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Dynamic theory for the mesoscopic electric circuit

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Abstract

The quantum theory for mesoscopic electric circuit with charge discreteness is briefly described. The minibands of quasi-energy in LC design mesoscopic electric circuit have been found. In the mesoscopic "pure" inductance design circuit, just like in the mesoscopic metallic rings, the quantum dynamic characteristics have been obtained explicitly. In the "pure" capacity design circuit, the Coulomb blockade had also been addressed.

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1. Introduction

Progress in the study of mesoscopic physics has been especially rapidly developed during the eighties and nineties due to the development of advanced crystal growth and lithography techniques which facilitate sophisticated experiments [1]. The electronic device community has been witnessing, for a number of years, a strong and definite trend in the miniaturization of intergrated circuits and components towards atomic scale dimensions [2]. Clearly, when the transport dimension reaches a characteristic dimension, namely, when the charge-carrier inelastic coherence length and charge-carrier confinement dimension approach the Fermi wavelength, the physics of classical devices, based on the motion of particles and ensemble averaging, is expected to be invalid. The wave nature of electron, discreteness of energy levels and specific properties must now be taken into account and quantum effects should become much more important [3]. Büttiker, Imry and Landauer were the first to understand the quantum effects, using the simple model of a one-dimensional (1D) ring with disorder [4]. They predicted Josephson like effects in

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small and strictly 1D rings of normal metal driven by an external magnetic flux, except that $2e$ is replaced by e. In mesoscopic systems, several manifestations of the Aharonov–Bohm effects and Aharonov–Casher effects have been predicted and verified [5]. The persistent current in mesoscopic metallic ring has been discussed in many different cases by using several methods [6]. In our previous papers [7], the mesoscopic metallic ring is regarded as a "pure" inductance (L) design electrical circuit. The persistent current formula has been proposed by using the quantum theory for mesoscopic electric circuits in accord with the discreteness of electric charge. In this Letter, we will discuss the quantum dynamic characteristics in mesoscopic electronic circuit which the source changes with time. In Section 3, the minibands of quasienergy in LC design mesoscopic electric circuit have been explained. In Section 4, pure inductance design circuit, just like a mesoscopic metallic rings, electrons driven by an external field show a variety of very interesting phenomena, including Bloch oscillations [8], Wannier–Stark ladders [9] and Landau–Zener tunneling [10]. These have been observed in high-quality superlattices, optical ring resonators and more recently on ultra cold atoms in accelerating optical potentials [11]. The same effects had been predicted by Likharev et al., in the mesoscopic Josephson junction [12]. In Section 5, pure capacity circuit, just like quantum dot, Coulomb blockade had also been addressed by quantum theory. In Section 2, we at first briefly describe a quantum mechanical theory for mesoscopic electric circuit based on the discreteness of charge, and consider the external source as a time function [13].

2. Quantization of the mesoscopic electric circuit

The classical equation of motion for an electric circuit of LC design is the same as that for a harmonic oscillator, where the "coordinate" means electric charge [14]. The quantization of the circuit was carried out in the same way as that of a harmonic oscillator [15]. In order to take into account the discreteness of electronic charge, we must impose that the eigenvalues of the self-adjoint operator $\hat{q}$ (electric charge) take discrete values.

$$\hat{q}|n\rangle = nq_e|n\rangle, \tag{1}$$

where $n \in Z$ (set of integers) and $q_e = 1.602 \times 10^{-19}c$, the elementary electric charge [7]. Since the spectrum of charge is discrete, the inner product in charge representation will be a sum instead of the usual integral and the electric current operator will be defined by the discrete derivatives $\nabla_{q_e}, \bar{\nabla}_{q_e}$.

$$\nabla_{q_e} = (\hat{Q} - 1)/q_e, \qquad \bar{\nabla}_{q_e} = \left(1 - \hat{Q}^+\right)/q_e, \tag{2}$$

where $\hat{Q} = e^{iq_e\hat{p}/h}$ is a minimum "shift operator". Then we can write down the "momentum" operator which is also the "current" operator apart from the inductance factor

$$\hat{P} = \frac{\hbar}{2i}(\nabla_{q_e} + \bar{\nabla}_{q_e}) = \frac{\hbar}{2iq_e}\left(\hat{Q} - \hat{Q}^+\right). \tag{3}$$

It is easy to check that $\nabla^+_{q_e} = -\bar{\nabla}_{q_e}$. Thus for the mesoscopic quantum electric circuit one will obtain a finite differential Schrödinger equation.

