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# Anti-correlation and subsector structure in financial systems

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**Abstract** – With the random matrix theory, we study the spatial structure of the Chinese stock market, the American stock market and global market indices. After taking into account the signs of the components in the eigenvectors of the cross-correlation matrix, we detect the subsector structure of the financial systems. The positive and negative subsectors are anti-correlated with respect to each other in the corresponding eigenmode. The subsector structure is strong in the Chinese stock market, while somewhat weaker in the American stock market and global market indices. Characteristics of the subsector structures in different markets are revealed.

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**Introduction.** – In recent years, much attention of physicists has been attracted to the financial dynamics, which exhibits various collective behaviors [1–9]. Statistical properties of price fluctuations and cross-correlations between individual stocks are of great interest, not only for quantitatively unveiling the complex structure of the financial systems, but also practically for the asset allocation and portfolio risk estimation [10–12]. The probability distribution of price returns usually exhibits a power-law tail, and represents the robust characteristics in stock markets [13–15], while the higher-order time correlations and interactions between stocks are less universal [5–7,16]. In some cases, price returns may also show a Poisson-like distribution [17,18].

It is an important and challenging topic to explore the “spatial” structure in financial systems. For example, the hierarchical structure of stock markets has been investigated through the minimal spanning tree method and its variants [19–23]. With the random matrix theory (RMT), business sectors and topology communities may be identified [16,24–26]. The RMT method was firstly developed in the complex quantum systems where the interactions between subunits are unknown [27,28]. The structure of business sectors has been examined for mature markets such as the New York Stock Exchange (NYSE) and the Korean Stock Exchanges [24,25,29–32], and also for some emerging markets such as the National Stock Exchange in India [16]. Very recently, the RMT method was applied to identify the dominant eigenmodes in the indices of the industrial production [33]. In particular, one has investigated the structure of interactions between stocks for the

Chinese stock market based on the RMT method [6]. As an important emerging market, the Chinese market exhibits stronger cross-correlations than the mature ones. At the same time, the effect of the standard business sectors is weak in the Chinese market. Instead, unusual sectors such as ST and Blue-chip sectors are detected.

In this paper, with the RMT method, we aim at further understanding of the spatial structure. Our observation is that the components in an eigenvector of the cross-correlation matrix may show positive and negative signs. To the best of our knowledge, what role the signs of the components play has not been explored. Our main finding is that the signs of the components in an eigenvector may classify a sector into two subsectors, which are anti-correlated with respect to each other within this eigenmode. This goes beyond what one may gain with standard methods such as the minimal spanning tree and its variants in the analysis of the “spatial” structure in financial systems [19–23].

**Methods and basics.** – We have collected the daily data of 259 stocks traded in the Shanghai Stock Exchange (SSE) from January, 1997 to November, 2007, in total, 2633 days. The daily data of 259 stocks in the NYSE are from January, 1990 to December, 2006, in total, 4286 days. Meanwhile, we have collected the daily data of a set of 66 financial indices, including 57 indices in stock markets and 9 treasury bond rates in US from September, 1997 to October, 2008, in total, 2669 days. We name the 66 indices the *global market indices* (GMI). The data of the SSE are taken from “Wind Financial Database” <http://www.wind.com.cn> and the data of the NYSE and GMI are from “Yahoo Finance” <http://finance.yahoo.com>.

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We assume that the price is the same as the preceding day [34], if the price of a stock is absent in a particular day. It has been pointed out that the missing data do not result in artifacts [16]. All markets concerned in this paper have normal trading sessions in all days of the week, except for Saturdays, Sundays and holidays declared in advance, excluding the Egyptian and Tel Aviv Stock Exchange. For the latter two stock markets, trading takes place from Sunday to Thursday. For the alignment of the time series, we simply move the data on Sunday to Friday for these two markets. For comprehensive understanding of the cross-correlation of financial markets, the bond rates in US are also added in our analysis, which include 9 indices ranging from the 3 month to 20 year rates.

