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Low temperature properties of one-dimensional SU(4) Hubbard-like model at low concentration

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Abstract. On the basis of Bethe ansatz solution of the one dimensional SU(4) Hubbard-like model, we have studied its thermodynamic properties by means of the Yang-Yang thermodynamic Bethe ansatz. The Landé g factors for both spin and orbit are taken into account so as to describe electrons with orbital degeneracy. The free energy at low temperature is given and the specific heat both in the strong coupling and weak coupling limits are obtained. At the same time, we also point out the three possible critical fields, which may cause a quantum phase transition.

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1 Introduction

The study of integrable models has a long and rich history in condensed matter physics beginning with Bethe's solution of the one-dimensional Heisenberg Model and extending nowadays to a variety of soluble models providing the paradigms that enrich much of our physical intuition. Recently there has been much interest in the studies on the $3d$ electrons in transitional metal oxides [1] because of the existence of orbital degrees of freedom [2,3] in addition to the spin one. Since the SU(4) symmetry was first pointed out [4], there has been various studies, such as critical properties in photoemission spectra for the one-dimensional Mott insulator with orbital degeneracy [5]. One dimensional solutions of the model in the insulating limit [6,7] as well as the Hubbard-like model [8] were presented. There were discussions of the thermodynamics based on the SU(N) generalized Bethe ansatz equation [13], but the physical meaning of the external field is ambiguous there because it is not associated with spin and orbital degree of freedom directly.

In present paper, we study the low-temperature thermodynamics of the SU(4) Hubbard-like model for electrons with 2-fold orbital degeneracy at low concentration. The thermal equilibrium is discussed exactly on the basis of the known Bethe ansatz equations by taking account of the Landé g factor. In the next section we first briefly present the Bethe ansatz equation under consideration, then discuss the thermodynamics of the model by considering a string hypothesis. In Section 3, we calculate the formal expressions of the thermodynamic quantities, such as free energy etc... In Section 4, the case of low temperature is discussed extensively.

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2 The model and its spectrum in thermodynamic limit

The model Hamiltonian reads

$$\mathcal{H} = -t\sum_{i,a}\mathcal{P}(C^{+}_{i,a}C_{i+1,a}+C^{+}_{i+1,a}C_{i,a})\mathcal{P}+U\sum_{i,a<a'}n_{i,a}n_{i,a'} \quad (1)$$

where $i = 1, 2, \ldots, L$ identify the lattice site, and $a = 1, 2, ..., 4$ labels the four states of the spin and orbitals [4]. The $\mathcal{P}$ projects the Hilbert space onto the sector so that the sites are only occupied by at most two electrons. The internal degree of freedom (1) is specified to the spin and orbitals in the present model. The Bethe ansatz equation for the spectrum was suggested [9,8] as

$$\begin{aligned}
\mathrm{e}^{\mathrm{i}k_j L} &= \prod_{b=1}^{M}\frac{\sin k_j - \lambda_b + \mathrm{i}\eta}{\sin k_j - \lambda_b - \mathrm{i}\eta},\\
\prod_{l=1}^{N}\frac{\lambda_a - \sin k_l + \mathrm{i}\eta}{\lambda_a - \sin k_l - \mathrm{i}\eta} &= -\prod_{b=1}^{M}\frac{\lambda_a-\lambda_b+\mathrm{i}2\eta}{\lambda_a-\lambda_b-\mathrm{i}2\eta}\prod_{c=1}^{M'}\frac{\mu_c-\lambda_a+\mathrm{i}\eta}{\mu_c-\lambda_a-\mathrm{i}\eta},\\
\prod_{b=1}^{M}\frac{\mu_a-\lambda_b+\mathrm{i}\eta}{\mu_a-\lambda_b-\mathrm{i}\eta} &= -\prod_{c=1}^{M'}\frac{\mu_a-\mu_c+\mathrm{i}2\eta}{\mu_a-\mu_c-\mathrm{i}2\eta}\prod_{d=1}^{M''}\frac{\nu_d-\mu_a+\mathrm{i}\eta}{\nu_d-\mu_a-\mathrm{i}\eta},\\
\prod_{b=1}^{M'}\frac{\nu_a-\mu_b+\mathrm{i}\eta}{\nu_a-\mu_b-\mathrm{i}\eta} &= -\prod_{c=1}^{M''}\frac{\nu_a-\mu_c+\mathrm{i}2\eta}{\nu_a-\mu_c-\mathrm{i}2\eta}.
\end{aligned} \quad (2)$$

