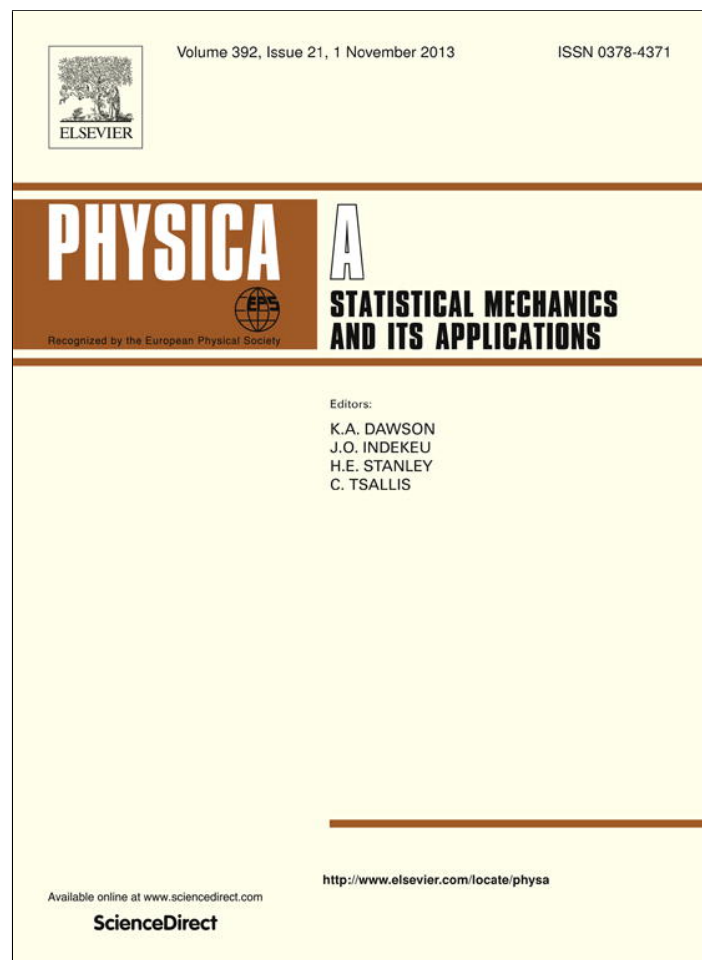


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## Time-reversal asymmetry in financial systems



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## H I G H L I G H T S

- We investigate large-fluctuation dynamics in financial markets.
- The dynamics is time-reversal symmetric in time scales of minutes.
- The dynamics is time-reversal asymmetric at daily time scales.
- Time-reversal asymmetry is induced by exogenous events.

## A R T I C L E I N F O

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## A B S T R A C T

We investigate the large-fluctuation dynamics in financial markets, based on the minute-to-minute and daily data of the Chinese Indices and the German DAX. The dynamic relaxation both before and after the large fluctuations is characterized by a power law, and the exponents  $p_{\pm}$  usually vary with the strength of the large fluctuations. The large-fluctuation dynamics is time-reversal symmetric at the time scale in minutes, while asymmetric at the daily time scale. Careful analysis reveals that the time-reversal asymmetry is mainly induced by external forces. It is also the external forces which drive the financial system to a non-stationary state. Different characteristics of the Chinese and German stock markets are uncovered.

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## 1. Introduction

Financial markets are complex systems which share common features with those in traditional physics. In recent years, large amounts of high-frequency data have piled up in stock markets. This allows an analysis of the fine structure and interaction of the financial dynamics, and many empirical results have been documented [1–11]. Although the price return of a financial index is short-range correlated in time, the volatility exhibits a long-range temporal correlation [2,3]. The dynamic behavior of volatilities is an important topic in econophysics [2,3,12,13].

Assuming that a financial market is in a stationary state, one may analyze its static statistical properties. For a comprehensive understanding of the financial market, however, it is also important to investigate the non-stationary dynamic properties. A typical example is the so-called financial crash [14,6]. Lillo and Mantegna study three huge crashes of the stock market, and find that the rate of volatilities larger than a given threshold after such market crashes decreases by a power law with certain corrections in shorter times [15]. This dynamic behavior is analogous to the classical Omori law, which describes the aftershocks following a large earthquake [16]. Selcuk analyzes the daily data of the financial indices from 10 emerging stock markets and also observed the Omori law after the two largest crashes [17]. Mu and Zhou extend such an analysis to the stock market in Shanghai [18]. Recently, Weber et al. demonstrate that the Omori law holds also after “intermediate shocks”, and the memory of volatilities is mainly related to such relaxation processes [19].