We define the logarithmic price return of the  $i$ -th stock over a time interval  $\Delta t$  as

$$R_i(t) \equiv \ln P_i(t + \Delta t) - \ln P_i(t), \quad (1)$$

where  $P_i(t)$ , represents the price of the stock price at time  $t$ . To ensure different stocks with an equal weight, we introduce the normalized price return

$$r_i(t) = \frac{R_i(t) - \langle R_i(t) \rangle}{\sigma_i}, \quad (2)$$

where  $\langle \dots \rangle$  is the average over time  $t$ , and  $\sigma_i = \sqrt{\langle R_i^2 \rangle - \langle R_i \rangle^2}$  denotes the standard deviation of  $R_i$ . Then, the elements of the cross-correlation matrix  $C$  are defined by the equal-time correlations

$$C_{ij} \equiv \langle r_i(t)r_j(t) \rangle. \quad (3)$$

By the definition,  $C$  is a real symmetric matrix with  $C_{ii} = 1$ , and  $C_{ij}$  is valued in the domain  $[-1, 1]$ .

The mean value  $\overline{C}_{ij}$  of the elements for the SSE is 0.37, much larger than 0.16 and 0.26 for the NYSE and GMI, respectively. It confirms that stock prices in emerging markets are more correlated than mature ones [16,35,36]. The correlation between financial indices in the GMI is smaller than that of the SSE, but bigger than that of the NYSE.

We now compute the eigenvalues of the cross-correlation matrix  $C$ , in comparison with those of the so-called *Wishart* matrix, which is derived from non-correlated time series. Assuming  $N$  time series with length  $T$ , and in the large- $N$  and large- $T$  limit with  $Q \equiv T/N \geq 1$ , the probability distribution  $P_{rm}(\lambda)$  of the eigenvalue  $\lambda$  for the Wishart matrix is given by [37,38]

$$P_{rm}(\lambda) = \frac{Q}{2\pi} \frac{\sqrt{(\lambda_{max}^{ran} - \lambda)(\lambda - \lambda_{min}^{ran})}}{\lambda}, \quad (4)$$

with the upper and lower bounds

$$\lambda_{min(max)}^{ran} = \left[ 1 \pm \left( 1/\sqrt{Q} \right) \right]^2. \quad (5)$$

For a dynamic system, large eigenvalues of the cross-correlation matrix, which deviate from  $P_{rm}(\lambda)$ , imply that there exist non-random interactions. In fact, in both mature and emerging stock markets, the bulk of the

Table 1:  $\lambda_{min(max)}^{ran}$  denote the lower (upper) bound of the eigenvalues of the Wishart matrix, while  $\lambda_{min}^{real}$ ,  $\lambda_0$ ,  $\lambda_1$  and  $\lambda_2$  represent the lower bound of the eigenvalues and the three largest eigenvalues of the real systems, respectively.

	$\lambda_{min}^{ran}$	$\lambda_{max}^{ran}$	$\lambda_{min}^{real}$	$\lambda_0$	$\lambda_1$	$\lambda_2$	$\overline{C}_{ij}$
SSE	0.47	1.73	0.18	97.3	4.17	3.35	0.37
NYSE	0.54	1.55	0.20	45.6	8.71	6.24	0.16
GMI	0.72	1.33	0.00	21.5	6.65	5.40	0.26

eigenvalue spectrum  $P(\lambda)$  of the cross-correlation matrix is similar to  $P_{rm}(\lambda)$  of the Wishart matrix, but some large eigenvalues deviate significantly from the upper bound  $\lambda_{max}^{ran}$ . This scenario looks similar for the GMI. Let us arrange the large eigenvalues in the order of  $\lambda_\alpha > \lambda_{\alpha+1}$ . As shown in table 1, the largest eigenvalue  $\lambda_0$  of the SSE (China) is 97.33, about 56 times as large as the upper bound  $\lambda_{max}^{ran}$  of  $P_{rm}(\lambda)$ , while  $\lambda_0$  of the NYSE (US) and GMI is 45.61 and 21.53, about 29 and 16 times as large as  $\lambda_{max}^{ran}$ , respectively.