where $\eta = U/4t$. In continuous limit, equations (2) reduce to the Bethe ansatz equations obtained by Sutherland [10]. Unlike the conventional SU(2) Hubbard model the SU(4) generalization (2) of Lieb-Wu solution [11] is valid at low temperature and low concentration [12,13].

For the ground state (*i.e.*, at zero temperature), k, λ, μ, ν are real roots of the Bethe ansatz equation (2). However, for the excited state, they can be complex roots. We will not take account of the complex roots in the charge space k for repulsive interaction [14]. The complex roots of λ, μ, ν always form a "bound state" with several similar kinds of rapidity which arise from the consistency of both sides of the Bethe ansatz equations. The complex roots are defined as

$$\begin{aligned} \Lambda_a^{nj} &= \lambda_a^n + (n+1-2j)\mathrm{i}\eta, \quad j = 1,2\ldots,n, \\ U_b^{mj} &= \mu_b^m + (m+1-2j)\mathrm{i}\eta, \quad j = 1,2\ldots,m, \\ V_c^{lj} &= \nu_c^l + (l+1-2j)\mathrm{i}\eta, \quad j = 1,2\ldots,l, \end{aligned} \tag{3}$$

up to the order $O(\mathrm{e}^{-L})$ which vanishes in the thermodynamic limit. Substituting those strings into equation (2) and taking the logarithm of it, we get

$$\begin{aligned} 2\pi I_j &= k_j L + 2\sum_{an} \tan^{-1}\Big(\frac{\sin k_j - \lambda_a^n}{n\eta}\Big), \\ 2\pi J_a^n &= 2\sum_{l} \tan^{-1}\Big(\frac{\lambda_a^n - \sin k_l}{n\eta}\Big) \\ &\quad - 2\sum_{blt} A_{nlt} \tan^{-1}\Big(\frac{\lambda_a^n - \lambda_b^l}{l\eta}\Big) \\ &\quad - 2\sum_{clt} B_{nlt} \tan^{-1}\Big(\frac{\mu_c^l - \lambda_a^n}{t\eta}\Big), \\ 2\pi K_a^n &= 2\sum_{blt} B_{nlt} \tan^{-1}\Big(\frac{\mu_a^n - \lambda_b^l}{t\eta}\Big) \\ &\quad - 2\sum_{clt} A_{nlt} \tan^{-1}\Big(\frac{\mu_a^n - \mu_c^l}{t\eta}\Big) \\ &\quad - 2\sum_{dlt} B_{nlt} \tan^{-1}\Big(\frac{\nu_d^l - \mu_a^n}{t\eta}\Big), \\ 2\pi Q_a^n &= 2\sum_{blt} B_{nlt} \tan^{-1}\Big(\frac{\nu_a^n - \mu_b^l}{t\eta}\Big) \\ &\quad - 2\sum_{clt} A_{nlt} \tan^{-1}\Big(\frac{\nu_a^n - \nu_c^l}{t\eta}\Big). \end{aligned} \tag{4}$$

where

$$A_{nlt} = \begin{cases} 1, \text{ for } t = n+l, |n-l|, \\ 2, \text{ for } t = n+l-2, \cdots, |n-l|+2, \\ 0, \text{ otherwise} \end{cases}$$

and

$$B_{nlt} = \begin{cases} 1, \text{ for } t = n+l-1, n+l-3, ..., |n-l|+1, \\ 0, \text{ otherwise.} \end{cases}$$