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Stimulated by these works, we systematically analyze the *large-fluctuation* dynamics in financial markets, based on the minute-to-minute and daily data of the Chinese Indices and German DAX. In our study, a large fluctuation is identified when its volatility is sufficiently large compared with the average one. At the time scale in minutes, those large volatilities may have nothing to do with real financial crashes or *rallies*, and represent only extremal fluctuations at the microscopic level. Even at the daily time scale, a large volatility maybe also not yet corresponding to a real financial crash or rally. But as the magnitude of the large volatility increases, it approaches a real financial crash or rally. In fact, the dynamic behavior of rallies has not been touched to our knowledge.

The purpose of this paper is multi-fold. We investigate the dynamic relaxation both *before* and *after* the large fluctuations. We focus on the time-reversal symmetry or asymmetry at different time scales. To achieve more reliable results, we introduce the remnant and anti-remnant volatilities to describe the large-fluctuation dynamics, different from those in Refs. [15,17,18]. More importantly, we examine the dynamic behavior of different categories of large fluctuations, and explore the origin of the time-reversal asymmetry at the daily time scale. We reveal how the financial system is driven to a non-stationary state by exogenous events, and compare the results of the mature German market and emerging Chinese market.

## 2. Large-fluctuation dynamics

In this paper, we have collected the daily data of the German DAX from 1959 to 2009 with 12 407 data points, and the minute-to-minute data from 1993 to 1997 with 360 000 data points. The daily data of the Shanghai Index are from 1990 to 2009 with 4 482 data points, and the minute-to-minute data are from 1998 to 2006 with 95 856 data points. The daily data of the Shenzhen Index are from 1991 to 2009 with 4 435 data points, and the minute-to-minute data are from 1998 to 2003 with 50 064 data points. The minute-to-minute data are recorded every minute in the German market, while every 5 min in the Chinese market. A working day is about 450 min in Germany while exactly 240 min in China. In our terminology, the results of the “Chinese Indices” are the averages of the Shanghai Index and Shenzhen Index.

Denoting a financial index at time  $t$  as  $P(t)$ , the return and volatility are defined as  $R(t) \equiv \ln P(t+1) - \ln P(t)$  and  $|R(t)|$  respectively. Naturally, the dynamic properties of volatilities may depend on the time scale. To study the dynamic relaxation after and before the large fluctuations, we introduce the remnant and anti-remnant volatilities,

$$v_{\pm}(t) = [\langle |R(t' \pm t)| \rangle_c - \sigma] / Z, \tag{1}$$

where  $Z = \langle |R(t')| \rangle_c - \sigma$ ,  $\sigma$  is the average volatility, and  $\langle \cdot \cdot \cdot \rangle_c$  represents the average over those  $t'$  with specified large volatilities. In our analysis, the large volatilities are selected by the condition  $|R(t')| > \zeta$ , and the threshold  $\zeta$  is well above  $\sigma$ , e.g.,  $\zeta = 2\sigma, 4\sigma, 6\sigma$ , and  $8\sigma$ . At the daily time scale, when  $\zeta$  is sufficiently large, the selected events correspond to the financial crashes or rallies. At the time scale in minutes, the events may be only extremal fluctuations. The remnant volatility  $v_+(t)$  describes how the system relaxes from a large fluctuation to the stationary state, while the anti-remnant volatility  $v_-(t)$  depicts how it approaches a large fluctuation.

Large shocks in volatilities are usually followed by a series of aftershocks. Thus we *assume* that both  $v_+(t)$  and  $v_-(t)$  obey a power law,

$$v_{\pm}(t) \sim (t + \tau_{\pm})^{-p_{\pm}}, \tag{2}$$

where  $p_{\pm}$  are the exponents and  $\tau_{\pm}$  are positive constants. In most cases studied in this paper, the constants  $\tau_{\pm}$  are rather small. For reducing the fluctuations, we integrate Eq. (2) from 0 to  $t$ . Thus the cumulative function of  $v_{\pm}(t)$  is written as

$$V_{\pm}(t) \sim \left[ (t + \tau_{\pm})^{1-p_{\pm}} - \tau_{\pm}^{1-p_{\pm}} \right] \tag{3}$$

