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On return-volatility correlation in financial dynamics

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Abstract – With the daily and minutely data of the German DAX and Chinese indices, we investigate how the return-volatility correlation originates in financial dynamics. Based on a retarded volatility model, we may eliminate or generate the return-volatility correlation of the time series, while other characteristics, such as the probability distribution of returns and long-range time correlation of volatilities etc., remain essentially unchanged. This suggests that the leverage effect or anti-leverage effect in financial markets arises from a kind of feedback return-volatility interactions, rather than the long-range time correlation of volatilities and asymmetric probability distribution of returns. Further, we show that large volatilities dominate the return-volatility correlation in financial dynamics.

Financial markets are complex systems with many-body interactions. The possibility of accessing large amounts of historical financial data have spurred the interest of physicists, to analyze the financial dynamics with physical concepts and methods. Some "stylized facts" of the financial markets are revealed [1–8]. Different models and theoretical approaches have been proposed to describe and reproduce the features of the financial dynamics [9–22].

A complex system is often characterized by time correlations and spatial correlations. A famous stylized fact of the financial dynamics is the "volatility clustering", *i.e.*, the long-range time correlation of volatilities, though the price return itself is short-range correlated in time [2,3,15]. Meanwhile recent researches are concerned with the cross-correlations between different stocks and their statistical properties in different stock markets [8,23–29]. To further understand the financial dynamics, one may consider the higher-order time correlations. It was first observed by Black [30,31] that past negative returns increase future volatilities, *i.e.*, the return-volatility correlation is *negative*. This is the *leverage effect* in financial markets. In the past years many literatures have been devoted to the leverage effect, and various relevant correlation coefficients have been measured within GARCH-like models [32–35]. Recently Bouchaud *et al.* quantitatively computed the return-volatility correlation function with the daily data of several financial markets, and observed that it decays by an exponential law [4,36]. More recently, Zheng *et al.* discovered a *positive* return-volatility correlation in Chinese financial markets [7,37], *i.e.*, the so-called *anti-leverage effect*. Further, it is shown that both the leverage effect in German markets and the anti-leverage effect in Chinese markets can be detected on both *daily* and *minutely* time scales [7,37].

How does the return-volatility correlation originate in financial dynamics? The economic interpretation of this phenomenon is still controversial [32,35]. According to Black, a price drop increases the risk of a company to go bankrupt, and its stock therefore becomes more volatile. This induces the leverage effect. Different models have been proposed to explain the leverage effect with certain success [4,38–43]. The retarded volatility model is a good example [4]. The core thought of this model is that the reference price used to set the scale for price updates is not the instantaneous price but rather a moving average of the price over a past period of time. In fact, the retarded volatility model may generate both the leverage and anti-leverage effects by selecting appropriate coupling parameters $K(t)$ [4,7,37]. More recently, there have been discussions whether the long-range time correlation of volatilities may play an important role in the origination of the leverage effect [40,44,45]. Especially, it is argued that *both* the long-range time correlation of volatilities and asymmetric probability distribution of returns are *necessary* in order to have a leverage effect [45].

By the definition of the return-volatility correlation function, the long-range time correlation of volatilities and asymmetric probability distribution of returns *together*

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may indeed induce or alter the leverage effect or anti-leverage effect. The question is only whether it is really a dominating mechanism *in financial markets*. In fact, the shuffling procedure (*i.e.*, randomly changing the time ordering of returns) naturally destroy not only the long-range time correlation of volatilities, but also the return-volatility correlation. Similarly, randomly changing the sign of the return removes also both the asymmetry of the probability distribution of returns and the return-volatility correlation. These arguments provided in ref. [45] should not be the evidences in this respect.

In this paper, we investigate how the leverage and anti-leverage effects originate in financial markets, with the daily and minutely data of the German DAX and Chinese indices, and based on a retarded volatility model. The German DAX exhibits a standard leverage effect. To the best of our knowledge, the Chinese financial market is the unique one with an anti-leverage effect. We collect the daily data of the German DAX from 1959 to 2009 with 12407 data points, and the minutely data from 1993 to 1997 with 360000 data points. The daily data of the Shanghai Index are from 1990 to 2009 with 4482 data points, and the minutely data are from 1998 to 2006 with 95856 data points. The daily data of the Shenzhen Index are from 1991 to 2009 with 4435 data points, and the minutely data are from 1998 to 2003 with 50064 data points. The minutely data are recorded every minute in the German DAX, while every 5 minutes in the Chinese indices. A working day is about 450 minutes in Germany while exactly 240 minutes in China. The dynamic behavior of the Shenzhen Index is similar to that of the Shanghai Index. Sometimes we show only the results of the latter.