$$i\hbar\frac{\partial}{\partial t}\left|\Psi(t)\right\rangle = \left[-\frac{\hbar^2}{2q_eL}(\nabla_{q_e} - \bar{\nabla}_{q_e}) + \frac{1}{2C}\hat{q}^2 - \epsilon(t)\hat{q}\right]\left|\Psi(t)\right\rangle, \tag{4}$$

where L stands for inductance, C for the capacity and $\epsilon(t)$ for the voltage of an electric source which is always the time function. $H_0 = -\frac{\hbar^2}{2q_eL}(\nabla_{q_e} - \bar{\nabla}_{q_e})$ is the field free part which describes the "pure" L design circuit. After some calculation, we obtain the following commutation relations for the charge $\hat{q}$ and the "current" $\hat{P}$ with the free Hamiltonian H_0.

$$[\hat{H}_0, \hat{P}] = 0, \qquad [\hat{H}_0, \hat{q}] = i\hbar\hat{P},$$
$$[\hat{q}, \hat{P}] = i\hbar\left(1 + \frac{q_e^2}{\hbar^2}H_0\right). \tag{5}$$

In order to solve the finite differential Schrödinger equation, the charge representation has been used

$$\left|\Psi(t)\right\rangle = \sum_{n=-\infty}^{+\infty} C_n(t)|n\rangle. \tag{6}$$

Inserting (6) into (4), we get the equation

$$i\hbar\frac{\partial}{\partial t}C_n(t) = -\frac{\hbar^2}{2q_e^2L}\left[C_{n+1}(t) + C_{n-1}(t)\right] + \frac{\hbar^2}{q_e^2L}C_n(t) + \frac{n^2q_e^2}{2C}C_n(t) - \epsilon(t)nq_eC_n(t). \tag{7}$$

If the voltage source is a Faraday law generator

$$\epsilon(t) = -\frac{d\Phi}{dt}, \tag{8}$$

we may write

$$u(\theta,t) = \sum_{n=-\infty}^{+\infty} C_n(t) \exp\left\{in\left(\theta + \frac{q_e\Phi}{\hbar}\right)\right\}, \tag{9}$$

and Eq. (7) becomes

$$i\hbar\frac{\partial}{\partial t}u(\theta,t) = \left[-\frac{q_e^2}{2C}\frac{\partial^2}{\partial\theta^2} + \frac{\hbar^2}{q_e^2 L}\left[1-\cos\left(\theta + \frac{q_e\Phi}{\hbar}\right)\right]\right]u(\theta,t), \tag{10}$$

i.e., the effective Hamiltonian for a voltage biased mesoscopic electric circuit reads

$$H_{\text{eff}}(\Phi) = -\frac{q_e^2}{2C}\frac{\partial^2}{\partial\theta^2} + \frac{\hbar^2}{q_e^2 L}\left[1-\cos\left(\theta + \frac{q_e\Phi}{\hbar}\right)\right], \tag{11}$$

where the applied voltage obeys Eq. (8). If it is defined a current biased mesoscopic circuit, $I_{\text{ex}} = \frac{dQ}{dt}$, the effective Hamiltonian for current biased mesoscopic electric circuit reads

$$H_{\text{eff}}(Q) = \frac{1}{2C}\left(-iq_e\frac{\partial}{\partial\theta} - Q\right)^2 + \frac{\hbar^2}{q_e^2 L}(1-\cos\theta). \tag{12}$$

The equation is merely a time-varying canonical transformation of the effective Hamiltonian for a voltage biased mesoscopic electric circuit [16]. It is shown the duality between charge Q and flux Φ also present the Josephson like effects. After gauge transformation, Eqs. (11) and (12) are turned into the well-known Mathieu equation, its eigenvalues and eigenfunctions had also been show [7,17]. Energy bands, Bloch oscillations and Coulomb blockade in Eqs. (11) and (12) of mesoscopic Josephson junction had been discussed more detailly [12]. In the next section, we will discuss the quantum dynamic characteristics in a LC design mesoscopic circuit, the minibands of quasienergy have been explained.

3. Minibands of quasienergy in LC design mesoscopic electric circuit

Let us consider the effective Hamiltonian Eq. (12) for current biased mesoscopic electric circuit. The Schrödinger equation is

$$i\hbar\frac{\partial}{\partial t}\Psi(\theta,t) = \left[\frac{1}{2C}\left(-iq_e\frac{\partial}{\partial\theta} - Q(t)\right)^2 + \frac{\hbar^2}{q_e^2 L}(1-\cos\theta)\right]\Psi(\theta,t), \tag{13}$$

its potential with period 2π. If the current source is the time function $I_{\text{ex}} = I_0\sin(\omega t)$. Thus $Q(t) = -\frac{I_0}{\omega}\cos(\omega t)$.

