



Modeling interactions of trading volumes in financial dynamics

F. Ren^{a,b,c,*}, B. Zheng^b, P. Chen^b

^a School of Business, East China University of Science and Technology, Shanghai 200237, China

^b Zhejiang Institute of Modern Physics, Zhejiang University, Hangzhou 310027, China

^c Research Center for Econophysics, East China University of Science and Technology, Shanghai 200237, China

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ABSTRACT

A dynamic herding model with interactions of trading volumes is introduced. At time t , an agent trades with a probability, which depends on the ratio of the total trading volume at time $t - 1$ to its own trading volume at its last trade. The price return is determined by the volume imbalance and number of trades. The model can reproduce the power-law distributions of the trading volume, number of trades and price return, and the probable relation between them. The exponents are tunable by adjusting the values of the parameters, but show slight deviation from those revealed in empirical studies. Moreover, the time series generated are long-range correlated. We demonstrate that the results are rather robust, and do not depend on the particular form of the trading probability.

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1. Introduction

In the statistical analysis of financial markets, much attention has been drawn to the study of the stock prices [1–13]. Denoting as $p(t)$ the price of a given stock or financial index, the price return $r(t)$ is defined as the change of the logarithmic price in a time interval Δt , i.e., $r(t) \equiv \ln p(t) - \ln p(t - \Delta t)$. A power-law tail with an exponent $\xi_r \cong 3.0$ is found in the cumulative probability distribution of price returns [6–8], which indicates that the large price fluctuation is more common than one might naively expect. Another important statistical property basically observed in financial markets is the long-range correlation of the volatility which is simply defined as the magnitude of the price return [9,10]. Many efforts have been devoted to the understanding of the financial markets along this direction, with both phenomenological analysis and microscopic multi-agent models [3,6,7,10,14–22].

Recent empirical studies show that the trading volume is highly correlated with the price return and volatility [8,23–25], and this confirms the famous saying that it takes trading volume to move stock prices. A positive linear correlation is revealed on the basis of the data analysis at time scales larger than one minute [26–28]. For the high-frequency data at the microscopic transaction level, the volume–return relation follows a scaling behavior. Lillo et al. found a master curve with a scaling form using the Trade and Quote database of US stocks [29], and the scaling function is found to be a power-law form for large volumes. Lim and Zhou found similar scaling behavior in Australian and Chinese stocks [30,31]. Due to the significant importance of the trading volume, its statistical properties are worthy of careful analysis.

Like price returns, the cumulative probability distributions of trading volumes also show power-law behaviors. Plerou et al. reported the scaling exponent $\xi_v \cong 1.5$ [7,25,32,33], but there was criticism from Rać et al. indicating that they may have underestimated the tail exponent [34]. In an effort to understand the power-law distributions of the price returns, Gabaix et al. promoted a scaling theory [8] based on the measurements of empirical relations between trading volumes,

* Corresponding address: School of Business, East China University of Science and Technology, 130 Meilong Road, P.O. Box 114, Shanghai 200237, China. Tel.: +86 21 64253634.

E-mail address: fren@ecust.edu.cn (F. Ren).

numbers of trades and price returns. They claimed that large price returns are caused by large trading volumes. In contrast, there are other controversial opinions about the formation of large price returns. In contrast, from phenomenological analysis of order book data, Farmer suggested that large price movements are driven by fluctuations in liquidity [35,36], i.e., variations in the response to changes in supply and demand. Weber and Rosenow confirmed the conclusion that the price returns are substantially dominated by liquidity from different points of view [37]. In fact, it is difficult to offer a complete answer as regards how large price movements occur. Nevertheless, it remains very important to fully understand the statistical properties of the financial fluctuations, such as the power-law distributions of different quantities, and their relations, as well as the long-range time correlations.

