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Entanglement of two-qutrit system

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Abstract. In this paper, von Neumann entropy is used to study three kinds of bipartite qutrit model, which represent SU(3) strongly correlated model, three-level Lipkin-Meshkov-Glick model, and spin-1 model. The relation between the ground-state entanglement and the quantum phase transition in those models is exhibited and the connection between the entanglement extremes and the symmetries is also discussed.

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1 Introduction

Quantum entanglement [1–3] is an important prediction in quantum mechanics, which constitutes a valuable resource in quantum information process. In recent years, there has been considerable progress in understanding the nonclassical correlations in the ground state of a many-body systems. Particularly, the study on the characteristics of the ground-state entanglement [4] has absorbed much attention, for example, the role of entanglement played in quantum phase transitions (QPT). The origin of the correlations in physical systems exhibiting a quantum phase transition is argued to be related to quantum entanglement [5]. It was found that the next-nearest neighbor entanglement is a maximum at the critical point [6] for transverse Ising model. The pairwise quantum entanglement in systems of fermions itinerant in a lattice [7] and the anisotropic Heisenberg model [8] have been investigated deeply. In these works, the role of entanglement played in quantum phase transition were suggested to consider. Quantum phase transition occurs at absolute zero which is usually driven by quantum fluctuations. With the development of long-range correlations and a nonzero expectation value for an ordered parameter [6], the QPT in quantum many-body system strongly influences the behavior of the system near the critical point. The quantum phase diagram plotted versus the change of an external parameter or coupling constant is an important topics in correlated systems. Recently, the quantum entanglement in one-dimensional correlated fermionic system was shown [9] to identify quantum phase transitions in fermionic systems. It is therefore interesting to investigate those relations in various physical models.

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In this paper, we study the bipartite system which carries out $SU(3)$ representation. In realistic systems, there are several models that carry out $SU(3)$ representation. In next section we study a model with $SU(3)$ symmetry which may be relevant to quark model in particle physics. Another example of qutrit system is the three-level Lipkin-Meshkov-Glick model [10] which is discussed in Section 3. In the last section, we study a spin-1 bipartite model. The relation between the entanglement and symmetry are all discussed.

2 SU(3) model

As we know, the $SU(3)$ group is a exponential mapping of the A_2 algebra, which has three positive roots: α_1, α_2, and $\alpha_1+\alpha_2$. Here α_1 and α_2 are simple roots. In a non-orthogonal coordinate system, $\alpha_1 = (1, -1/2)$ and $\alpha_2 = (-1/2, 1)$. On the other hand, A_2 has two commutative generators, denoted by H_1, H_2, which span the Cartan subalgebra. In the fundamental representation of $su(3)$ algebra, there are 3 bases denoted by $|1\rangle$, $|2\rangle$, and $|3\rangle$. In terms of these bases, the Cartan generators can be written as follows,

$$H_1 = \frac{1}{2}(|1\rangle\langle 1| - |2\rangle\langle 2|), \quad H_2 = \frac{1}{2}(|2\rangle\langle 2| - |3\rangle\langle 3|). \quad (1)$$

Accordingly, there are 3 raising generators and 3 lowering generators as follows,

$$\begin{aligned} &E_{\alpha_1} = |1\rangle\langle 2|, E_{-\alpha_1} = |2\rangle\langle 1|, \\ &E_{\alpha_2} = |2\rangle\langle 3|, E_{-\alpha_2} = |3\rangle\langle 2|, \\ &E_{\alpha_1+\alpha_2} = |1\rangle\langle 3|, E_{-(\alpha_1+\alpha_2)} = |3\rangle\langle 1|. \end{aligned}$$