The equation has a very interesting property: it is periodic in both space and time [18]. Using the Kramers–Henneberger transformation [19]. The ansatz is

$$\Psi(\theta,t) = \exp\left(\frac{i}{\hbar}\int_0^t d\tau\frac{q_e}{C}Q(\tau)p\right)\chi(\theta,t). \tag{14}$$

Transforms the Schrödinger equation (13) into

$$i\hbar\frac{\partial}{\partial t}\chi(\theta,t) = \left[-\frac{q_e^2}{2C}\frac{\partial^2}{\partial\theta^2} + \frac{1}{2C}Q^2(t) + \frac{\hbar^2}{q_e^2 L}\left(1-\cos\left[\theta - \alpha\sin(\omega t)\right]\right)\right]\chi(\theta,t), \tag{15}$$

with $\alpha = \frac{q_e I_0}{\hbar C\omega^2}$. We separate

$$\frac{1}{2C}Q^2(t) = \frac{1}{2C}\frac{I_0^2}{\omega^2}\cos^2(\omega t) = \frac{1}{4C}\frac{I_0^2}{\omega^2}\big(1+\cos(2\omega t)\big), \tag{16}$$

and remove the periodic part of Q^2 by defining

$$\chi_1(\theta,t) = \exp\left(-\frac{i}{\hbar}\int_0^t d\tau\frac{I_0^2}{4C\omega^2}\cos(2\omega t)\right)\chi(\theta,t). \tag{17}$$

Of course, the "obvious" transformation

$$\chi_1(\theta,t) = \exp\left(-\frac{i}{\hbar}\int_0^t d\tau\frac{I_0^2}{4C\omega^2}\cos(2\omega t)\right)\chi(\theta,t)$$

is not periodic in time and, therefore, does not leave the quasienergy spectrum invariant. If we then assume that the frequency ω is large, we can average the moving potential $\frac{\hbar^2}{q_e^2 L}(1-\cos[\theta-\alpha\sin(\omega t)])$ over one cycle and obtain

$$i\hbar\frac{\partial}{\partial t}\chi_1(\theta,t) = \left(-\frac{q_e^2}{2C}\frac{\partial^2}{\partial\theta^2}+\frac{I_0^2}{4C\omega^2}+V_{\mathrm{av}}(\theta)\right)\chi_1(\theta,t), \tag{18}$$

with

$$V_{\mathrm{av}}(\theta)=\frac{1}{T}\int_0^T dt\,\frac{\hbar^2}{q_e^2 L}\bigl(1-\cos[\theta-\alpha\sin(\omega t)]\bigr).$$

The operator on the r.h.s. in (18) now is time-independent. This means that the quasienergy of the original problem can, in the limit of high frequencies, be calculated as the energy for the potential

$$V(\theta)=\frac{I_0^2}{4C\omega^2}+V_{\mathrm{av}}(\theta), \tag{19}$$

in particular, the cosine potential (13) leads to

$$V(\theta)=\frac{I_0^2}{4C\omega^2}+\frac{\hbar^2}{q_e^2 L}\bigl[1-V_0\cos(\theta)\bigr], \tag{20}$$

with a renormalized potential strength

$$V_0=\frac{\hbar^2}{q_e^2 L}J_0(\alpha), \tag{21}$$

where J_0 is a zero order Bessel function. To conclude, this example demonstrates that the "allowed" quasienergy for the periodically driven potential group together in bands exactly as the energies of the undriven system do, and according (21) the band width can be controlled by the strength I_0 and frequency ω of the driving force via the parameter α in the high frequency care. In the pure L design mesoscopic electric circuit, just as the mesoscopic metallic rings. The quasienergy band will be suppressed the same reason by using the voltage biased electric circuit theory [23].