We introduce the trading volume as an important factor of financial markets for a multi-agent model of trading activity, aiming at a full understanding of the statistical properties of the financial fluctuations including the power-law distributions of the price return, trading volumes and number of trades and the long-range time correlation of the volatility. In our model, agents are endowed with different trading volumes which are temporally correlated with their personal situation and the market circumstances, represented by their own trading volumes and the total trading volumes respectively. On the basis of the temporal correlation of trading volumes revealed by empirical studies, we construct a multi-agent model with agent-dependent and time-dependent trading volumes. In the literature, some stochastic models of trading activity, typically at the phenomenological level, have been analyzed for this purpose [38–40]. Certain aspects of the financial fluctuations could be reproduced. A multiplicative stochastic model of the time interval between two successive trades, for example, is able to reproduce the statistical properties of the number of trades [38], but it does not refer to the relation with the power-law distributions of the price return and trading volume. In the present paper, we construct our model on the basis of the microscopic structure and interactions, to capture fundamental mechanisms in the financial dynamics.

The paper is organized as follows. In Section 2, the dynamic herding model with interactions of trading volumes is introduced. In Section 3, numerical results from the model are presented. Section 4 contains the conclusion.

2. The model definition

The concept of percolating or herding is important in describing the financial markets [16,18,41]. The dynamic version of the static percolation model, the so-called EZ herding model, shows certain attractive features [18], e.g., the herding structure is dynamically generated in a simple but robust way. The EZ herding model captures the power-law distribution of the price return, but the volatility is short-range correlated in time. To achieve the long-range time correlation of the volatility, a feedback interaction should be introduced [20,42]. Up to now, however, it is still far from realistic and harmonic. For example, only the volatility is considered. The trading volume and number of trades have not been touched. The price return is calculated from the volatility with random ± 1 signs, and this should not describe the financial dynamics realistically.

In this section, we develop a dynamic herding model including the price return, trading volume, and number of trades. In fact, it is not easy to build such a model. We have probed many possible variations of the microscopic structure and interaction, and finally come to the present form.

2.1. The standard EZ herding model

To start, let us first consider the EZ herding model. The system consists of M agents, which form clusters during dynamic evolution. Initially, each agent is a cluster. The dynamics evolves in the following way:

- (1) At a time step t , select an agent i (and thus its cluster) at random.
- (2) With a probability $1 - a$, i remains inactive in trading; select another agent j randomly. If i and j are in different clusters, combine the two clusters into one.
- (3) With a probability a , i becomes active and makes a trade. Then all agents in the cluster follow. After that, this cluster is broken into a state where each agent is a separate cluster. The size of this cluster is recorded as $s(t)$.

Here the probability a is a constant, and controls the dynamic evolution. Since one does not define buying or selling of the trade, only the magnitude of the price return defined as $|r(t)| = s(t)$ is essentially generated. Step (2) represents transmission of information. Considering the time between two actions as the time unit, $1/a$ is the rate of transmission of information. If a is small, for example, transmission of information is fast, and agents tend to form larger clusters and act collectively. Numerical simulations [18] show that for a certain value of a , the probability distribution $P(s)$ obeys a power law.

2.2. The herding model interacted with the trading volume

However, the EZ herding model does not exhibit long-range time correlation of the volatility. Furthermore, we need to include the price return, trading volume and number of trades [8,23,25]. Therefore, we assume that each agent trades with an individual trading volume $v_i(t) > 0$. Denoting buying and selling with $\sigma_i(t) = +1$ and -1 respectively, all agents in a cluster are given the same trade sign $\sigma_i(t)$. Initially, each agent i with $v_i(0) = 1$ is a cluster, and randomly selects a trade

sign $\sigma_i(0) = \pm 1$. The total trading volume is set to $V(0) = 1$. We construct the dynamics as follows:

(1) At a time step t , select an agent i (and thus its cluster) at random, and calculate the trading probability

$$a_i(t) = \frac{1}{1 + bV(t-1)/v_i(t-t')}, \quad t' \geq 1. \quad (1)$$

Here $v_i(t-t')$ is the trading volume of i at its last trade, $V(t-1)$ is the total trading volume at $t-1$, and the parameter b is a measure of the sensitivity to the rational $V(t-1)/v_i(t-t')$ and is set to be a positive value. The larger b is, the more affected the trading probability is by its former private trading volume and the total trading volume.