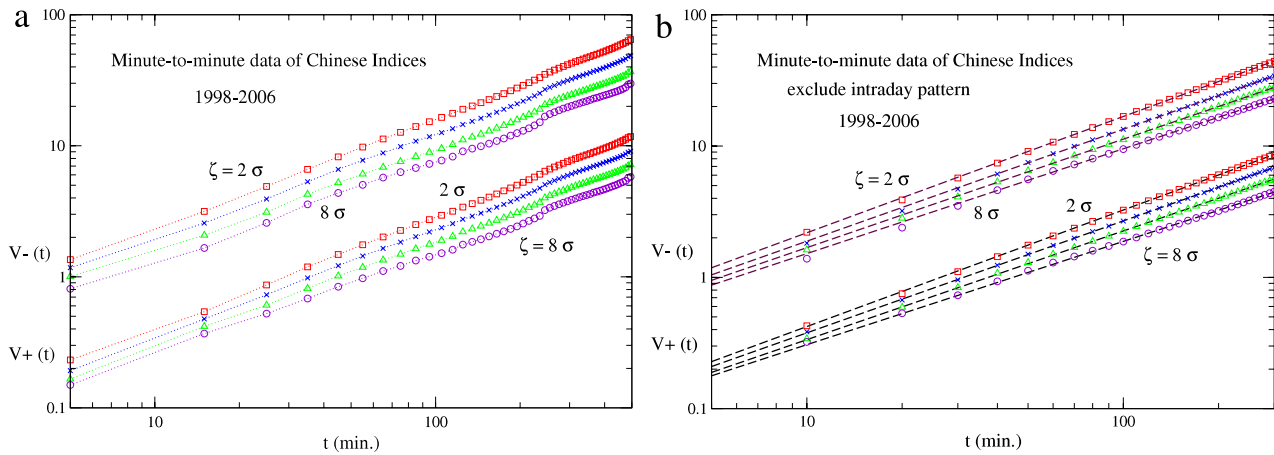
for  $p_{\pm} \neq 1$ . The power-law behavior of  $v_{\pm}(t)$  just represents the long-range temporal correlation of volatilities. Such a power-law behavior has been well understood in dynamic critical phenomena, even in the case far from equilibrium [20,21].

Our main motivation in this paper is to explore different characteristics of the large-fluctuation dynamics at the daily time scale and the time scale in minutes, especially the time-reversal symmetry or asymmetry. We first analyze the minute-to-minute data of the Chinese Indices and German DAX. Now  $|R(t)|$  is calculated in units of five minutes for the Chinese Indices and one minute for the German DAX. For the minute-to-minute data, a large volatility may not indicate a real macroscopic crash or rally, and it only possibly brings the dynamic system to a *microscopic* non-stationary state. In Fig. 1(a),  $V_{\pm}(t)$  of the Chinese Indices are displayed on a log–log scale. Due to the so-called intra-day pattern [22,23,3,8], the curves periodically fluctuate at a working day, i.e.,  $t \sim 240$  minutes. Such a kind of intra-day patterns should be removed. Following the procedure in Refs. [3,24], the intra-day pattern  $D(t'_{day})$  is defined as

$$D(t'_{day}) = \frac{1}{N} \sum_{j=1}^N |R_j(t'_{day})|, \tag{4}$$

where  $j$  runs over all the trading days  $N$ , and  $t'_{day}$  is the time in a trading day. To remove this intra-day pattern, we normalize the volatility at time  $t' = t'_{day}$  by

$$r(t'_{day}) = |R(t'_{day})| / D(t'_{day}). \tag{5}$$



**Fig. 1.** (a)  $V_{\pm}(t)$  for the minute-to-minute data of the Chinese Indices. From above, the threshold is  $\zeta = 2\sigma, 4\sigma, 6\sigma$  and  $8\sigma$  respectively. (b) The same as (a), but the intra-day pattern has been removed. Dashed lines show the power-law fits with Eq. (3). The  $\zeta$ -dependent exponents  $p_- = p_+$ , and  $\tau_{\pm} = 0$ .

**Table 1**

$p_{\pm}$  are measured with the minute-to-minute and daily data of the Chinese Indices (CHN) and German DAX. For CHN(min), DAX(min) and CHN(day),  $\tau_{\pm} = 0$ . In the fifth sector,  $p_{\pm}$  are estimated for DAX(day) from the tails of the curves in Fig. 2.