We denote the price of a stock index at time t' as $P(t')$, then its logarithmic price return over a time interval Δt is

$$R(t', \Delta t) \equiv \ln P(t' + \Delta t) - \ln P(t'). \tag{1}$$

For comparison of different financial indices, we introduce the normalized return

$$r(t') \equiv [R(t', \Delta t) - \langle R(t', \Delta t)\rangle]/\sigma, \tag{2}$$

where $\langle \ldots \rangle$ represents the average over time t', and $\sigma = \sqrt{\langle R^2\rangle - \langle R\rangle^2}$ is the standard deviation of $R(t', \Delta t)$. Following ref. [4], the return-volatility correlation function is defined by

$$L(t) = [\langle r(t')|r(t'+t)|^2\rangle - L_0]/Z, \tag{3}$$

with $Z = \langle |r(t')|^2\rangle^2$ and $L_0 = \langle r(t')\rangle\langle |r(t')|^2\rangle$. In general, $L(t)$ depends on Δt. In this paper, we take Δt to be the minimum time unit, *i.e.*, one day for the daily data, and one minute and five minutes for the minutely data of the German DAX and Chinese indices, respectively.

In fig. 1, the return-volatility correlation function is displayed for the daily and minutely data of both the German DAX and Chinese indices. $L(t)$ of the German DAX shows negative values up to at least 20 days. This is the well-known leverage effect. $L(t)$ of the Chinese indices (the average of the Shanghai Index and Shenzhen Index) takes positive values at least up to 10 days. That is the so-called anti-leverage effect [7,37]. For the minutely data, the original return-volatility correlation functions are rather noisy due to the high-frequency mode. To extract the dynamic behavior of the slow mode, we have performed an average over a 4-day window. This is the finding in refs. [7,37].

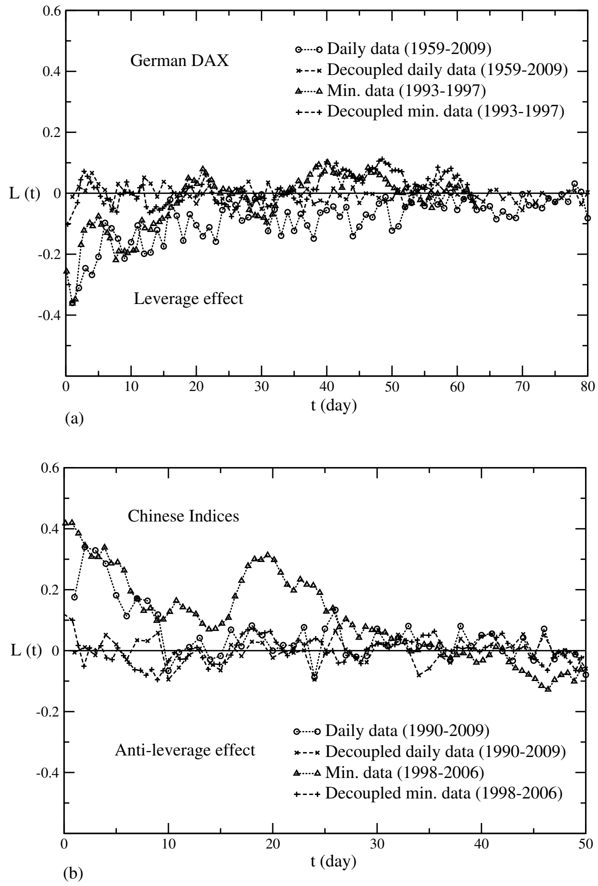

Fig. 1: The return-volatility correlation is displayed for the daily and minutely data of both the German DAX and Chinese indices. After the decoupling interactions are introduced, the leverage effect of the German DAX in (a) and the anti-leverage effect of the Chinese Indices in (b) are eliminated. For the curves of the minutely data, one day is 450 minutes for the German DAX and 240 minutes for the Chinese indices.