In the following, we will discuss a system which has anisotropy structure and non-uniform external fields are imposed. The Hamiltonian can be written in terms of the generators of $su(3)$ algebra,

$$\left.\begin{aligned}\mathcal{H} = \sum_{\langle i,j\rangle}\Bigg\{ & \sum_{\alpha\in\Delta_R} E_\alpha(i)E_{-\alpha}(j) + g^{mn}H_m(i)H_n(j) \\ & +\Delta\left(H_1(i)H_1(j)+H_2(i)H_2(j)\right) \\ & +\tilde{\Delta}\left(H_1(i)H_2(j)+H_2(i)H_1(j)\right) \\ & +b_1\left(H_1(i)+H_1(j)\right)+b_2\left(H_2(i)+H_2(j)\right)\end{aligned}\right\} \quad (2)$$

where i, j denote nearest-neighbor sites, Δ_R stands for the set of all roots of A_2 algebra with the metric (g^{mn})

$$(g^{mn}) = \frac{1}{3}\begin{pmatrix} 8 & 4 \\ 4 & 8 \end{pmatrix} \quad (3)$$

and Δ, $\tilde{\Delta}$ are the anisotropy factors; b_1 and b_2 stand for non-uniform external fields. Here the coupling constant is set to unit. It is obvious that the first line of the equation (2) denotes the isotropic part of the Hamiltonian. When the model is isotropic in the absent of external fields, it has $SU(3)$ symmetry. When the model is translationally invariant, for example, the chain becomes a ring, one can study the pairwise entanglement of the any neighbor particles. The concurrence vector [11] is expected to measure the entanglement of systems beyond qubits. However, for bipartite model, one only needs to calculate the von Neumann entropy directly,

$$E(\rho) = -tr(\rho_1 \log_2 \rho_1) \quad (4)$$

where $\rho_1 = \mathrm{Tr}_2(|\Psi\rangle\langle\Psi|)$ is the reduced density matrix. For a $d\otimes d$-dimensional pure state $|\Psi\rangle \in H_d \otimes H_d$, the entropy satisfies the inequality

$$0 \leq E(|\Psi\rangle) \leq \log_2 d, \quad (5)$$

where the lower (upper) bound is reached if and only if $|\Psi\rangle$ is a product state (maximally entangled state). For simplicity, we will refer to von Neumann entropy simply as entropy in this paper.

2.1 Anisotropic coupling in the absence of external field

Firstly, we consider a simple case. The system is isotropic in the absent of external fields, i.e., Δ, $\tilde{\Delta}$ and b_1, b_2 are all zero. The ground state is highly degenerate. There are only 2 different eigenvalues $-4/3$ and $2/3$. The former is 3-fold degenerate and the latter is 6-fold degenerate.

Then, we consider a little more complicated case in which the system becomes anisotropic with absent of external field, i.e., $b_1 = b_2 = 0$. The degeneracy is partly destroyed. In the bases $|ij\rangle$, $i, j = 1, 2, 3$, equation (2) can be rewritten in the form of 9×9 matrix. From the Hamiltonian, one can obtain the ground state, which is related to the anisotropy factor and external magnetic fields. Some of them are still degenerate. When $\Delta < -16/3$ and $8 + 2\Delta < \tilde{\Delta} < \Delta/2$, the ground state energy $(8 + 3\Delta)/12$ corresponds to the state $|33\rangle$ and $|11\rangle$. They are both non-entangled. Thus, the realistic state can be given by the superposition of the above states,

$$|\psi\rangle = a|33\rangle + b|11\rangle, \quad (6)$$

where a and b are coefficients with the restriction $|a|^2 + |b|^2 = 1$. Due to equation (4), the entropy is $E = -(|a|^2 \log_2 |a|^2 + |b|^2 \log_2 |b|^2)$. Considering the normalization condition $|a|^2 + |b|^2 = 1$, one know that when $|a| = |b| = 1/\sqrt{2}$, the entropy reaches maximal value 1. Since the entropy of maximally entangled state is $\log_2 3$ under the definition of equation (4), the state is not maximally entangled. One then can know the average value of the entropy $\langle E(|\psi\rangle)\rangle = 0.61$ as a and b are varied. When Δ and $\tilde{\Delta}$ do not satisfy the above conditions, the ground state is $|22\rangle$ with the energy $(4 + 3\Delta - 3\tilde{\Delta})/6$. This state is non-degenerate and non-entangled.