Now we consider the question in the thermodynamic limit $N, L, M, M', M'' \rightarrow \infty$ with a fixed concentration $D = N/L$ by introducing the distribution of roots and holes for k, λ, μ, ν respectively:

$$\begin{aligned} \frac{1}{L}\frac{\mathrm{d}I(k)}{\mathrm{d}k} &= \rho(k) + \rho^h(k), \\ \frac{1}{L}\frac{\mathrm{d}J_n(\lambda)}{\mathrm{d}\lambda} &= \sigma_n(\lambda) + \sigma_n^h(\lambda), \\ \frac{1}{L}\frac{\mathrm{d}K_n(\mu)}{\mathrm{d}\mu} &= \omega_n(\mu) + \omega_n^h(\mu), \\ \frac{1}{L}\frac{\mathrm{d}Q_n(\nu)}{\mathrm{d}\nu} &= \tau_n(\nu) + \tau_n^h(\nu). \end{aligned} \tag{5}$$

We obtain the following coupled integral equations.

$$\begin{aligned} \rho + \rho^h &= \frac{1}{2\pi} + \sum_n \cos k \int K_n(\sin k - \lambda)\sigma_n(\lambda)\mathrm{d}\lambda, \\ \sigma_n + \sigma_n^h &= \int K_n(\lambda - \sin k)\rho(k)\mathrm{d}k \\ &\quad - \sum_{ml} A_{nml} \int K_l(\lambda - \lambda')\sigma_m(\lambda')\mathrm{d}\lambda' \\ &\quad + \sum_{lt} B_{nlt} \int K_t(\lambda - \mu)\omega_l(\mu)\mathrm{d}\mu, \\ \omega_n + \omega_n^h &= \sum_{lt} B_{nlt} \int K_t(\mu - \lambda)\sigma_l(\lambda)\mathrm{d}\lambda \\ &\quad - \sum_{lt} A_{nlt} \int K_t(\mu - \mu')\omega_l(\mu')\mathrm{d}\mu' \\ &\quad + \sum_{lt} B_{nlt} \int K_t(\mu - \nu)\tau_l(\nu)\mathrm{d}\nu, \\ \tau_n + \tau_n^h &= \sum_{lt} B_{nlt} \int K_t(\nu - \mu)\omega_l(\mu)\mathrm{d}\mu \\ &\quad - \sum_{lt} A_{nlt} \int K_t(\nu - \nu')\tau_l(\nu')\mathrm{d}\nu' \end{aligned} \tag{6}$$

where $K_n(x) = \pi^{-1}n\eta/(n^2\eta^2 + x^2)$, and the integration limits are defined by

$$\begin{aligned} \frac{N}{L} &= \int_{-Q_0}^{Q_0} \rho(k)\mathrm{d}k, \\ \frac{M}{L} &= \sum_n n \int_{-B_1}^{B_1} \sigma_n(\lambda)\mathrm{d}\lambda, \\ \frac{M'}{L} &= \sum_n n \int_{-B_2}^{B_2} \omega_n(\mu)\mathrm{d}\mu, \\ \frac{M''}{L} &= \sum_n n \int_{-B_3}^{B_3} \tau_n(\nu)\mathrm{d}\nu. \end{aligned} \tag{7}$$

Since $|1\rangle = |\underline{\uparrow}\rangle$, $|2\rangle = |\overline{\uparrow}\rangle$, $|3\rangle = |\underline{\downarrow}\rangle$ and $|4\rangle = |\overline{\downarrow}\rangle$, the

z-components of the total spin and orbital momentum are given by,

$$\frac{T_z}{L} = \frac{1}{2}\int \rho \mathrm{d}k + \sum_n n \int \omega_n \mathrm{d}\mu - \sum_n n \int \sigma_n \mathrm{d}\lambda - \sum_n n \int \tau_n \mathrm{d}\nu,$$
$$\frac{S_z}{L} = \frac{1}{2}\int \rho \mathrm{d}k - \sum_n n \int \omega_n \mathrm{d}\mu. \quad (8)$$

The energy is given by

$$\frac{E}{L} = -2t \int \cos k \rho(k) \mathrm{d}k, \quad (9)$$

and the magnetization by

$$M_z = g_s S_z + g_t T_z, \quad (10)$$

where g_s, g_t are Landé g factors that we know $g_s = 2, g_t = 1$, which was ignored in previous literature because of the inability to related it to the spin-orbital model.