4. The pure L design mesoscopic electronic circuit

The Schrödinger equations for a L design both in the presence of an adiabatic power source and in the absence of source are solved exactly [7]. A gauge field is introduced and a formula for persistent current that is a periodic function of the magnetic flux is obtained. It provides a formulation of the persistent current in the mesoscopic ring from a different point of view. We can discuss the dynamic characteristics of the mesoscopic metallic ring regarded the ring as a "pure" L design. We should point out that "pure" means that the energy of L is much larger than that of C so that we can ignore the capacity energy in Eqs. (4) and (7). One should have in mind, that the capacity always exists in the mesoscopic metallic ring due to structure, impurities or some other reasons. Therefore, we can use the perturbation theory to discuss the mesoscopic metallic rings only using the Hamiltonian of inductance part. We can read the Schrödinger equation for a L design in the presence of an external source with time from Eq. (7)

$$i\hbar\frac{\partial}{\partial t}C_n(t)=-\frac{\hbar^2}{2q_e^2 L}\bigl[C_{n+1}(t)+C_{n-1}(t)\bigr] + \frac{\hbar^2}{q_e^2 L}C_n(t)-\epsilon(t)nq_eC_n(t). \tag{22}$$

In solving Eq. (22), we first perform a discrete Fourier transform over the charge labeled n by multiplying (6) by $\exp(-ikn)$ and summing over all n, where k is the same as θ in last section,

$$C_k(t)=\sum_n \exp(-ikn)C_n(t) \times\exp\left(i\left[\frac{q_e}{\hbar}\Phi(t)+\frac{\hbar^2}{q_e^2 L}t\right]\right). \tag{23}$$

The form of Eq. (22) in k space (or current representation) is then

$$i\hbar\frac{\partial}{\partial t}C_k(t)=-\frac{\hbar^2}{q_e^2 L}\cos\left[k+\frac{q_e}{\hbar}\Phi(t)\right]C_k(t). \tag{24}$$

After integration, we get

$$C_k(t)=C_k(0)\exp\left[i\frac{\hbar^2}{q_e^2 L}\cos k\mu(t)\right] \times\exp\left[-i\frac{\hbar^2}{q_e^2 L}\sin k\nu(t)\right], \tag{25}$$

where

$$\mu(t)=\int_0^t\cos\left(\frac{q_e}{\hbar}\Phi(t)\right)dt,$$

$$\nu(t)=\int_0^t \sin\left(\frac{q_e}{\hbar}\Phi(t)\right)dt.$$

By using of the identities

$$\exp\left(i\frac{\hbar^2}{q_e^2L}\cos k\mu(t)\right)=\sum_{p=-\infty}^{+\infty}\exp\left(ip\frac{\pi}{2}\right)\exp(-ipk)J_p\left[\frac{\hbar^2}{q_e^2L}\mu(t)\right], \tag{26}$$

$$\exp\left(i\frac{\hbar^2}{q_e^2L}\sin k\nu(t)\right)=\sum_{q=-\infty}^{+\infty}\exp(-iqk)J_q\left[\frac{\hbar^2}{q_e^2L}\nu(t)\right], \tag{27}$$

where J is the Bessel function, $p\in Z$ and $q\in Z$, we get the solution

$$C_m(t)=\sum_{np}C_n(0)\exp\left(-i\left[n\frac{q_e}{\hbar}\Phi(t)+\frac{\hbar^2}{q_e^2L}t\right]\right)\times\exp\left(ip\frac{\pi}{2}\right)J_p\left[\frac{\hbar^2}{q_e^2L}\mu(t)\right]\times J_{n-m-p}\left[\frac{\hbar^2}{q_e^2L}\nu(t)\right]. \tag{28}$$

Using Graf's addition theorem for Bessel function [20], we can simplify to

$$C_m(t)=\sum_n C_n(0)\exp\left(i\left(-n\frac{q_e}{\hbar^2}\Phi(t)-\frac{\hbar}{q_e^2L}t\right)\right)\times(-\lambda)^{n-m}J_{n-m}\left(\frac{\hbar^2}{q_e^2L}\left[\mu(t)^2+\nu(t)^2\right]^{1/2}\right) \tag{29}$$

where $\lambda=([\nu^2(t)-i\mu^2(t)]/[\nu^2(t)+i\mu^2(t)])^{1/2}$. The resulting expression for the probability propagator $|C_m(t)|^2$ is

$$\left|C_m(t)\right|^2=J_m^2\left[\frac{\hbar^2}{q_e^2L^2}\left[\mu^2(t)+\nu^2(t)\right]\right]. \tag{30}$$

The mean-square electric charge corresponding to is obtained immediately with the help of the identity $\sum_m J_m^2(z)m^2=z^2/2$,

$$\langle\hat{q}^2\rangle=\frac{\hbar^2}{q_e^2L^2}\left[\mu^2(t)+\nu^2(t)\right]. \tag{31}$$