According to the previous works [6,16,24,39], the large eigenvalues deviating from the bulk correspond to different modes of motion in stock markets. The components in the eigenvector of the largest eigenvalue  $\lambda_0$  are uniformly distributed. Therefore, the largest eigenvalue represents the market mode, which is driven by interactions that are common for stocks in the entire market. The components in the eigenvectors of other large eigenvalues are localized. A particular eigenvector is dominated by a sector of stocks, usually associated to a business sector. By  $u_i(\lambda_\alpha)$ , we denote the component of the  $i$ -th stock in the eigenvector of  $\lambda_\alpha$ . To identify the sector, one may introduce a threshold  $u_c$ , to select the dominating components in the eigenvector by  $|u_i(\lambda_\alpha)| \geq u_c$  [6]. The threshold  $u_c$  is determined by two criteria. Firstly, if the matrix is random,  $\langle |u(\lambda)| \rangle \sim 1/\sqrt{N}$  for every eigenmode. Therefore,  $u_c$  should be larger than  $1/\sqrt{N}$ . Secondly,  $u_c$  should not be too large, otherwise there would be not so many stocks in each sector.

In this paper, we show that the components in an eigenvector may carry positive and negative signs, and the components with opposite signs are anti-correlated within this eigenmode. Inspired by this observation, we investigate the subsector structure of the financial markets, by taking into account the signs of the components. In other words, we separate a sector into two subsectors by two thresholds  $u_c^\pm = \pm u_c$ :  $u_i(\lambda_\alpha) \geq u_c^+$  and  $u_i(\lambda_\alpha) \leq u_c^-$ , which correspond to the positive and negative subsectors, respectively.

**Subsectors.** – According to ref. [6], standard business sectors can hardly be detected in the SSE (China). Instead, one finds that there exists three unusual sectors, *i.e.*, the ST, Blue-chip and SHRE sectors, corresponding to the second, third and fourth largest eigenvalues, respectively. What are the dominating stocks for the eigenvectors of other large eigenvalues remains puzzling.

In the SSE, a company will be specially treated if its financial situation is abnormal. Then a prefix of

Table 2: The subsectors in the SSE. The fraction is the number of well-identified stocks over the total number of stocks in the subsector. Null: no obvious category; ST: specially treated; Trad: traditional industry; Tech: high technology; SHRE: Shanghai real estate; Weak: weakly cyclical industry; Stro: strongly cyclical industry; Fin: finance; IG: industrial goods; Util: utility; Basic: basic materials; Heal: health care; CG: daily consumer goods; Serv: services.

	$\lambda_1$		$\lambda_2$		$\lambda_3$		$\lambda_4$		$\lambda_5$		$\lambda_6$	
Sign	+	-	+	-	+	-	+	-	+	-	+	-
Sector	Null	ST	Trad	Tech	ST	SHRE	Weak	Stro	Fin	Null	IG	Null
$u_c^\pm = \pm 0.08$	26	31/35	22/23	23/25	24/27	27/27	23/26	24/26	14/18	25	15/17	25
$u_c^\pm = \pm 0.10$	7	20/23	16/17	12/13	11/12	20/20	13/15	15/16	10/14	17	8/9	18

Table 3: The subsectors in the NYSE. The abbreviations can be seen in the caption of table 2.

$\lambda_i$	$\lambda_1$		$\lambda_2$		$\lambda_3$		$\lambda_4$		$\lambda_5$		$\lambda_6$	
Sign	+	-	+	-	+	-	+	-	+	-	+	-
Sector	Util	Tech	CG	Basi	Tech	CG	Null	Null	Basi	Null	Null	Fin
$u_c^\pm = \pm 0.08$	26/26	3/4	9/16	23/26	15/26	14/32	26	25	6/6	9	14	16/20
$u_c^\pm = \pm 0.10$	25/25	0/0	0/0	19/21	6/13	13/19	18	12	0/0	8	8	16/18
$u_c^\pm = \pm 0.12$	21/21	0/0	0/0	19/21	5/7	5/6	7	6	0/0	4	2	8/11

$\lambda_i$	$\lambda_7$		$\lambda_8$		$\lambda_9$		$\lambda_{10}$		$\lambda_{11}$	
Sign	+	-	+	-	+	-	+	-	+	-
Sector	Tech	Serv	Serv	Heal	Fin	Serv	Null	CG	Null	Serv
$u_c^\pm = \pm 0.08$	11/24	12/29	9/19	9/16	11/25	13/24	12	2/4	7	9/12
$u_c^\pm = \pm 0.10$	7/13	10/18	8/11	8/13	6/11	11/19	6	2/3	6	9/10
$u_c^\pm = \pm 0.12$	4/7	8/11	5/5	7/7	4/6	10/10	2	2/3	1	7/8

the acronym ‘‘ST’’ will be added to the stock ticker. The acronym ‘‘ST’’ will be removed when the financial situation becomes normal. In ref. [6], the so-called ST sector consists of the ‘‘ST’’ stocks. On the other hand, the Blue-chip sector is referred to those companies with a national reputation, and with good performance, *i.e.*, a reasonable positive profit in a period of time. Meanwhile, the SHRE represents the companies registered in Shanghai with the real estate business.