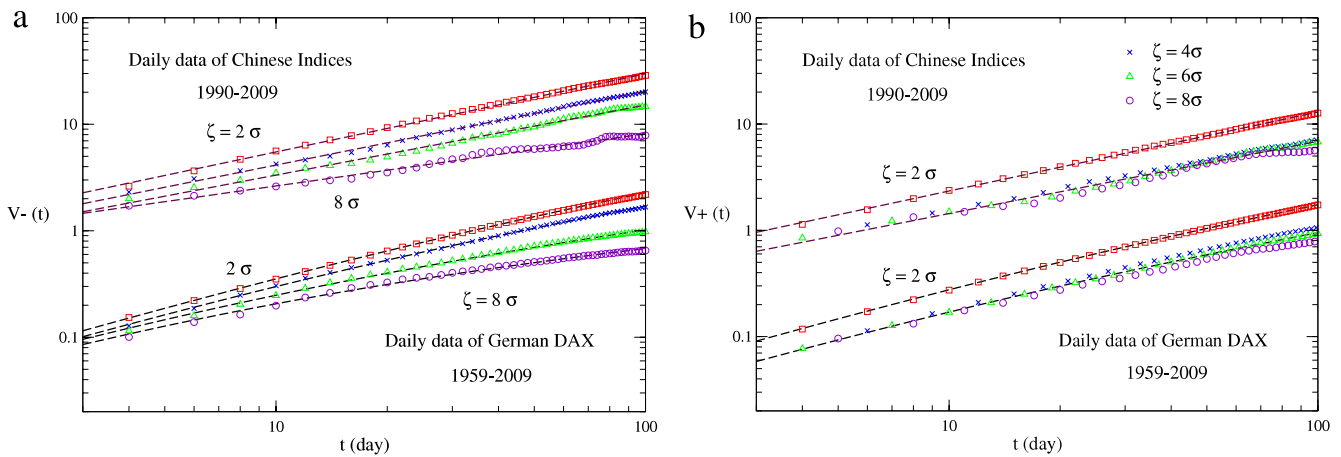
$\zeta$	$2\sigma$	$4\sigma$	$6\sigma$	$8\sigma$
CHN (min)				
$p_-$	0.11 (1)	0.15 (1)	0.17 (1)	0.20 (1)
$p_+$	0.11 (1)	0.15 (1)	0.18 (1)	0.22 (1)
DAX (min)				
$p_-$	0.16 (1)	0.23 (1)	0.27 (1)	0.29 (1)
$p_+$	0.16 (1)	0.22 (1)	0.26 (1)	0.29 (1)
CHN (day)				
$p_-$	0.27 (3)	0.31 (4)	0.36 (4)	0.51 (6)
$p_+$	0.26 (2)	0.32 (3)	0.33 (4)	0.36 (5)
DAX (day)				
$\tau_-$	13.11	9.06	4.07	3.78
$p_-$	0.41 (3)	0.47 (4)	0.60 (5)	0.77 (7)
$\tau_+$	10.66	9.23	7.28	3.98
$p_+$	0.40 (2)	0.42 (3)	0.45 (5)	0.46 (5)
$p_-$	0.28 (3)	0.30 (3)	0.50 (5)	0.61 (5)
$p_+$	0.25 (2)	0.28 (2)	0.31 (3)	0.35 (4)

With  $r(t')$ , we then recalculate  $V_{\pm}(t)$ . This is shown in Fig. 1(b). Now an almost perfect power-law behavior is observed for both  $V_-(t)$  and  $V_+(t)$ , starting from  $t \sim 5$  minutes. The curves of  $V_{\pm}(t)$  of the German DAX look very similar to those of the Chinese Indices, with the intra-day pattern around  $t \sim 450$  minutes.

Fitting the curves of  $V_{\pm}(t)$  of the minute-to-minute data to Eq. (3), we obtain the exponents  $p_{\pm}$  summarized in the first and second sectors of Table 1. For both the Chinese Indices and German DAX, both  $p_+$  and  $p_-$  increase with the threshold  $\zeta$ . Especially,  $p_+$  and  $p_-$  of every  $\zeta$  are equal within statistical errors. In other words, the dynamic behavior at the microscopic time scale, typically in minutes, is symmetric before and after large volatilities. The exponents  $p_{\pm}$  of the German DAX are somewhat larger than those of the Chinese Indices.

Due to the overnight information in financial markets, the dynamic properties of the overnight returns may be different from the usual ones [25]. To study the possible influence of the overnight returns, we may remove the data points of the overnight returns in the calculations of  $V_{\pm}(t)$ . The results show that the exponents  $p_{\pm}$  remain almost the same.

To further understand the large-fluctuation dynamics of financial markets, we have also calculated  $V_{\pm}(t)$  with the daily data of the Chinese Indices and German DAX. As shown in Fig. 2, the dynamic behavior of  $V_{\pm}(t)$  can also be described by Eq. (3), although the curves look slightly fluctuating, compared with those of the minute-to-minute data. From the fitting, we obtain  $\tau_{\pm} \approx 0$  for the Chinese Indices, while  $\tau_{\pm} \neq 0$  for the German DAX. The exponents  $p_{\pm}$  are listed in the third and fourth sectors of Table 1.  $p_{\pm}$  of the daily data also vary with  $\zeta$ , similar to those of the minute-to-minute data. However, the  $\zeta$ -dependence of  $p_+$  becomes obviously weaker, i.e.,  $p_- \neq p_+$ . In other words, the time-reversal symmetry before and after the large fluctuations is violated at the daily time scale, for both the Chinese Indices and German DAX. Again  $p_{\pm}$  of the German DAX are larger than those of the Chinese Indices. But a large part of the difference looks effectively induced by the short-time behavior. If one directly estimates  $p_{\pm}$  from the slopes of the tails of the curves,  $p_{\pm}$  of the German DAX are less