2.2 Isotropic coupling in the presence of external field

In this section, we will discuss the model in which $\Delta = \tilde{\Delta} = 0$ and external fields are present. In this case, all the degeneracies are destroyed. In the same way, we can get the following results:

(1). The ground state is $(-|23\rangle + |32\rangle)/\sqrt{2}$ when

$$\begin{aligned}&(a)\quad 0 < b_1 \leq 4/3, \quad -b_1 < b_2 < 2b_1,\\ &(b)\quad b_1 > 4/3, \quad (b_1 - 4)/2 < b_2 < (4 + b_1)/2.\end{aligned}$$

We can know the entropy of this state is 1 and the state is not maximally entangled. The corresponding energy is $(-8 - 3b_1)/6$.

(2). The ground state is $|11\rangle$ with the energy $2/3 + b_1$ when $b_1 < -8/3$ and $4 + 2b_1 < b_2 < -4 - b_1$. $E(|\Psi\rangle) = 0$.

(3). The ground state has the form $|33\rangle$ with the energy $2/3 - b_2$, when

$$\begin{aligned}&(a)\quad b_1 \leq 4/3, \quad b_2 > 4 - b_1,\\ &(b)\quad b_1 > 4/3, \quad b_2 > (4 + b_1)/2.\end{aligned}$$

This is a product state and $E(|\Psi\rangle) = 0$.

(4). The ground state is $(-|13\rangle + |31\rangle)/\sqrt{2}$ with the energy $(-8 + 3b_1 - 3b_2)/6$ when

$$\begin{aligned}&(a)\quad b_1 \leq -8/3, \quad -4 - b_1 < b_2 < 4 - b_1,\\ &(b)\quad -8/3 < b_1 \leq 0, \quad b_1/2 < b_2 < 4 - b_1,\\ &(c)\quad 0 < b_1 < 4/3, \quad 2b_1 < b_2 < 4 - b_1.\end{aligned}$$

The entropy $E(|\Psi\rangle) = 1$.

(5). The ground state is $(-|12\rangle + 21\rangle)/\sqrt{2}$ with the energy $(-8 + 3b_2)/6$ when

$$\begin{aligned}&(a)\quad b_1 \leq -8/3, \quad -4 + 2b_1 < b_2 < 4 + 2b_1,\\ &(b)\quad -8/3 < b_1 \leq 0, \quad -4 + 2b_1 < b_2 < b_1/2,\\ &(c)\quad 0 < b_1 < 4/3, \quad -4 + 2b_1 < b_2 < -b_1.\end{aligned}$$

The entropy $E(|\Psi\rangle) = 1$.

(6). The ground state is $|22\rangle$ with the energy $(2-3b_1+3b_2)/3$ when

$$(a)\ \ b_1 \le 4/3,\ \ b_2 < -4+2b_1,$$
$$(b)\ \ b_1 > 4/3,\ \ b_2 < (-4+b_1)/2.$$

This is also a product state.

From above discussions, we know that when external field is imposed, the degeneracy of the isotropic system is destroyed. The ground state energy is the function of non-uniform external fields. It is worthy of noticing that in each condition of above 6 cases, the ground state is dependent on the external field, and the entanglement of the system is independent of the external fields. The entropy is a constant. That is 0 or 1.

For the system with $\Delta = \tilde{\Delta}$ and $b_1 = b_2 = b$, we can obtain the relationship between entanglement, anisotropy factor, and external fields. The results are as follows:

(1). The ground state is $|33\rangle$ with the energy $(2-3b)/3$ when

$$b > 27/10,$$
$$\frac{3(2-b)+\sqrt{36-4b+b^2}}{2} < \Delta < 3(b-1) - \sqrt{9-2b+b^2}.$$

Obviously, this is a product state.