Now we introduce two thresholds  $u_c^\pm = \pm u_c$  to separate the dominating components in an eigenvector into two parts, *i.e.*,  $u_i(\lambda_\alpha) \geq u_c^+$  and  $u_i(\lambda_\alpha) \leq u_c^-$ , which are referred to the *positive* and *negative subsectors*, respectively. With this method, we are able to identify the subsectors of the SSE up to the seventh largest eigenvalue  $\lambda_6$ , and to achieve deeper understanding of the unusual sectors such as the ST and Blue-chip sectors. The results are shown in table 2. The market mode described by the largest eigenvalue  $\lambda_0$  is not included in the table, where all components in the eigenvector possess a same sign.

The negative components in the eigenvector of the second largest eigenvalues  $\lambda_1$  are dominated by the ST stocks. With the threshold  $u_c^- = -0.10$ , for example, 23 dominating stocks are selected, and 20 of them are the ST stocks. Therefore, this subsector is called the ST subsector. For the positive components in the eigenvector of  $\lambda_1$ , we could not identify a common feature for the dominating stocks. In fact, as the threshold  $u_c^+$  increases, the number of the dominating stocks shrinks. For example, with the threshold  $u_c^- = -0.10$ , there are only 7 dominating stocks, and half are also the ST stocks. In ref. [6], therefore,

the whole sector of  $\lambda_1$  is called the ST sector. The negative components in the eigenvector of the third largest eigenvalue  $\lambda_2$  well define the high-technology subsector, while the positive ones are dominated by the traditional industry stocks. Stocks in both subsectors are the Blue-chip stocks. Therefore, these two subsectors together are ascribed to the Blue-chip sector in ref. [6]. For the fourth largest eigenvalue  $\lambda_3$ , the SHRE sector detected in ref. [6] splits into two subsectors, *i.e.*, the SHRE and ST subsectors. Consistent with the result in ref. [6], half of the ST stocks are also the SHRE stocks. But the ST stocks here are different from those for  $\lambda_1$ .

In ref. [6], the sector structure is explored only up to  $\lambda_3$ . With the exploration of the subsector structure, we are able to step further. For  $\lambda_4$ , the positive and negative subsectors are identified to be the weakly and strongly cyclical industry, respectively. The former includes the stocks which fluctuate little with the economic cycle, such as the daily consumer goods and services, while the latter is blooming or depressing with the economic cycle, including the basic materials and energy resources. The positive components in the eigenvectors of  $\lambda_5$  and  $\lambda_6$  are dominated by the finance and non-daily consumer subsectors, although the negative ones remain unknown.

Taking into account the signs of the components in the eigenvector, one may explore the subsector structure in the SSE up to  $\lambda_6$ . A number of standard business subsectors such as the high technology and finance are also observed. But the SSE is indeed dominated by unusual sectors and subsectors such as the ST, Blue-chip, traditional industry, SHRE, weakly and strong cyclical industry. In China, the

Table 4: The subsector structure in the GMI. The thresholds are  $u_c^\pm = \pm 0.15$ . The bold italic items are those not belonging to the areas. NorA refers to the North America, and b3m and b1y are the 3-month and 1-year bonds.