In fig. 2(a), the probability distributions of positive and negative returns are plotted for the daily data of both the German DAX and Chinese indices on a log-log scale. Obviously, the positive and negative tails are not asymmetric. The exponent governing the power law decay is about 3.80 [37]. Similar behavior is also observed for the minutely data. In other words, neither the leverage effect of the German DAX nor the anti-leverage effect of the Chinese indices is generated by the asymmetric probability distribution of returns together with the long-range time correlations of volatilities as suggested in ref. [45].

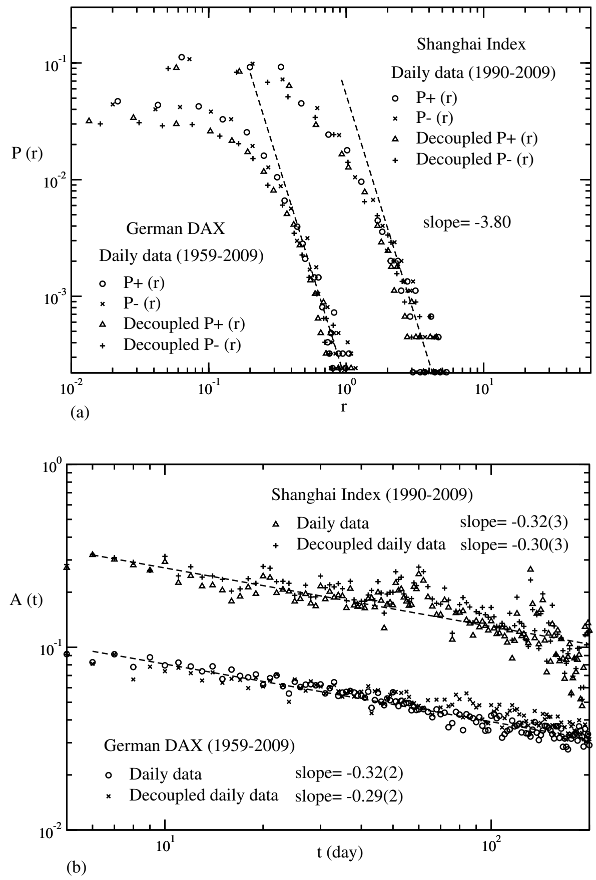

Fig. 2: (a) Probability distributions $P_{\pm}(r)$ of positive and negative returns are displayed for the daily data of the German DAX and Shanghai Index. $P_{\pm}(r)$ remain almost unchanged after the decoupling interactions are introduced. For clarity, $P_{\pm}(r)$ of the German DAX have been shifted to the left by 4.5 units. (b) The time correlation function of volatilities is displayed for the daily data of both the German DAX and Shanghai Index. The long-range time correlation remains after the decoupling interactions are introduced.

We now consider a retarded volatility model [4,7], which assumes that the price return at time t' replies on those in the past times

$$r(t') = \left[1 - \sum_{t=1}^{\infty} K(t) r(t'-t)\right] \sigma(t')\epsilon(t'), \tag{4}$$

where $\epsilon(t')$ is a Gaussian white noise and $\sigma(t')$ is the reference volatility [4,7]. Following ref. [4], one may approximately derive that the return-volatility correlation function $L(t)$ is about $-2K(t)$ if $\sigma(t')$ is the order of 1. More generally,

$$K(t) = -\frac{C}{2} L(t), \tag{5}$$

with C being a positive constant. Simply speaking, the feedback return-volatility interaction described by $K(t)$ in eq. (4) could generate a non-zero $L(t)$ according to eq. (5). Is this the real dynamic mechanism in financial markets?

Our thought is to introduce a decoupling return-volatility interaction to eliminate the non-zero return-volatility correlation from the time series of returns in real financial markets. If other characteristics of the time series remain unchanged, one may catch to some extent how the leverage and anti-leverage effects originate. For this purpose, we reformulate eq. (4) as