**Fig. 2.**  $V_{\pm}(t)$  for the daily data of the Chinese Indices and German DAX. From above, the threshold is  $\zeta = 2\sigma, 4\sigma, 6\sigma$  and  $8\sigma$  respectively. Dashed lines show the power-law fits with Eq. (3).  $\tau_{\pm} = 0$  for the Chinese Indices and  $\tau_{\pm} \neq 0$  for the German DAX. In (a),  $p_{-}$  varies with  $\zeta$ . In (b), the  $\zeta$ -dependence of  $p_{+}$  is weak. For clarity, some curves have been shifted downwards or upwards.

different from those of the Chinese indices. This is shown in the fifth sector of Table 1. In fact, as an emerging market, the Chinese market shares common features with the Western markets in basic statistical properties [24,26], while exhibiting its own characteristics in the return-volatility correlation and spatial structure [9–11].

At this stage one may ask whether the power law in Eq. (3) provides the best fit to the empirical data in financial markets. With the method proposed by Preis et al. in Ref. [27], for example, the power-law behavior of  $V_{\pm}(t)$  passes the Kolmogorov–Smirnov test for the minute-to-minute data with any thresholds  $\zeta$ , and for the daily data with  $\zeta = 2\sigma$  and  $4\sigma$ . For the daily data with  $\zeta = 6\sigma$  and  $8\sigma$ , the curves  $V_{\pm}(t)$  are somewhat fluctuating, and one could not make a clear judgment. In this paper, however, our main purpose is to study the time-reversal symmetry or asymmetry before and after the large fluctuations, and we simply assume the power law in Eq. (3) and fit it to the empirical data in financial markets.

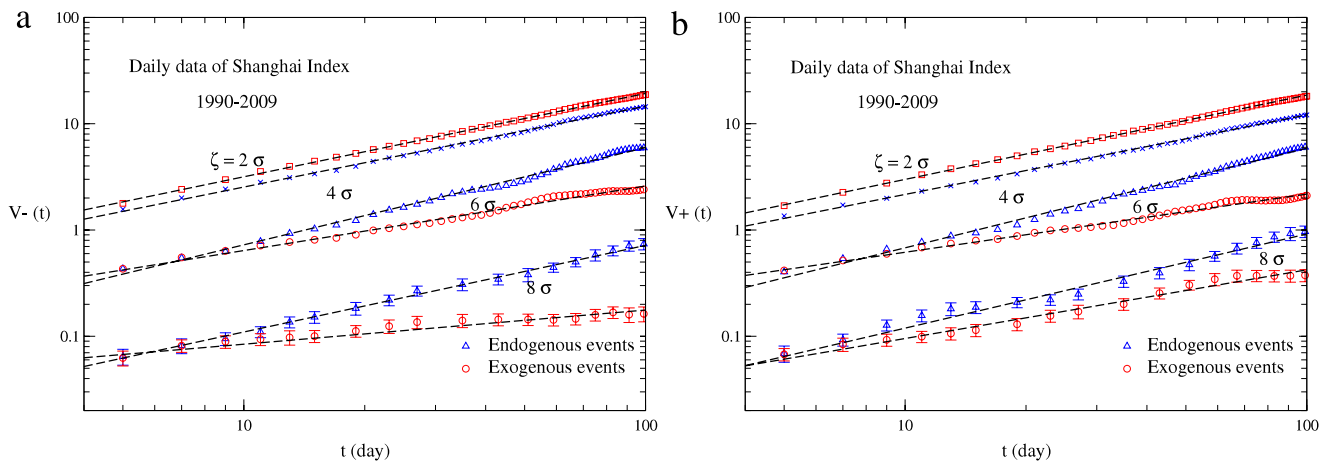
In Refs. [15,28], the dynamic relaxation after a financial crash, which is an event corresponding to an extremely large  $\zeta$  and with  $R(t') < 0$  in our terminology, has been investigated. The observable  $N_{+}(t)$ , which is the number of times that the volatility exceeds a certain threshold  $\zeta_1$  in the time  $t$  after the financial crash, decays by a power law. For comparison, we have also performed such an analysis. To reduce the fluctuations, we choose a large but not extremely large threshold  $\zeta = 2\sigma$  to gain some samples for average. Additionally we extend the calculations to both  $N_{+}(t)$  and  $N_{-}(t)$ . For the minute-to-minute data, we observe that  $N_{\pm}(t)$  of  $\zeta_1 = 2\sigma$  to  $5\sigma$  could be fitted by Eq. (3). The exponents  $p_{\pm}$  are weakly  $\zeta_1$ -dependent, and with  $p_{-} = p_{+}$ , i.e., similar to those for  $V_{\pm}(t)$  in Table 1. For the daily data, the fluctuations are large, although the asymmetric behavior between  $N_{+}(t)$  and  $N_{-}(t)$  could be qualitatively observed. The weak point of this analysis is that there are two thresholds  $\zeta$  and  $\zeta_1$ .