3 Thermal equilibrium

Yang and Yang [15] first introduced a definition of entropy in terms of the distribution of roots and holes for Bethe ansatz solvable systems. This method was recently carefully compared with the transfer matrix approach, the results of both approaches were shown to be in agreement with each other [16]. In order to obtain thermal equilibrium at a finite temperature, we should maximize the free energy $\Omega = E + E_J - T\mathcal{S} - \mathcal{A}N$ where $E_J = -HM_z$ is the Zeemann energy due to the external magnetic field and $\mathcal{A}$ is the chemical potential. The entropy is given by

$$\begin{aligned}\frac{\mathcal{S}}{L} = &\int \left[(\rho + \rho^h)\ln(\rho + \rho^h) - \rho \ln \rho - \rho^h \ln \rho^h\right] \mathrm{d}k \\ &+ \sum_n \int \left[\left(\sigma_n + \sigma_n^h\right)\ln(\sigma_n + \sigma_n^h)\right. \\ &\left.-\sigma_n \ln \sigma_n - \sigma_n^h \ln \sigma_n^h\right] \mathrm{d}\lambda \\ &+ \sum_n \int \left[\left(\omega_n + \omega_n^h\right)\ln(\omega_n + \omega_n^h) - \omega_n \ln \omega_n\right. \\ &\left.-\omega_n^h \ln \omega_n^h\right] \mathrm{d}\mu + \sum_n \int \left[\left(\tau_n + \tau_n^h\right)\ln(\tau_n + \tau_n^h)\right. \\ &\left.-\tau_n \ln \tau_n - \tau_m^h \ln \tau_n^h\right] \mathrm{d}\nu. \end{aligned} \quad (11)$$

We obtain the Helmholtz free energy from the condition $\delta\Omega = 0$ that

$$F = -\frac{TL}{2\pi}\int \ln[1 + \mathrm{e}^{-\epsilon}]\mathrm{d}k, \quad (12)$$

where ϵ should be solved from the following equations,

$$\begin{aligned}\epsilon = &-\frac{1}{T}\Big[2t\cos k + \mathcal{A} + \frac{1}{2}(g_s + g_t)H\Big] \\ &- \sum_n \int K_n(\lambda - \sin k)\ln\left[1 + \mathrm{e}^{-\theta_n(\lambda)}\right]\mathrm{d}\lambda, \\ \theta_n = &\, g_t n H/T \\ &- \int \cos k K_n(\sin k - \lambda)\ln\left[1 + \mathrm{e}^{-\epsilon(k)}\right]\mathrm{d}k \\ &+ \sum_{lt} A_{nlt}\int K_t(\lambda - \lambda')\ln\left[1 + \mathrm{e}^{-\theta_l(\lambda')}\right]\mathrm{d}\lambda' \\ &- \sum_{lt} B_{nlt}\int K_t(\mu - \lambda)\ln\left[1 + \mathrm{e}^{-\zeta_l(\mu)}\right]\mathrm{d}\mu, \\ \zeta_n = &\,(g_s - g_t)nH/T \\ &- \sum_{lt} B_{nlt}\int K_t(\lambda - \mu)\ln\left[1 + \mathrm{e}^{-\theta_l(\lambda)}\right]\mathrm{d}\lambda \\ &+ \sum_{lt} A_{nlt}\int K_t(\mu - \mu')\ln\left[1 + \mathrm{e}^{-\zeta_l(\mu')}\right]\mathrm{d}\nu' \\ &- \sum_{lt} B_{nlt}\int K_t(\nu - \mu)\ln\left[1 + \mathrm{e}^{-\xi_l(\nu)}\right]\mathrm{d}\nu, \\ \xi_n = &\, g_t n H/T \\ &- \sum_{lt} B_{nlt}\int K_t(\mu - \nu)\ln\left[1 + \mathrm{e}^{-\zeta_l(\mu)}\right]\mathrm{d}\mu \\ &+ \sum_{lt} A_{nlt}\int K_t(\nu - \nu')\ln\left[1 + \mathrm{e}^{-\xi_l(\nu')}\right]\mathrm{d}\nu'. \end{aligned} \quad (13)$$