(2) With a probability $1 - a_i(t)$, the agent i remains inactive in trading; select another agent j randomly. If i and j are in different clusters, combine the two clusters into one. The agents in the new cluster share a common strategy and their trade sign is taken to be the same as the larger cluster of the previous two.

(3) With a probability $a_i(t)$, all agents in the cluster which i belongs to become active, and trade with a common decision, i.e., buy or sell. Another cluster is randomly selected to trade with the active cluster and make trades according to their own trade sign. In our model, all the agents in one cluster trade following their common strategy. From this point of view, the model retains the herding nature of its trading behavior. After that, these two clusters are broken into a state where each agent is a separate cluster with a trade sign selected randomly.

Empirical studies have revealed that the trading volumes in financial markets are long-term correlated [43,25,32,44]. Our assumption of trading probability $a_i(t)$ can approximately explain this phenomenon. Supposing a large preceding trading volume $V(t-1)$, the trading probability $a_i(t)$ is small according to Eq. (1). Most of the agents remain inactive and aggregate to form large clusters. We further assume $v_i(t) = 1/a_i(t)$ to be the trading volume of agent i , on the basis of a plausible observation that if an agent is collecting a lot of information, i.e., with a small $a_i(t)$, it will make a large trade. $V(t) = \sum_i v_i(t)$ is the total trading volume. Once two separate large clusters are picked out to perform trades, the agents in these two clusters trade with large volumes on average. This will consequently lead to a large trading volume in the following trading step t . Let $N(t)$ denote the number of agents in the two active clusters; we define it as the number of trades.

To determine the price return, we need more careful consideration. Empirical studies show that the trading volume seems to have a square root impact on the price return, and the price return saturates at extremely large trading volumes [8,23]. Further, the correlation between the price return and trading volume is largely due to the number of trades [25,45]. Following the square root price impact function, we assume that the price return is determined by the volume imbalance $Q(t) = \sum_i v_i(t)\sigma_i$ and the number of trades. The volume imbalance reflects the difference between supply and demand [35,37]. Quantitatively, we define the price return

$$r(t) = \text{Sign}(Q(t)) \frac{\sqrt{|Q(t)|}}{\sqrt{|Q(t)|} + A} \sqrt{N(t)}. \quad (2)$$

The parameter A is taken to be a large positive value, such that $r(t) \sim \sqrt{|Q(t)|}\sqrt{N(t)}$ at relatively small $|Q(t)|$, and $r(t) \sim \sqrt{N(t)}$ at extremely large $|Q(t)|$.

The key ingredient in our model is the time-dependent probability $a_i(t)$. In Eq. (1), we assume that $a_i(t)$ depends on the ratio of the total trading volume at time $t-1$ to its individual trading volume at its last trade. In financial markets, a large trading volume is usually accompanied by the strong fluctuation of the price return [8,23,25]. This inversely leads to large trading volumes in the following time steps. Therefore, $a_i(t)$ is taken to be inversely proportional to the trading volume $V(t-1)$. If $V(t-1)$ is large, transmission of information is fast, the probability of combining two clusters is high, and then the number of the trades increases on average, and finally leads to large trading volumes. Such a dynamic feedback interaction of the trading volume essentially generates the long-range time correlation of the volatility. On the other hand, $a_i(t)$ is taken to be proportional to the individual trading volume $v_i(t-t')$ at its last trade, on the basis of the empirical assumption that an agent with a large trading volume in its last trade may be more active in trading in the following time steps. Further, the thermodynamic limit is well defined due to the ratio $V(t-1)/v_i(t-t')$ in Eq. (1).

3. Simulation results

3.1. The probability distribution function

In our model, the only tunable parameter is b . In calculating the price return, we fix the value $A = 50$. For each value of b , we take an average over 10^8 iterations, after 10^6 iterations for equilibration. To detect the finite size effect, we perform extensive simulations with different total numbers of agents, and find that the results become stable for $M \geq 40\,000$ as shown in Fig. 1(a). Therefore, we report the results with $M = 80\,000$.