(2). The ground state is $|11\rangle$ with the energy $(8+12b+3\Delta)/12$ when

$$(a)\ \ b \le -6,$$
$$\frac{24b+18b^2}{2+3b} < \Delta < -\frac{5}{3}(1+b) - \frac{1}{3}\sqrt{25+2b+b^2},$$
$$(b)\ \ -6 < b < -\frac{8}{3},\ \ \frac{24b+18b^2}{2+3b} < \Delta < \frac{-8b-2b^2}{2+b}.$$

The state is not entangled.

(3). When the ground state is

$$\frac{1}{N_-}\left(\frac{\Delta-\sqrt{64+\Delta^2}}{8}|13\rangle + |31\rangle\right), \tag{7}$$

and the entropy is given by

$$E(|\psi\rangle) = -\left(\frac{\Delta-\sqrt{64+\Delta^2}}{8N_-}\right)^2 \log_2\left(\frac{\Delta-\sqrt{64+\Delta^2}}{8N_-}\right)^2 - \frac{1}{N_-^2}\log_2\frac{1}{N_-^2}. \tag{8}$$

where N_- is the normalization parameter:

$$N_{\mp}^2 = 1 + \left(\frac{\sqrt{\Delta^2+64}\mp\Delta}{8}\right)^2.$$

In this case, the Δ and b should satisfy

$$(a)\ \ b \le -6,\ \ \Delta > -5(1+b)/3 - \sqrt{25+2b+b^2}/3,$$
$$(b)\ \ -6 < b \le 0,\ \ \Delta > -b,$$
$$(c)\ \ 0 < b \le 27/10,\ \ \Delta > 2b/3,$$
$$(d)\ \ b > 27/10,\ \ \Delta > 3(b-1) - \sqrt{9-2b+b^2}.$$

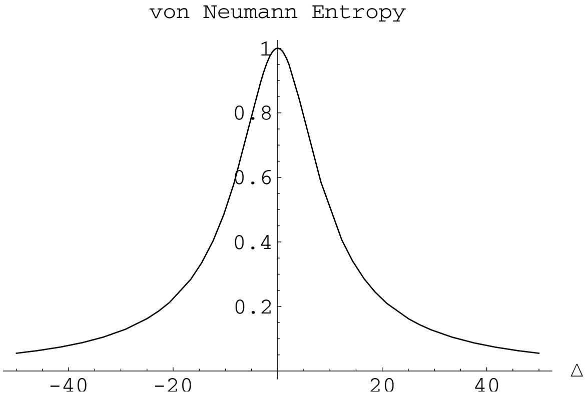

Fig. 1. Von Neumann entropy versus anisotropy factor Δ of $SU(3)$ model.

The ground state energy is $(-8-9\Delta-3\sqrt{64+\Delta^2})/24$.

The ground state entanglement changes as the anisotropy factor Δ is varied. It can be plotted in Figure 1. In the figure, one can see when $\Delta = 0$, the entropy reaches maximal value 1. However, it is not maximally entangled. In fact, when $\Delta = 0$ and $b \neq 0$, equation (2) only has $SU(2)$ symmetry. When the system becomes anisotropy, the symmetry is broken. One can evaluate the correlation function in the vicinity of the phase point. One can obtain

$$\langle O\otimes O\rangle - \langle O\rangle\langle O\rangle = \frac{16(\sqrt{64+\Delta^2}-\Delta)}{64+(\Delta-\sqrt{64+\Delta^2})^2}, \tag{9}$$

where $O = E_{\alpha_1+\alpha_2} - E_{-\alpha_1-\alpha_2}$. One can obtain at the point $\Delta = 0$, the correlation function reaches maximum 1. This is an evidence that critical point corresponds to the situation where the lattice is most entangled. The symmetry breaking present in the ground state is a key feature of the quantum phase transition.