	Sign		Area
$\lambda_0$		VLIC XMI DJA DJI DJT DJX IIX IXIC MID NDX NWX OEX PSE RUA RUI RUT SML SPC <b><i>GDAXI GSPTSE</i></b>	US
$\lambda_1$		AORD HSI HSNC HSNF HSNP HSNU JKSE KS11 N225 NZ50 PSI STI <b><i>ATX</i></b>	Asia
$\lambda_2$		b6m b1y b2y b3y b5y b7y b20y b20y	Bond
$\lambda_3$	+	AEX BFX FCHI FTSE GDAXI MIB- TEL SSMI	EU
	-	SHA SZA SHB SZB	China
$\lambda_4$	+	HSI HSNC HSNP	HK
	-	AEX BFX FCHI FTSE GDAXI MIB- TEL SSMI <b><i>SHA SZA SHB SZB</i></b>	EU
$\lambda_5$	+	IIX IXIC NDX NWX PSE SOXX	US
	-	XMI DJA DJI DJU DJX <b><i>b3m</i></b>	US
$\lambda_6$	+	b3m b6m b1y	Bond
	-	b7y b10y b20y <b><i>HSNU</i></b>	Bond
$\lambda_7$	+	AORD JKSE KS11 N225 NZ50 PSI TWII	Asia
	-	HSI HSNC HSNF HSNP HSNU <b><i>b3m</i></b>	HK
$\lambda_8$	+	BVSP IPSA MERV MXX <b><i>XAX</i></b> <b><i>GSPTSE</i></b>	NorA

companies are not operated strictly within the registered business. Therefore, standard business subsectors are rarely observed. From the view of the behavioral psychology, the investors in China are extraordinarily looking at the performance of the companies and the dominating business and areas, etc. Therefore, unusual sectors such as the ST, Blue-chip, and SHRE emerge.

For comparison, we also apply this method to study the subsector structure in the NYSE (US). The results are listed in table 3. The subsector structure in the NYSE is somewhat different from that in the SSE. From general belief, the standard business subsectors should dominate the eigenvectors of the large eigenvalues. Additionally, it would be expected that there exists only one dominating subsector in an eigenvector, probably under certain conditions, *e.g.*, when the total number of stocks is sufficiently large. To clarify these issues, our results are presented up to the thresholds  $u_c^\pm = \pm 0.12$ . As shown in table 3, most subsectors are indeed the standard business subsectors. For  $\lambda_1$ ,  $\lambda_2$ ,  $\lambda_6$  and  $\lambda_{11}$ , only one dominating subsector remains for sufficiently large thresholds  $u_c^\pm$ . For  $\lambda_3$ ,  $\lambda_7$ ,  $\lambda_8$  and  $\lambda_9$ , however, there are two dominating subsectors. For our dataset of the NYSE, our method does also provide a deeper understanding on the spatial structure.

Finally, as shown in table 4, the subsectors in the GMI can be identified with the threshold  $u_c = \pm 0.15$ , exclusively in terms of the *areas* to which the indices

belong. Different from the SSE and NYSE, the eigenvector of the largest eigenvalue  $\lambda_0$  of the GMI does not describe the so-called “market mode”, which represents the global motion of the financial system. This may reflect the fact that all the financial markets in the world have not been in such a unified status. The first, second and third largest eigenvalues correspond to the US, Asia-Pacific and Bond sectors, with only a single dominating subsector. The US sector mainly consists of the indices in US, except for the GSPTSE from Canada and GDAXI from Germany. This result reflects that US is the dominating economy in the world. From  $\lambda_3$  to  $\lambda_7$ , there emerge two dominating subsectors. One important feature of the subsector structure is that the indices in the mainland of China or in Hong Kong always form an independent subsector. On the other hand, the US bond rates do not mix with the indices in stock markets. For  $\lambda_6$ , the short-term bond rates and long-term bond rates are separated into the positive and negative subsectors, respectively.

**Anti-correlation between subsectors.** – *What is the physical meaning of the positive and negative subsectors?* The cross-correlation between two stocks can be written as

$$C_{ij} = \sum_{\alpha=1}^N \lambda_{\alpha} C_{ij}^{\alpha}, \quad C_{ij}^{\alpha} = u_i^{\alpha} u_j^{\alpha}, \quad (6)$$

where  $\lambda_{\alpha}$  is the  $\alpha$ -th eigenvalue,  $u_i^{\alpha}$  is the  $i$ -th component in the eigenvector of  $\lambda_{\alpha}$ , and  $C_{ij}^{\alpha}$  represents the cross-correlation in the  $\alpha$ -th eigenmode. In other words, the cross-correlation between two stocks can be decomposed into those from different eigenmodes. Since the eigenvalue  $\lambda_{\alpha}$  is always positive, it gives the weight of the  $\alpha$ -th eigenmode, and the sign of  $C_{ij}^{\alpha}$  is essential in the sum. According to eq. (6),  $C_{ij}^{\alpha}$  is positive if the components  $u_i^{\alpha}$  and  $u_j^{\alpha}$  have the same sign in a particular eigenmode. Otherwise, it is negative. When  $C_{ij}^{\alpha}$  is negative, two stocks are referred to be *anti-correlated in this eigenmode*: when the price return of the  $i$ -th stock is positive, the price return of the  $j$ -th stock tends to be negative in the statistical sense. Therefore, all stocks in a same subsector are positively correlated in this eigenmode, while the stocks in different subsectors are anti-correlated. This is the physical meaning of the subsectors. For the NYSE, however, only a number of sectors split into two subsectors. This suggests that the spatial structure and interactions among the stocks in the SSE are more complicated.