The principal results are valid for any time dependence of the electric field. Of special interest is the case of the sinusoidal field, i.e., $\Phi(t)=\frac{\epsilon_0}{\omega}\cos(\omega t)$. By taking $\omega\to 0$ in Eqs. (17) and (18) [20], we obtained

$$\left|C_n(t)\right|^2=J_n^2\left[\left(-\frac{2}{q_eL}\right)\sin\left(\frac{q_e\epsilon_0t}{2\hbar}\right)\right]. \tag{32}$$

The mean-square electric charge is

$$\langle\hat{q}^2\rangle=\frac{2}{(q_eL\epsilon_0)^2}\sin^2\left(\frac{q_e\epsilon_0t}{2\hbar}\right). \tag{33}$$

The propagator expression (32) has been briefly mentioned earlier in the literature on Stark ladders [20]. The argument of the Bessel function in (32) is itself an oscillatory function of time, the oscillation frequency being proportional to the magnitude of the electric field corresponds to a Bloch wave oscillation [21]. Eq. (32) shows this "localization" explicitly [22]. The quasienergy band is suppressed by the driving extend field if the ratio of Bloch frequency to the extend field frequency is the root of ordinary Bessel function of order zero [23]. The mean square charge does not grow without bound but oscillates sinusoidally. Since a mesoscopic metal ring is a natural "pure" L design, the formula (32) and (33) is valid for quantum effects on a single mesoscopic ring. We will find useful the form of the current operator in our investigations below. It is straightforward to show that the current operator is given by Eq. (3), its mean-square value can be calculated by using Eqs. (3) and (6)

$$\langle\hat{P}^2\rangle=\frac{\hbar^2}{2q_e^2}. \tag{34}$$

It indicates that the maximum quantum noise in mesoscopic rings takes a finite value if the elementary charges q_e should not be considered as the infinitesimal. The result is consistent with our calculation in LC design mesoscopic electric circuit [7].

5. The pure C design mesoscopic electronic circuit

We observe the Schrödinger equation for a LC design. The mesoscopic capacity may be relatively very small (about 10^{-8} F), but the inductance of a macroscopic circuit connecting a source is relatively large because the inductance of a circuit is proportional to the area which the circuit spans. We can neglect the term reversely proportional to L in Eq. (7) and study the equation for a pure C design circuit as

$$i\hbar\frac{\partial}{\partial t}C_n(t)=\left[\frac{n^2q_e^2}{2C}-nq_e\epsilon(t)\right]C_n(t). \tag{35}$$

We can get the energy for the eigenstate $|n\rangle$ and C_n by integrated (35),

$$E(t)=\frac{[nq_e-C\epsilon(t)]^2}{2C}-\frac{C}{2}\epsilon^2(t), \tag{36}$$

$$\ln C_n(t)=\frac{1}{i\hbar}\int_0^t\left[\frac{n^2q_e^2}{2C}-nq_e\epsilon(t')\right]dt'. \tag{37}$$

Eq. (36) involves both the charge quantum number and the voltage source. The Coulomb blockade can be explained by the semiclassic method [22]. From the quantization of mesoscopic electric circuit, the current operator had been defined in Eq. (3) and its average value for this state $\Psi(t)$ can be easy calculated as

$$\begin{aligned}&\langle\Psi(t)|P|\Psi(t)\rangle\\&\quad=-\frac{\hbar}{q_e}2\pi\sum_{n\in Z}\delta\left(t-\frac{4\pi n\hbar C}{q_e^2}\right)\\&\qquad\times\sin\left[\frac{q_e^2}{2C}t-q_e\int_0^t\epsilon(t')\,dt'\right].\end{aligned} \tag{38}$$

Clearly, the average current is of the form of sharp pulses as a δ function which occurs periodically according to the changes of voltage. The voltage difference between two pulses are q_e/C [24]. This is the called Coulomb blockade phenomena caused by the charge discreteness.

6. Conclusions and discussions

Taking the charge discreteness into account, we studied the quantization of LC design mesoscopic electric circuit with a time-dependant external source. The minibands of quasienergy have been explained. The mesoscopic pure L design electric circuit, for example the mesoscopic metallic rings, the quantum dynamic characteristics just as Bloch wave oscillation, dynamic localization and Wannier–Stark ladders have been obtained explicitly. As Y. Imry pointed out that many interesting things occur in the mesoscopic metallic rings when the flux changes with time, we see that the propagator expression (33) will oscillate with a Josephson-type frequency [8]. When the changes of flux is not slow enough, Zener-type transitions may occur among the energy bands. This necessitates a dynamical treatment which we shall do in the future. In mesoscopic pure C design electrical circuit, Coulomb blockade had been addressed by the quantum theory.

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