(4). The ground state is

$$\frac{1}{N_+}\left(\frac{-(\Delta+\sqrt{64+\Delta^2})}{8}|12\rangle + |21\rangle\right) \tag{10}$$

with the energy $(-8+12b+3\Delta-3\sqrt{64+\Delta^2})/24$ when

$$(a)\ \ -6 < b \le -\frac{8}{3},\ \ \frac{-8b-2b^2}{2+b} < \Delta < -b,$$
$$(b)\ \ -\frac{8}{3} < b < 0,\ \ 4b < \Delta < -b.$$

It is easy for us to obtain the entropy

$$E(|\psi\rangle) = -\left(\frac{\Delta+\sqrt{64+\Delta^2}}{8N_+}\right)^2 \log_2\left(\frac{\Delta+\sqrt{64+\Delta^2}}{8N_+}\right)^2 - \frac{1}{N_+^2}\log_2\frac{1}{N_+^2}. \tag{11}$$

Obviously, when $\Delta = 0$, the entropy reaches its maximal value 1. The correlation function

$$\langle O\otimes O\rangle - \langle O\rangle\langle O\rangle = \frac{16(\Delta+\sqrt{64+\Delta^2})}{64+(\Delta+\sqrt{64+\Delta^2})^2} \tag{12}$$

where $O = E_{\alpha_1} - E_{-\alpha_1}$. It reaches its maximum 1 at the point $\Delta = 0$.

(5). The ground state is

$$\frac{1}{N_+}\left(-\frac{1}{8}(\sqrt{64+\Delta^2}+\Delta)|23\rangle + |32\rangle\right)$$

with the energy $(-8-12b+9\Delta-3\sqrt{64+\Delta^2})/24$ when

$$(a)\ \ b \le -\frac{8}{3},\ \ \Delta < \frac{24b+18b^2}{2+3b},$$

$$(b)\ \ -\frac{8}{3} < b \le 0,\ \ \Delta < 4b,$$

$$(c)\ \ 0 < b \le \frac{27}{10},\ \ \Delta < \frac{2b}{3},$$

$$(d)\ \ b > \frac{27}{10},\ \ \Delta < \frac{3}{2}(2-b) + \frac{1}{2}\sqrt{36-4b+b^2}.$$

The entropy is the same as equation (11). One also can obtain the correlation function

$$\langle O\otimes O\rangle - \langle O\rangle\langle O\rangle = \frac{16(\Delta+\sqrt{64+\Delta^2})}{64+(\Delta+\sqrt{64+\Delta^2})^2}, \qquad (13)$$

where $O = E_{\alpha_2} - E_{-\alpha_2}$. Similarly, the correlation function reaches its maximum 1 at the phase point $\Delta = 0$.

In the same way, we can obtain the entanglement of the ground state of the general case, in which the system is anisotropic and is interacted with non-uniform external fields. The result will be more complex. From above, it can be known that as the external fields or anisotropy factor are varied, the entanglement of the ground state is changed accordingly. At the energy critical point, the system is degenerate.

Here we only study a bipartite system. For a many-body system, it will be much more complex. From above calculation, we can guess even for an N-body system, the total entanglement will reach its maximum when the system is maximally symmetric, i.e., each particle has isotropic structure.

3 Three-level Lipkin-Meshkov-Glick model

In this section the three-orbital Lipkin-Meshkov-Glick (LMG) model is considered. This is a nontrivial model, analytically soluble in a few simple cases and numerical soluble in others, which mimics the shell-model pictures of the nucleus. The model has M particles that are distributed among three energy orbitals, each of which is M-fold degenerate.