From empirical studies of the stock time series, the probability distribution of the trading volume V obeys a power law

$$P(V) \sim V^{-(1+\xi_V)}, \quad (3)$$

with $\xi_V \cong 1.5$, while that of the number of trades N obeys

$$P(N) \sim N^{-(1+\xi_N)}, \quad (4)$$

with $\xi_N \cong 3.4$. The exponents of these two power-law distributions appear to have an approximate relation $\xi_N \cong 2\xi_V$ [8].

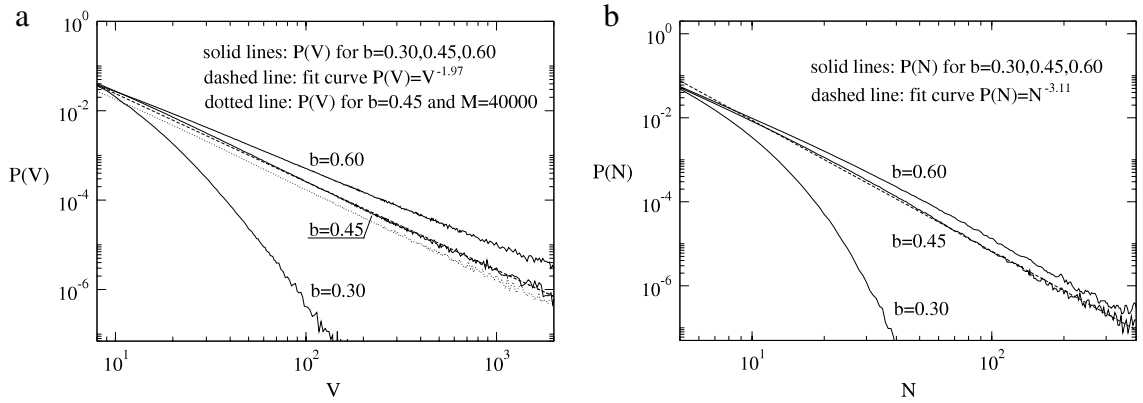


Fig. 1. (Color online) Probability distributions of the trading volume and number of trades are plotted for $b = 0.30, 0.45$ and 0.60 in (a) and (b). The results are obtained with a total number of agents $M = 80\,000$. For comparison, a curve for $b = 0.45$ with $M = 40\,000$ is also displayed in (a).

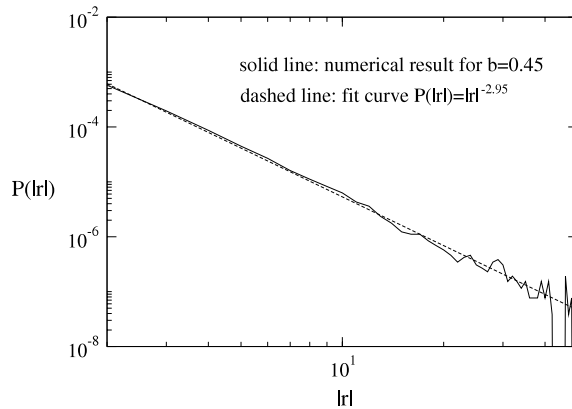


Fig. 2. (Color online) Probability distribution of the volatility for $b = 0.45$ and $M = 80\,000$.