In getting the above expressions, we have introduced the notations:

$$\mathrm{e}^{\epsilon} = \frac{\rho^h}{\rho}, \quad \mathrm{e}^{\theta_n} = \frac{\sigma_n^h}{\sigma_n}, \quad \mathrm{e}^{\zeta_n} = \frac{\omega_n^h}{\omega_n}, \quad \mathrm{e}^{\xi_n} = \frac{\tau_n^h}{\tau_n}\cdot \quad (14)$$

4 The case at low temperature

At very low temperature, the density of roots undergo a slight modification from that of the ground state, and the right hand side of the equations (14) are approximately zero below Fermi surface. Then we have

$$\begin{aligned}
\epsilon =& -\frac{1}{T}\Big[2t\cos k + \mathcal{A} + \frac{1}{2}(g_s+g_t)H\Big]\\
&+\int_{-B_1}^{B_1} K_1(\lambda-\sin k)\theta(\lambda)\mathrm{d}\lambda,\\
\theta =& g_t H/T + \int_{-Q_0}^{Q_0}\cos k K_1(\sin k-\lambda)\epsilon(k)\mathrm{d}k\\
&-\int_{-B_1}^{B_1} K_2(\lambda-\lambda')\theta(\lambda')\mathrm{d}\lambda'\\
&+\int_{-B_2}^{B_2} K_1(\mu-\lambda)\zeta(\mu)\mathrm{d}\mu,\\
\zeta =& (g_s-g_t)H/T + \int_{-B_1}^{B_1} K_1(\lambda-\mu)\theta(\lambda)\mathrm{d}\lambda\\
&-\int_{-B_2}^{B_2} K_2(\mu-\mu')\zeta(\mu')\mathrm{d}\mu'\\
&+\int_{-B_3}^{B_3} K_1(\nu-\mu)\xi(\nu)\mathrm{d}\nu,\\
\xi =& g_t H/T + \int_{B_2}^{B_2} K_1(\mu-\nu)\zeta(\mu)\mathrm{d}\mu\\
&-\int_{-B_3}^{B_3} K_2(\nu-\nu')\xi(\nu')\mathrm{d}\nu', \qquad (15)
\end{aligned}$$

which, in the continuous limit, reduces to the result of Schlottmann [17] when considering the SU(4) case. It is plausible to take $B_1, B_2, B_3 = \infty$ in our present case. By making a Fourier transform of the second, third and fourth equation of equation (15), we have

$$\begin{aligned}
\tilde{\theta} =& g_t\tilde{h} + \frac{1}{2\pi}\int_{-Q_0}^{Q_0}\cos k\mathrm{e}^{-\eta|w|-\mathrm{i}w\sin k}\epsilon(k)\mathrm{d}k\\
&-\mathrm{e}^{-2\eta|w|}\tilde{\theta} + \mathrm{e}^{-\eta|w|}\tilde{\zeta},\\
\tilde{\zeta} =& (g_s-g_t)\tilde{h} + \mathrm{e}^{-\eta|w|}\tilde{\theta} - \mathrm{e}^{-2\eta|w|}\tilde{\zeta} + \mathrm{e}^{-\eta|w|}\tilde{\xi},\\
\tilde{\xi} =& g_t\tilde{h} + \mathrm{e}^{-\eta|w|}\tilde{\zeta} - \mathrm{e}^{-2\eta|w|}\tilde{\xi} \qquad (16)
\end{aligned}$$

where $\tilde{h} = H\delta(w)/T$. It is not difficult to derive an integral equation for ϵ:

$$\begin{aligned}
\epsilon =& -[2t\cos k\\
&+\mathcal{A}]/T + \frac{1}{\eta}\int_{-k_F}^{k_F}\cos k'\epsilon(k')R_3\Big(\frac{\sin k-\sin k'}{\eta}\Big)\mathrm{d}k \qquad (17)
\end{aligned}$$

where k_F is the Fermi momentum and

$$R_n(x) = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\sinh(nw)}{\sinh(4w)}\mathrm{e}^{\mathrm{i}wx-|w|}\mathrm{d}w. \qquad (18)$$