Up to now, we always average over all the selected large fluctuations in computing  $V_{\pm}(t)$  (or  $N_{\pm}(t)$ ). However, the large fluctuations could originate differently, and the dynamic relaxation may depend on the category of large fluctuations. Especially, it is puzzling *how the time-reversal asymmetry arises at the daily time scale?* Our first thought is to classify the large fluctuations  $|R(t')|$  by  $R(t') < 0$  and  $R(t') > 0$ , i.e., the so-called “crashes” and “rallies”. Such a classification is not illustrating at all at the time scale in minutes, and the exponents  $p_{\pm}$  remain the same for both the crashes and rallies. For the daily data of the Chinese Indices,  $p_{\pm}$  of the crashes and rallies are also only marginally different. The situation is subtle for the German DAX, and will be clarified below.

The large fluctuations at the daily time scale could be also classified into endogenous events and exogenous events [6,29]. An exogenous event is associated with the market’s response to external forces, and an endogenous event is generated by the dynamic system itself. Naturally, different stock markets may differently respond to the external forces. Looking carefully at the history of the Shanghai stock market, for example, we find that there are 9 exogenous events among the 16 large volatilities selected by the threshold  $\zeta = 8\sigma$ , as shown in Table 2. For the large volatilities corresponding to the thresholds such as  $\zeta = 2\sigma$  and  $4\sigma$ , it is not meaningful to naively identify the external forces. In Fig. 3,  $V_{\pm}(t)$  of  $\zeta = 6\sigma$  and  $8\sigma$  are displayed for the endogenous and exogenous events of the Shanghai Index. Obviously, the endogenous and exogenous events lead to different dynamic behaviors for both  $V_{+}(t)$  and  $V_{-}(t)$ . The dynamic relaxation of the exogenous events is faster. For the Shenzhen Index, we obtain qualitatively the same results.

For the German DAX, we have worked hard to identify the exogenous events, in spite of our unfamiliarity of the history of the German stock market. The findings are listed in Table 3. In Fig. 4,  $V_{\pm}(t)$  of  $\zeta = 6\sigma$  and  $8\sigma$  are displayed for the endogenous and exogenous events of the German DAX. Obviously,  $V_{-}(t)$  exhibits different dynamic behaviors for the endogenous and exogenous events, while  $V_{+}(t)$  remains almost the same. This behavior is different from that of the Shanghai Index. Although there exist some fluctuations, we may estimate the exponents  $p_{\pm}$  from the slopes of the curves, to quantify the findings in Figs. 3 and 4. The results are given in Table 4.

Our first observation is that the time-reversal asymmetry in the large-fluctuation dynamics at the daily time scale is mainly induced by the exogenous events, i.e.,  $p_{-} \approx p_{+}$  for the endogenous events and  $p_{-} > p_{+}$  for the exogenous events.



**Fig. 3.**  $V_{\pm}(t)$  for the daily data of the Shanghai Index. For  $\zeta = 6\sigma$  and  $8\sigma$ ,  $V_{\pm}(t)$  are displayed for endogenous and exogenous events separately. Error bars for  $\zeta = 8\sigma$  are given by standard deviations of all events, and those for  $\zeta = 2\sigma, 4\sigma$  and  $6\sigma$  are smaller than the symbols.

**Table 2**

The 9 exogenous events corresponding to  $\zeta = 8\sigma$  for the daily data of the Shanghai Index. The total number of the large volatilities for  $\zeta = 8\sigma$  is 16. IPOs: Initial Public Offerings; CSRC: China Security Regulatory Commission; MFSAT: Ministry of Finance and State Administration of Taxation; SASAC: State-owned Asset Supervision and Administration Commission.

Date	Event	
92.05.21	Rally	Chinese stock markets allowed free bidding transactions.
94.03.14	Rally	The State Council announced that income from stock transfer was exempt from tax this year, and banned the arbitrary right issue of listed companies.
94.08.01	Rally	CSRC decided to suspend new coming IPOs, to control the scale of right issues, and to develop mutual fund and fostered institution investors.
95.05.18	Rally	CSRC suspended the pilot program of the national debt and future trading.
95.05.23	Crash	CSRC declared large amount of deposits of IPOs in 1995.
96.12.16	Crash	CSRC set a limitation for the price change in a trading day in stock markets.
97.05.22	Crash	CSRC and Central Bank controlled the fund investing in stock markets.
01.10.23	Rally	CSRC declared stopping reduction of state-owned shares.
08.09.19	Rally	MFSAT declared reduction of the stamp tax rate in stock transactions; SASAC announced support for central enterprises and for holdings of listed companies to buy shares back.