One can define two-fermion operations that are symmetric under interchange of the particle labels,

$$G_{kl} \equiv \sum_{m=1}^{M} a^\dagger_{km} a_{lm}. \qquad (14)$$

The Hamiltonian can be written in terms of these operators only,

$$\mathcal{H} = \sum_{k=0}^{2} \epsilon_k G_{kk} - \frac{1}{2}\sum_{k,l=0}^{2} V(1-\delta_{kl})G^2_{kl}. \qquad (15)$$

These operators satisfy the commutation relation,

$$[G_{kl}, G_{k'l'}] = G_{kl'}\delta_{k'l} - G_{k'l}\delta_{kl'}. \qquad (16)$$

From equation (16) one can know the model has $SU(3)$ symmetry. Here ϵ_k is the energy for each ortibals. In our calculatins we place the orbitals symmetrically about zero, $\epsilon_2 = -\epsilon_0 \equiv \epsilon, \epsilon_1 = 0$. we also choose vanishing interactions for particles in different orbitals, i.e.,$V_{kl} = V(1-\delta_{kl})$. The noninteracting ground state has all M particles in the ground orbital, and is represented by $|00\rangle$. The other basis states are written using the symmetric raising operators

$$|bc\rangle = C(b,c)G^b_{10}G^c_{20}|00\rangle$$

where $C(b,c)$ is the normalization parameter. Because there is only a finite number of ways of putting M particles in three orbitals, there is a finite number of bases for the LMG model. The dimension of the symmetric basis is $N = (M+2)(M+1)/2$[12]. Here we only consider the simplest case in which $M = 2$. In the six bases $|00\rangle$, $|01\rangle$, $|02\rangle$, $|10\rangle$, $|11\rangle$, $|20\rangle$, the Hamiltonian can be written as

$$\frac{\mathcal{H}}{\epsilon} = \begin{pmatrix} -2 & 0 & -x & 0 & 0 & -x \\ 0 & 0 & 0 & 0 & 0 & 0 \\ -x & 0 & 2 & 0 & 0 & -x \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -x & 0 & -x & 0 & 0 & 0 \end{pmatrix} \qquad (17)$$

where $x = V/\epsilon$. From the Hamiltonian, we can obtain the ground state energy and corresponding eigenvectors. The entropy of the ground state can be plotted in Figure 2. When x approaches zero, the entropy also approaches zero. From Figure 3, one can see at the point $x = 1.051$, the first derivative of entropy dE/dx reaches its maximum 0.201. At the point $x = -0.946$, it reaches its minimum -0.0342. Here these points seems not correspond to any phase transition points.

4 Spin-1 bipartite system

In this section, we discuss the spin-1 bipartite system which is interacted with external magnetic fields. The Hamiltonian can be written as follows

$$\mathcal{H} = \sum_{\langle i,j\rangle} S^x_i S^x_j + S^y_i S^y_j + \Delta S^z_i S^z_j + h_1 S^z_i + h_2 S^z_j, \quad (18)$$

where h_1 and h_2 denote the external fields. In general, they are not uniform. In this paper, we only study the pairwise entanglement of two neighbor spins. In this section, the spin-1 state has the bases $|\uparrow\rangle$, $|0\rangle$ and $|\downarrow\rangle$. In such bases, equation (18) can be reexpressed as 9×9 matrix. We note

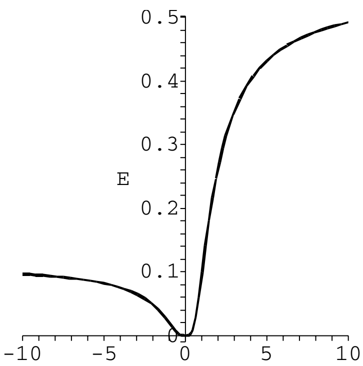

Fig. 2. Von Neumann entropy versus $x(= V/\epsilon)$.