In the absence of an external field ($H = 0$), equation (17) is rigorous. When the external field is weak, it is also plausible because the changes on the integral interval of spin and orbit rapidity is of order of $1/L$ in the thermodynamic limit. Moreover, even if we consider the contribution of the densities outside the Fermi surface, it is of order $\mathrm{e}^{-1/T}$. Clearly the presence of an external field has no explicit effect on the ratio of the density of charge roots and holes at low temperature, which manifests itself in the fact of spin-charge separation as well as orbit-charge separation in one dimension.

Once ϵ is solved, the free energy at low temperature can be evaluated by equation (12). Though an explicit expression can not be obtained from equation (17) in general case, it is easier for numerical calculation. Defining the density of roots at low temperature as $\rho = \rho_0 + \rho_1$, where ρ_1 is a slight deviation from the density of the ground state ρ_0 [8], we can obtain

$$\rho = \rho_0(1-\mathrm{e}^{\epsilon}) - \frac{1}{\pi}\int\frac{\cos k\eta\sigma_0 e^{\theta}\mathrm{d}\lambda}{\eta^2+(\sin k-\lambda)^2}. \qquad (19)$$

Substituting $\epsilon(k)$ of equation (17) into equation (12), we obtain

$$F = \frac{TL}{2\pi}\int\epsilon\mathrm{d}k - \frac{TL}{2\pi}\int\mathrm{e}^{\epsilon}\mathrm{d}k. \qquad (20)$$

Clearly when $T \to 0$ the free energy is just the ground state energy except for the additional chemical potential term $-\mathcal{A}L$ and the $\epsilon(k)$ just play a role of so called dressed energy.

Strong coupling limit

In the strong coupling limit $\eta \to \infty$, we have

$$\epsilon = -[2t\cos k + \mathcal{A}]/T. \qquad (21)$$

Since we consider the problem at low concentration, the Fermi surface is much less than π. Thus we can rewrite equation (21) as

$$\epsilon = [tk^2 - \kappa]/T, \qquad (22)$$

where $\kappa = 2t + \mathcal{A}$. And the distribution of the roots becomes

$$\rho = \frac{1}{2\pi}\frac{1}{1+\mathrm{e}^{[tk^2-\kappa]/T}}, \qquad (23)$$

with the condition $\int_{-k_F}^{k_F}\rho(k)\mathrm{d}k = D$, and when $T = 0$, it gives $k_F = \sqrt{\kappa_0/t}$. We obtain

$$\kappa = \kappa_0\left[1 - \frac{\pi^2T^2}{24\kappa_0^2}\right]^{-2}, \qquad (24)$$

where $\kappa_0 = t\pi^2D^2$ is the value of κ at zero temperature. Obviously the system's energy becomes

$$\frac{E}{L} \doteq \frac{\kappa_0^{3/2}}{3\pi\sqrt{t}}\left[1 + \frac{\pi^2T^2}{4\kappa_0^2}\right]. \qquad (25)$$

The specific heat and the entropy of the system are calculated as

$$C_V = \mathcal{S} = \frac{T}{6tD}, \qquad (26)$$

which is Fermi-liquid like.

Magnetic properties

In the presence of an external field, the densities of spin and orbit rapidities will change. We rewrite them as

$$\sigma = \sigma_0(1-\mathrm{e}^{\theta}),\quad \omega = \omega_0(1-\mathrm{e}^{\zeta}),\quad \tau = \tau_0(1-\mathrm{e}^{\xi}) \tag{27}$$

where θ, ζ, ξ are all negative and proportional to H/T. Approximately, they can be written as

$$\theta = \frac{1}{2}(g_s+g_t)\frac{H}{T}+\theta_0,\quad \zeta = g_s\frac{H}{T}+\zeta_0,\quad \xi = \frac{1}{2}(g_s+g_t)\frac{H}{T}+\xi_0. \tag{28}$$