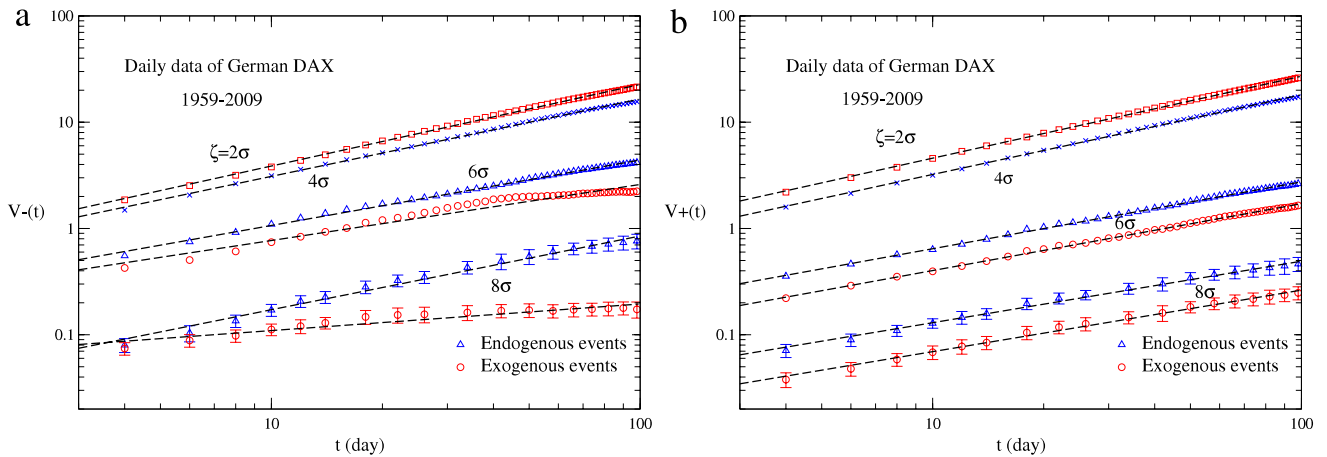
**Table 3**

The 16 exogenous events corresponding to  $\zeta = 8\sigma$  for the daily data of the German DAX. The total number of the large volatilities for  $\zeta = 8\sigma$  is 27.

Date	Event	
62.05.29	Crash	The big crash in NYSE in 05.28.
87.10.19	Crash	Black Monday all over the world.
87.10.22	Crash	
87.10.26	Crash	
87.10.28	Crash	
87.11.10	Crash	
89.10.16	Crash	Honecker in East Germany was forced to resign.
91.01.17	Rally	Gulf War started.
91.08.19	Crash	The coup against Gorbachev in Soviet Union.
97.10.28	Crash	Asian Financial Crisis.
01.09.12	Crash	Sep. 11 attack in US.
08.01.21	Crash	Subprime mortgage crisis.
08.10.06	Crash	
08.10.10	Crash	
08.10.13	Crash	
08.11.06	Crash	

In particular,  $p_{\pm}$  of the endogenous events are almost independent of the threshold  $\zeta$ . For the German DAX,  $p_{\pm}$  look increasing weakly with  $\zeta$ , possibly because we might still miss some exogenous events. It is due to the fluctuations that for the endogenous events with  $\zeta = 6\sigma$ ,  $p_{-} = 0.18$  and  $p_{+} = 0.16$  of the Shanghai Index are somewhat smaller, and  $p_{-} = 0.37$  of the German DAX is slightly larger.

In financial markets, a large fluctuation does not necessarily indicate that the dynamic system already jumps to a non-stationary state, for the probability distribution of volatilities is with a power-law tail, and allows extremal events. Since  $p_{\pm}$  are almost independent of  $\zeta$  for the endogenous events, the dynamic system probably remains still in the stationary state.



**Fig. 4.**  $V_{\pm}(t)$  for the daily data of the German DAX. For  $\zeta = 6\sigma$  and  $8\sigma$ ,  $V_{\pm}(t)$  are displayed for endogenous and exogenous events separately. Error bars for  $\zeta = 8\sigma$  are given by standard deviations of all events, and those for  $\zeta = 2\sigma, 4\sigma$  and  $6\sigma$  are smaller than the symbols.