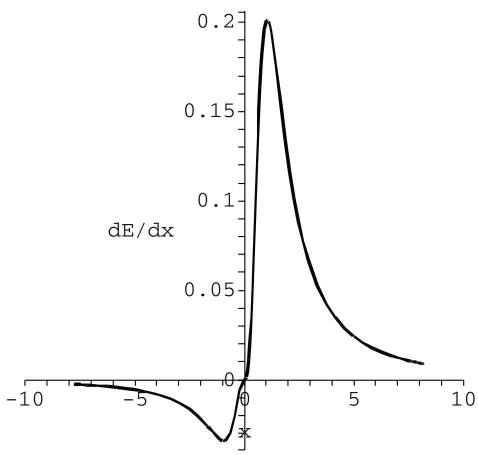

Fig. 3. dE/dx versus $x(= V/\epsilon)$, where E is the von Neumann entropy.

the spin-1 system carries out $SO(3)$ representation. Here, we only consider a simple case in which the external fields are uniform, i.e., $h_1 = h_2 = h$. One can obtain that the ground state is

$$|\psi\rangle = \frac{1}{N'}\left(|\uparrow\downarrow\rangle - \frac{1}{2}(\sqrt{8+\Delta^2} - \Delta)|00\rangle + |\downarrow\uparrow\rangle\right) \quad (19)$$

with the energy $(-\Delta - \sqrt{8+\Delta^2})/2$ when h and Δ satisfy

$$(a)\quad h \leq 0,\quad \Delta > \frac{1+2h-h^2}{h-1},$$

$$(b)\quad h > 0,\quad \Delta > \frac{h^2+2h-1}{h+1}.$$

One can obtain that the entropy is

$$E(|\psi\rangle) = -\frac{2}{N'^2}\log_2\frac{1}{N'^2} - \left(\frac{\sqrt{8+\Delta^2}-\Delta}{2N'}\right)^2 \log_2\left(\frac{\sqrt{8+\Delta^2}-\Delta}{2N'}\right)^2, \quad (20)$$

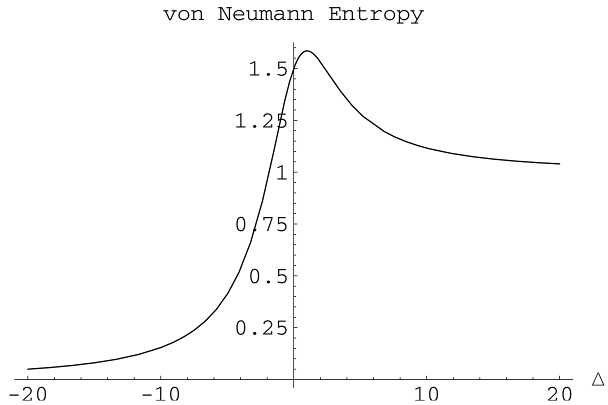

Fig. 4. Von Neumann entropy versus anisotropy factor Δ of spin-1 model.

where N' is the normalization parameter, and $N'^2 = (8 + (\sqrt{8+\Delta^2} - \Delta)^2)/4$.

The entropy can be plotted in Figure 4. From the figure, one can know the maximum of entropy is $\log_2 3$ at the point $\Delta = 1$, i.e., the model is isotropic. When the model is anisotropy, the symmetry is broken. The phase transition occurs. When the anisotropy factor Δ approaches positive infinity, the state becomes $(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ and the entropy approaches 1. It is obvious when Δ approaches negative infinity, the state is $|00\rangle$ and the entropy vanishes.

When $h > 0, \Delta < h - 1$ or $h < 0, \Delta < -1 - h$, the ground state is not entangled. When h and Δ do not satisfy above conditions, one can find that the entropy is 1. It is not maximally entangled.

5 Summary

In this paper, we have studied the three kinds of two-qutrit models. The SU(3) model (Eq. (2)) and the spin-1 model (Eq. (18)) are strongly correlated models but they have different symmetries. The former carries out $SU(3)$ representation and the latter $SO(3)$ representation. Due to the different symmetry, the entanglement shows different relationship between the phase transition and the symmetry. At the phase transition point, the entanglement of the model reaches its maximum and the correlation function reaches maximum too. It confirms that the large values of the correlation function imply a highly entangled ground state [6]. Critical points correspond to the situation where the lattice is most entangled. It is valid for high dimensional system. The LMG model has $SU(3)$ symmetry. We gave the relationship between the entropy and some parameters. Thus one can manipulate the entanglement via changing the parameters.

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