Then the magnetization of the system has the form

$$M_z/L = \frac{1}{2}(g_s+g_t)\int\rho\mathrm{d}k - g_t\int\sigma\mathrm{d}\lambda - (g_s-g_t)\int\omega\mathrm{d}\mu - g_t\int\tau\mathrm{d}\nu. \tag{29}$$

Concerning $M=3N/4, M'=N/2, M''=N/4$ at the ground state, we obtain

$$M_z/L = g_t\int\sigma_0\mathrm{e}^{\theta}\mathrm{d}\lambda + (g_s-g_t)\int\omega_0\mathrm{e}^{\zeta}\mathrm{d}\mu + g_t\int\tau_0\mathrm{e}^{\xi}\mathrm{d}\nu \tag{30}$$

and the susceptibility in the weak external field is

$$\chi = \frac{g_t}{2T}(g_s+g_t)\int\sigma_0\mathrm{e}^{\theta}\mathrm{d}\lambda + \frac{g_s}{T}(g_s-g_t)\int\omega_0\mathrm{e}^{\zeta}\mathrm{d}\mu + \frac{g_t}{2T}(g_s+g_t)\int\tau_0\mathrm{e}^{\xi}\mathrm{d}\nu. \tag{31}$$

At the ground state, if the external field is not zero, the system will be magnetized, which corresponds to the decreasing of M, M', M''. From equations (15), we can see that there exist three critical points H_c^1, H_c^2, H_c^3, which are determined by the condition $T\xi(0)=0, T\zeta(0)=0$ and $T\theta(0)=0$, respectively. Hence there exist four phases, which, in the language of Young tableau, can be characterized by $[N-M, M-M', M'-M'', M'']$, $[N-M, M-M', M']$, $[N-M, M]$, and $[N]$ respectively. The most interesting thing is, in the second phase, there may exist orbital antipolarization processes, which arise from the process $|\underline{\downarrow}\rangle \to |\overline{\uparrow}\rangle$. The contribution of orbital momentum to the magnetization M_z is negative, though that of spin to M_z is always positive, which results in the total magnetic susceptibility being positive. Thus the Landé factor of orbital g_t play a crucial role in the magnetization process [18]. Since for $g_t=0$, this process becomes much simpler [19], in which only one critical point and the final state is orbital singlet. In our case, the final state is described by $M=M'=M''=0$, and the densities of λ, μ, ν are all zero. Then we have

$$\rho+\rho^h = \rho(1+\mathrm{e}^{\epsilon}) = \frac{1}{2\pi}, \tag{32}$$

where $\epsilon(k)$ is given by

$$\epsilon(k)T = -\Big[2t\cos k + \mathcal{A} + \frac{1}{2}(g_s+g_t)H\Big],\quad g_t H_c^3 = -\int\cos k K_1(\sin k-\lambda)\epsilon(k)T\mathrm{d}k. \tag{33}$$

At large coupling constant the value of H_c^3 scales as $1/\eta$. Clearly, in the limit $\eta\to\infty$, a small value of H can flip all spins and orbits to upward. The density of charge roots becomes

$$\rho = \frac{1}{2\pi}\frac{1}{1+\mathrm{e}^{-[2t\cos k+2t]/T}} \tag{34}$$

which takes the value of $1/2\pi$ at the ground state. The density of charge roots in the strong coupling limit takes the same form as that in a sufficiently strong external field.

However, the analytic expression for H_c^1 and H_c^2 are still difficult to obtain.

5 Summary

In this paper, we have discussed the one-dimensional SU(4) Hubbard-like Model in the thermodynamic limit, which is reliable at low temperature and low concentration. We have studied the property of the thermal equilibrium and obtained the equilibrium equations. In the low temperature limit, the T-dependent density of roots and free energy, the specific heat in the strong coupling limit, as well as magnetic properties of the system are calculated. We have also discussed the magnetization process in the ground state and pointed out three critical points which may cause a quantum phase transition.

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