The probability distributions of the trading volume and the number of trades in our model are carefully investigated. In Fig. 1(a) and (b), $P(V)$ and $P(N)$ are plotted for $b = 0.30, 0.45, 0.60$ on a log–log scale. For a small b , e.g., $b \leq 0.3$, $P(V)$ and $P(N)$ decay rapidly, and do not show a power-law behavior. As b increases, both $P(V)$ and $P(N)$ show a power-law behavior at $b = 0.45$, at least for two orders of magnitude. In this sense, the system exhibits a ‘crossover’ behavior. Fitting the curves with the power laws in Eqs. (3) and (4), we estimate $\xi_V = 0.97$ and $\xi_N = 2.11$. These values of the exponents are consistent with the approximate relation $\xi_N \cong 2\xi_V$. For $b > 0.45$, $P(V)$ retains a power-law behavior over a broad range of b , but the exponent ξ_V changes with b and becomes smaller. On the other hand, $P(N)$ deviates from a power-law behavior up to a medium value N , while a power-law tail is still retained with an exponent ξ_N approximately the same as that at $b = 0.45$. In Fig. 1, the curves for $b = 0.60$ are displayed.

It is well known from the empirical analysis that the probability distribution of the price return exhibits a power-law tail

$$P(|r|) \sim |r|^{-(1+\xi_r)}, \tag{5}$$

with $\xi_r \cong 3.0$ [7]. In Fig. 2, the probability distribution of the price return of our model is plotted for $b = 0.45$ on a log–log scale. The curve can be nicely fitted with a power law, and the exponent ξ_r is estimated to be 1.95, close to $\xi_N = 2.11$. In summary, our dynamic herding model at $b = 0.45$ captures the power-law distributions of the trading volume, number of trades and price return. The corresponding exponents of these power-law distributions follow an approximate relation $\xi_r \cong \xi_N \cong 2\xi_V$. This is in agreement with that for the real markets reported in Ref. [8], though the exponents themselves are slightly different.

The specific form of $a_i(t)$ is not very important. Other functions may also work, if their behaviors are similar to that in Eq. (1). To verify this we also study the model with another $a_i(t)$ with a form

$$a_i(t) = 1 - ce^{-d \frac{V(t-1)}{v_i(t-t')}}, \quad t' \geq 1, \tag{6}$$

where c and d are two positive parameters. We retain the inverse relation between $a_i(t)$ and the ratio $V(t-1)/v_i(t-t')$, thus making it behave similarly to Eq. (1). An important characteristic of this model is the simple relation $v_i(t) = 1/a_i(t)$

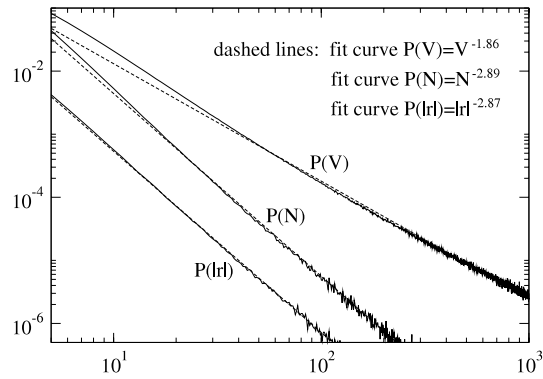


Fig. 3. (Color online) Probability distributions of the trading volume, number of trades, and price return for the model with $a_i(t)$ defined in Eq. (6). Results are obtained with $c = 1.0$, $d = 2.0$ and $M = 80\,000$.

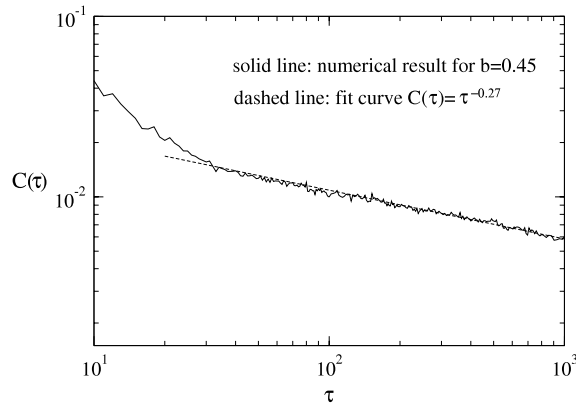


Fig. 4. (Color online) Auto-correlation function of the volatility for $b = 0.45$ and $M = 80\,000$.

between the trading volume and trading probability. This indicates that $v_i(t)$ are dynamic variables, interacting with each other through the trading probability $a_i(t)$, and evolving according to Eq. (6).

