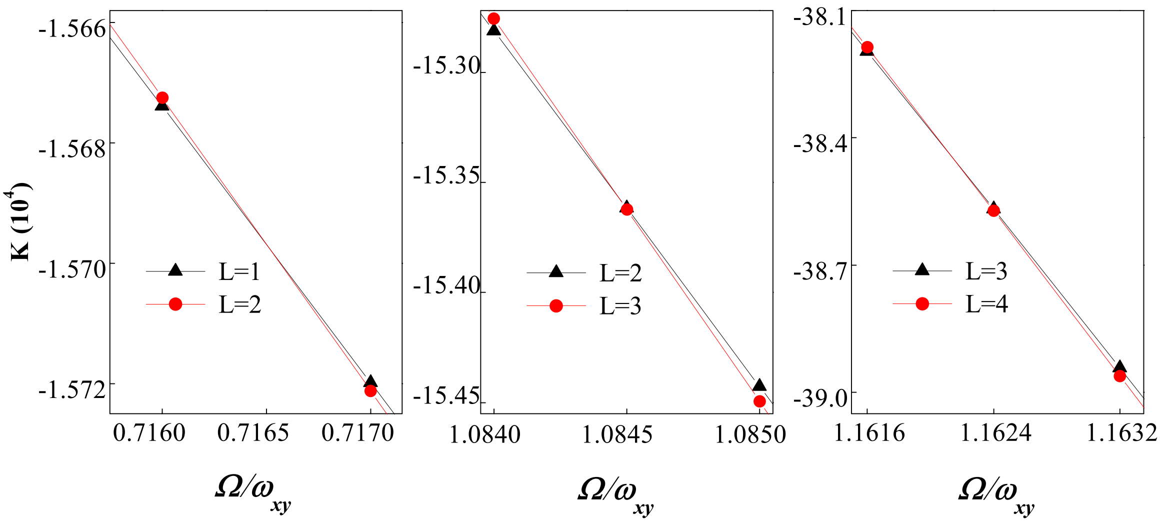

Fig. 2 (color online) The curves of K versus Ω/ω_{xy} in unit of $(2\hbar\omega_{xy})^{3/2}M^{-1/2}/(2\omega_z a_{sc})$ for different l with parameters $\lambda = 0.001$, $N = 3 \times 10^5$, $\omega_{xy} = 2\pi \times 65.6$, $\omega_z = 2\pi \times 11.0$, $a_{sc} = 53\text{Å}$.

regimes I, II and VI but not the other regimes. Thus the condition $\sigma^2 \geq a_\perp^2$ becomes $\tilde{\mu}(z) \geq \hbar\omega_{xy}[2.5l(l+1)\lambda + 3l(1-\Omega/\omega_{xy})]$. Note that for the same rotation frequency Ω with increasing l, the maximum extent of the condensate along the z-axis, R_z, will decrease:

$$R_z^2 \propto \mu/\hbar\omega_{xy} - \Omega/\omega_{xy} - 2.5l(l+1)\lambda - 3l(1-\Omega/\omega_{xy}). \tag{20}$$

For simplicity, we assume straight vortex lines [7] so that the (2D) density of vortex remains constant along the z-axis, then we obtain $K = 2R_zK(0)$.

We plot the values of K versus Ω/ω_{xy} for different l in Fig.2. We can see that with increasing Ω the state becomes a mixed phase with a multiply quantized vortex (hole) at the center of the cloud and singly-quantized vortices around it [15]. This assures that a fast rotating Bose-Einstein condensate confined in a quadratic-plus-quartic potential can generate a hole in the center of the gas which is in agreement with the theoretical and experimental studies [6, 8]. Our calculation shows that if Ω increases slowly, an array of singly quantized vortices is formed at first (just like in harmonic trap); then a hole carrying two units of angular momentum is generated when $\Omega_h/\omega_{xy} \approx 0.716$, which keeps until it reaches $\Omega/\omega_{xy} \approx 1.084$. With increasing Ω the hole will absorb the singly-quantized vortices around it whose angular momentum becomes larger and larger. Looking at the positions of intersections in the three graphs in Fig.2, it is easy to find that the velocity of absorbing the singly-quantized vortices becomes larger and larger, and finally a giant vortex carrying all of the angular momentum will be formed. When $\Omega/\omega_{xy} > 1.084$, every configuration of the vortex lattice remains for a short time so the fragility of the vortex lattice increases. However, recent theoretical study [23] found that much longer times were required for $\Omega \simeq \omega_{xy}$ to reach a well-ordered lattice. Based on this argument it is reasonable that vortices can not be detected for $\Omega \simeq \omega_{xy}$ in the experiment [6].

A useful quantity is $N_v = \left[\frac{\pi\langle r^2\rangle}{v}\right]_{z=0} = \left[l(\frac{\sigma^2}{a_\perp^2} - 1)\right]_{z=0}$, the number of vortices inside the extension of the condensate at the plane of $z = 0$. With increasing Ω, N_v becomes larger and larger. For $\lambda = 0.001$, the extension of the condensate is much larger than the size of a vortex when $\Omega/\omega > 0.7$. In this case, the average-vortex approximation is valid. For some values of Ω, we find that the number of vortices N_v is very close to both numerical and experimental results[6, 24].

5 Conclusion

Employing the approach of Ref. [5], we obtained the new form of the condensate wave function which is essentially the Lowest Landau Level (LLL) wave function with a regular lattice of vortices multiplied by a slowly varying envelope function. For fast rotating Bose-Einstein condensates, many vortices can be generated. For this system, too many parameters are included in the variational condensate wave function $\Phi = \sum_{m=0} c_m \Phi_m$ (c_m are variational parameters) which was used in the earlier paper. So there are difficulties in the analytic calculation if we use such a variational condensate wave function. However, assuming that the number of vortices is infinite, we obtained a variational wave function with only two variational parameters. It is reasonable for the fast rotating BEC system and simplifies the mathematical calculation. Using the variational trial function, we obtained the critical value Ω_h and proved that the fragility of the vortex lattice could be enhanced with increasing Ω. This result is in agreement with the experiment result [25] where the gas was trapped in a harmonic potential. In the experiment [25], the vortex lattice remains ordered, but its elastic shear strength is drastically reduced by increasing Ω. This implies that even a minor perturbation to the cloud can cause the lattice to melt. In the regime with very small λ, the quartic term also provides a perturbation to destroy the vortex lattice; it is therefore invisible when Ω reaches some large value [6].

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