Let us examine some examples in the SSE. The sector of  $\lambda_2$  is composed of the traditional industry and high-technology subsectors. The former represents those traditional industry companies with a long-term and stable interest, but a lower asset risk and expected revenue, while the latter includes the high-technology companies with novel business and conceptions, but a higher asset risk and potential profit. In a particular period, for example, the stock market is uncertain, and investors prefer the traditional industries with a lower risk, then their stock



prices rise up higher than those of the high-technology companies. In another period, however, the stock market is booming, and the situation is reverse. Thus, these two subsectors are anti-correlated in the eigenmode of  $\lambda_2$ . The sector of  $\lambda_4$  consists of the weakly and strongly cyclical industry subsectors. Both subsectors are unusual, but their anti-correlation seems obvious. The weakly and strongly cyclical industries are weakly and strongly correlated with the macro-economy environment, respectively. Thus, investors prefer the strongly cyclical industry when the macro-economy is booming. Instead, investors rather choose the weakly cyclical industry when the macro-economy declines. In ref. [6], the sector of  $\lambda_3$  is identified as the SHRE sector. Now this sector splits into the ST and SHRE subsectors. In fact, half of the ST stocks also belongs to the SHRE stocks. This suggests that the investors care much about the normal and abnormal financial situation, even for the SHRE companies.

In the NYSE, the subsector structure of  $\lambda_3$ ,  $\lambda_7$  and  $\lambda_9$  is understandable. The daily consumer goods and services are considered as the traditional industries, while the high technology and finance belong to another category. These two sorts of stocks may show an anti-correlation, consistent with the subsector structure of  $\lambda_2$  in the SSE. For  $\lambda_2$  with the threshold  $u_c^\pm = 0.08$ , a weak subsector structure is observed in the NYSE. From their intrinsic properties, the daily consumer goods and basic materials are classified as the weakly and strongly cyclical industries, respectively. This is similar to the case of  $\lambda_4$  in the SSE. For  $\lambda_8$ , the subsectors may be also explained along the lines above.

In the GMI, two examples are typical. The first one is the subsectors of  $\lambda_5$ , where all components except for one are the indices in the American stock markets. One subsector is composed of the IIX, IXIC, NDX, NWX, PSE and SOXX. Most of these indices are related to the information technology, semiconductor industry, internet industry, etc., with a potentially high payoff and asset risk. The other subsector consists of the XMI, DJA, DJI, DJU and DJX. Most of them are for the weighted and traditional companies, which share the general feature of a stable currency flow and mature business mode, but a lower profit. These two subsectors are anti-correlated in this eigenmode. The second example are the subsectors of  $\lambda_6$ , which are obviously anti-correlated for they are just short-term and long-term bond rates in US. For  $\lambda_3$ ,  $\lambda_4$  and  $\lambda_7$ , the subsector structure indicates that the stock markets in China are somewhat special.

To quantitatively measure the anti-correlation between the positive and negative subsectors, we construct the combinations of stocks in the two subsectors,  $I_\alpha^\pm(t) = \sum_i u_i^\pm(\alpha) r_i(t)$ , and compute the cross-correlation

$$C_{+-}(\alpha) = \langle I_\alpha^+(t) I_\alpha^-(t) \rangle. \quad (7)$$

Here  $u_i^\pm(\alpha)$  is the  $i$ -th positive or negative component in the  $\alpha$ -th eigenmode selected by the threshold, *e.g.*,  $u_c^\pm = \pm 0.08$ . In fig. 1, the cross-correlation  $C_{+-}(\alpha)$  is