**Table 4**  
 $p_{\pm}$  of the endogenous (EN.) and exogenous (EX.) events for the daily data of the Shanghai Index (SHI) and DAX.

$\zeta$	$2\sigma$	$4\sigma$	$6\sigma$	$8\sigma$	
SHI (day)					
$p_{-}$	0.22 (2)	0.24 (2)	0.18 (3)	0.20 (6)	En.
			0.54 (4)	0.80 (7)	Ex.
$p_{+}$	0.23 (2)	0.25 (3)	0.16 (4)	0.21 (6)	En.
			0.52 (4)	0.51 (5)	Ex.
DAX (day)					
$p_{-}$	0.28 (3)	0.30 (3)	0.37 (3)	0.31 (4)	En.
			0.50 (4)	0.76 (6)	Ex.
$p_{+}$	0.25 (2)	0.28 (2)	0.31 (3)	0.34 (5)	En.
			0.32 (4)	0.35 (4)	Ex.

However, the exogenous events induced by the external forces drive the dynamic system to a non-stationary state, and lead to the  $\zeta$ -dependent  $p_{\pm}$  and time-reversal asymmetry.

The second observation is that  $p_{+}$  of the exogenous and endogenous events are almost equal for the German DAX, while different for the Shanghai Index. Keeping in mind that  $v_{-}(t)$  describes how a financial market approaches a large fluctuation, it is understandable that  $p_{-}$  of the exogenous events is larger than that of the endogenous events, since an exogenous event should emerge more radically than an endogenous event. However, it is subtle whether  $p_{+}$  of the exogenous and endogenous events are the same or different. In principle, stock markets in different countries may differently respond to the external forces, and different sorts of external forces may lead to different dynamic characteristics [30,31]. It seems that once exogenous or endogenous events occur, the German market does not distinguish between exogenous and endogenous events. Such a phenomenon indicates on the one hand, that the German market is mature, and on the other hand, the exogenous events in the Chinese and German markets may be different. In fact, as shown in Tables 2 and 3, the external forces could be grouped into two sorts: market-policy changing, or political and economic accidents. All the exogenous events in the Shanghai Index correspond to the market-policy changes, while those in the German DAX are induced by the political and economic accidents. Since the Chinese market is newly set up, it is sensitive to the policies of the government.

The third observation is that all exogenous events except for one in the German DAX are crashes, while those in the Shanghai Index are a mixture of crashes and rallies. On the other hand, the endogenous events in the Shanghai Index are 4 rallies and 3 crashes, while those in the German DAX are all rallies. In other words, the dynamic relaxation of the large fluctuations is highly asymmetric between crashes and rallies in the German market, different from that in the Chinese market. It should be also interesting and important that for the mature German market, rallies are usually created by the endogenous events.

In fact, the time-reversal asymmetry in financial systems has been addressed also in related contexts and aspects. Petersen et al. quantitatively describe the volatility dynamics before and after the interest-rate changes by the US Federal Open Market Commission, for both the individual stocks and S&P500 index [32,33]. Since these events are exogenous in the macroscopic sense, the time-reversal symmetry is broken, consistent with our results. Mu et al. study the dynamic behavior of the volatility before and after the extreme events selected by the combination of the large cumulative price change and volatility exceeding a given threshold [34]. These events are not classified into exogenous and endogenous ones. Probably the large cumulative price change is essential for the time-reversal asymmetry.

### 3. Summary

In summary, we have investigated the large-fluctuation dynamics in financial markets, based on the minute-to-minute and daily data of the Chinese Indices and German DAX. The dynamic relaxation before and after the large fluctuations is characterized by the power law in Eq. (3). At the time scale in minutes, the large volatilities may have nothing to do with real financial crashes or rallies, and represent only extremal fluctuations. Although the exponents  $p_{\pm}$  increase with the threshold  $\zeta$ , the large-fluctuation dynamics is time-reversal symmetric, i.e.,  $p_{-} = p_{+}$ . At the daily time scale, as  $\zeta$  increases, the large fluctuations gradually approach the financial crashes or rallies. The results indicate that the large-fluctuation dynamics is time-reversal asymmetric, i.e.,  $p_{-} \neq p_{+}$ .

Careful analysis reveals that not only the time-reversal asymmetry but also the  $\zeta$ -dependence of  $p_{\pm}$  are mainly induced by exogenous events. In this sense, the exogenous events induced by the external forces may drive the financial dynamics to a non-stationary state, while the endogenous events may not. Once exogenous or endogenous events occur, the German stock market does not distinguish between exogenous and endogenous events, while the Chinese stock market does. All the exogenous events in the Chinese market correspond to the market-policy changes, while those in the German market are induced by political and economic accidents. It is interesting that for the mature German market, rallies are usually created by the endogenous events.

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