We study the probability distributions of the trading volume, number of trades and price return for the model with $a_i(t)$ defined in Eq. (6). To make the probability $a_i(t)$ fixed between 0 and 1, we set $c = 1.0$. On adjusting the parameter d , one observes a ‘crossover’ behavior similar to that of the model with $a_i(t)$ defined in Eq. (1): for small c , no power-law behavior is observed, while for large c power-law behavior occurs. In Fig. 3, power behaviors of $P(V)$, $P(N)$ and $P(|r|)$ for a large $c = 2.0$ are plotted. By fitting the slopes of these curves, we estimate $\xi_V = 0.86$, $\xi_N = 1.89$ and $\xi_r = 1.87$. These exponents display a relation $\xi_r \cong \xi_N \cong 2\xi_V$ also consistent with that for the real markets.

3.2. The auto-correlation function

The long-range time correlation of the volatility is another important feature of the financial markets. Let us define the auto-correlation function of the magnitude of the price return as

$$C(\tau) \equiv \frac{\langle |r(t)||r(t+\tau)| \rangle - (\langle |r(t)| \rangle)^2}{\langle |r(t)|^2 \rangle - (\langle |r(t)| \rangle)^2}, \quad (7)$$

where $\langle \dots \rangle$ represents the average over the time t . From the empirical analysis, it obeys a power law,

$$C(\tau) \sim \tau^{-\lambda}, \quad (8)$$

with $\lambda \cong 0.3$ [10,9].

We calculate $C(\tau)$ for the model with $a_i(t)$ defined in Eq. (1), and observe that a power-law behavior is achieved only when b is in the immediate neighborhood of 0.45. In Fig. 4(b), $C(\tau)$ is plotted for $b = 0.45$ on a log–log scale. The curve can be nicely fitted with a power law, indicating a long-range time correlation of the volatility. The exponent λ is estimated to be 0.27, very close to that of the real markets. This improves on the result $\lambda = 0.90$ from a naive model [42]. A power law with an exponent 0.53 (not shown in the figure) is also observed for $C(\tau)$ for the model with another $a_i(t)$ defined in Eq. (6), but with a reactively bigger fluctuation.

In our model, a large price movement is induced by a large volume imbalance $|Q(t)|$ and a large number of trades $N(t)$ due to the price dynamics defined in Eq. (2). This may occur when large clusters exist in the system. Large clusters are formed after agents are active in collecting information for a period of time. However, a single large trading volume does not necessarily lead to a large price movement.

4. Conclusion

We introduce a dynamic herding model with interactions of trading volumes. At time t , each agent trades with a probability $a_i(t)$ as a linear function of the ratio of the total trading volume $V(t-1)$ at time $t-1$ to its own trading volume $v_i(t-t')$ at its last trade. Agents are endowed with trade signs $\sigma_i(t) = \pm 1$ denoting buying and selling, and the price return is determined by the volume imbalance and number of trades. Our model reproduces the power-law distributions of the trading volume, number of trades and price return for a wide range of the parameter b . We find that for a specific threshold $b = 0.45$, though the exponents differ slightly from those for the empirical data, the approximate relation $\xi_r \cong \xi_N \cong 2\xi_V$ is obtained. The volatilities generated are long-range correlated in time. We also investigate the model with another exponential form of $a_i(t)$, which retains its inverse relation with the ratio $V(t-1)/v_i(t-t')$, and similar power-law behaviors and long-range correlation are observed. This indicates the robustness of our model.

By simulating the agents' reactions to the market information through the interactions between their individual trading volumes and the total trading volumes, the model can explain the power-law behaviors of the trading volume, numbers of trades and price return, and their relations. We believe that our model captures certain essences of the financial markets. Further works should include better simulations of the power-law exponents of the empirical data, more general interactions between the price return, trading volume and number of trades, and understanding of the leverage and anti-leverage effects in western and Chinese financial markets [46–49], etc.

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