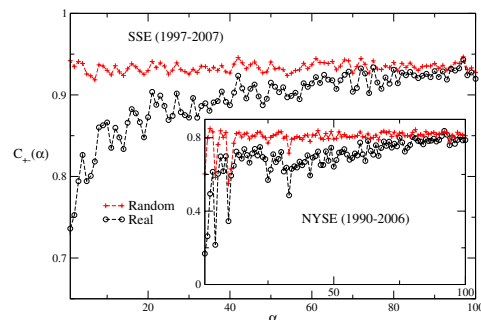


Fig. 1: (Colour on-line)  $C_{+-}(\alpha)$  for the SSE and NYSE are compared with that between two random combinations of stocks.

shown for the SSE and NYSE, in comparison with that between two random combinations of stocks. This result is not qualitatively sensitive to whether one introduces the thresholds  $u_c^\pm$  to select the dominating components.

In fig. 1, we observe that  $C_{+-}(\alpha)$  monotonically increases, and gradually approaches that for two random combinations of stocks.  $C_{+-}(\alpha)$  computed with  $I_\alpha^\pm(t)$  is smaller than that with two random combinations of stocks because of the anti-correlation between the positive and negative subsectors.

What matrix structure results in the subsector structure? Let us consider a  $4 \times 4$  cross-correlation matrix,

$$C_{4 \times 4} = \begin{pmatrix} 1 & 0.55 & 0.15 & 0.11 \\ 0.55 & 1 & 0.39 & 0.34 \\ 0.15 & 0.39 & 1 & 0.95 \\ 0.11 & 0.34 & 0.95 & 1 \end{pmatrix}, \quad (8)$$

which is taken from the  $\lambda_6$  sector of the GMI. The 1st and 2nd indices represent the 3-month and 6-month bond rates, respectively, identified as the positive subsector. The 3rd and 4th indices are the 10-year and 20-year bond rates, identified as the negative subsector. Obviously, the matrix elements  $C_{ij}$  within the same subsectors, *i.e.*, in the diagonal blocks, are larger than the ones between the positive and negative subsectors, *i.e.*, in the off-diagonal blocks.

To verify the anti-correlation more intuitively, therefore, we may calculate the average  $\bar{C}_{ij}$  within the positive or negative subsector, and between the positive and negative subsectors. The results for the NYSE are shown in fig. 2, and those for the SSE are similar. The average  $\bar{C}_{ij}$  within the positive or negative subsector is obviously much larger than that between the positive and negative subsectors, especially for small  $\alpha$ , *i.e.*, large eigenvalues. This strongly suggests that there indeed exists an anti-correlation between the positive and negative subsectors. However, we should keep in mind that the anti-correlation in a particular eigenmode is only a part of the cross-correlation between two stocks, as shown in eq. (6). How to make use of this anti-correlation theoretically and practically remains challenging.

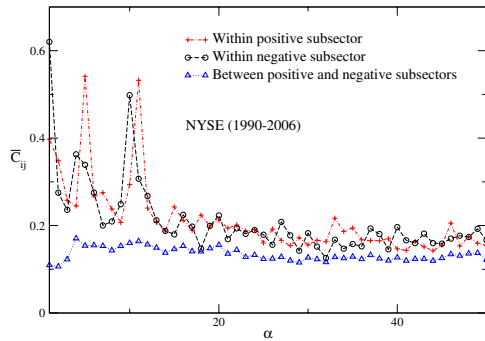


Fig. 2: (Colour on-line) The average cross-correlation  $\bar{C}_{ij}$  for the NYSE.

**Conclusion.** – With the RMT method, we have investigated the spatial structure of the SSE (China), NYSE (US) and GMI. Taking into account the signs of the components in the eigenvectors of the cross-correlation matrix, a sector may split into two subsectors, which are anti-correlated with respect to each other in the corresponding eigenmode. The results are shown in tables 2, 3 and 4. The NYSE is dominated by the standard business sectors and subsectors, and the GMI is controlled by the area sectors and subsectors, but without the market mode. In contrast to it, the SSE exhibits unusual sectors and subsectors.

The subsector structure is strong in the SSE, while somewhat weaker in the NYSE and GMI. The anti-correlation between the positive and negative subsectors in an eigenmode can be measured by  $C_{+-}(\alpha)$  in eq. (7) and the average  $\bar{C}_{ij}$  within the positive or negative subsector, and between the positive and negative subsectors, as shown in figs. 1 and 2.

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