$$r_0(t') = \left[1 + \sum_{t=1}^{\infty} K(t) r(t'-t)\right] r(t'), \tag{6}$$

where $r(t')$ is the original return of real financial markets, and $r_0(t')$ is the decoupled one. Here the decoupling interaction described by $K(t)$ in eq. (6) is with an opposite sign compared with the feedback interaction in eq. (4). In fact, eq. (4) is equivalent to eq. (6), if $\sigma(t')\epsilon(t')$ is replaced by $r_0(t')$. With eq. (4), one generates a time series $r(t')$ with a non-zero $L(t)$ from a time series $r_0(t')$ without a non-zero $L(t)$. With eq. (6), we just reverse this procedure. In practical simulations, we first calculate the return-volatility correlation function $L(t)$ from $r(t')$, then determine the parameter $K(t)$ according to eq. (5), and input it into eq. (6). Our finding is that the leverage or anti-leverage effect can be eliminated by adjusting the constant C in eq. (5) appropriately. In fig. 1, $L(t)$ calculated from the decoupled return $r_0(t')$ is shown. Obviously, it fluctuates around zero in all cases, *i.e.*, for the daily and minutely data of both the German DAX and Chinese indices. To obtain the decoupled data in fig. 1, the constant C is valued around 0.1 to 0.4 accordingly.

Now it is important to have a survey whether the decoupling interaction in eq. (6) changes other characteristics of the time series $r(t')$. The observation is that the decoupling interaction is only a *perturbation*, in the sense that $K(t)|r(t'-t)| \ll 1$ and $\sum_{t=1}^{\infty} K(t) r(t'-t) \ll 1$. Practically, we count the number of data with $K(t)|r(t'-t)| > 1$, and it is within 5 percent. Particularly, the interacting factor $1 + \sum_{t=1}^{\infty} K(t) r(t'-t)$ almost never changes the sign of the return, and it only alters the volatility. As a result, except for the return-volatility correlation, other characteristics of $r(t')$ remain unchanged.

In fig. 2(a), the probability distributions $P_{\pm}(r)$ of positive and negative returns are displayed for both the original daily returns $r(t')$ and decoupled daily returns $r_0(t')$ of the German DAX and Shanghai Index. Clearly, the decoupling interaction in eq. (6) does not modify either the power law tails or the shapes of $P_{\pm}(r)$. Similar results are also observed for the minutely data. Additionally, we have calculated the mean value $\langle r_0(t')\rangle$ for the decoupled daily and minutely returns, and it is the order of 10^{-3} to 10^{-5}, negligibly small.

The time correlation function of volatilities is defined as

$$A(t) = [\langle |r(t')||r(t'+t)|\rangle - \langle |r(t')|\rangle^2]/A_0, \tag{7}$$

and $A_0 = \langle |r(t')|^2\rangle - \langle |r(t')|\rangle^2$. It is well known that the volatility in financial dynamics is long-range correlated in time, *i.e.*, $A(t)$ decays by a power law. In fig. 2(b), $A(t)$ is plotted for both the original daily volatilities and decoupled daily volatilities of the German DAX and Shanghai Index. A power law behavior $A(t) \sim t^{-\beta}$ is detected, both before and after the decoupling interaction in eq. (6) is introduced. The exponent β remains almost the same after the leverage or anti-leverage effect is removed. For example, one estimates $\beta = 0.32(2)$ and 0.32(3) for the original daily data of the German DAX and Shanghai Index, and $\beta = 0.29(2)$ and 0.30(3) for the decoupled data [37].

Because of the intra-day pattern, $A(t)$ calculated with the minutely data shows a periodic oscillation, and this behavior also exists for the decoupled minutely data. We remove this intra-day pattern, *e.g.*, with the procedure in refs. [3,37], and estimate the exponents $\beta = 0.39(3)$ and 0.33(2) for the original minutely data of the German DAX and Shanghai Index, and $\beta = 0.35(3)$ and 0.34(2) for the decoupled data. The values of β of the German DAX are less accurate, because the length of a working day changes from time to time in Germany.

We could further explore important properties of the financial dynamics, *e.g.*, the persistence probability of returns or volatilities. The persistence probability of volatilities $P_+(t)$ (or $P_-(t)$) is defined as the probability that $|r(t'+\tilde{t})|$ has always been above (or below) $|r(t')|$ in time t, *i.e.*, $|r(t'+\tilde{t})| > |r(t')|$ (or $|r(t'+\tilde{t})| < |r(t')|$) for all $\tilde{t} < t$. The average is taken over t'. In general, $P_-(t)$ obeys a universal power law behavior, $P_-(t) \sim t^{-\theta_p}$ [6,46]. In fig. 3(a), the persistence probability $P_-(t)$ of volatilities is plotted for both the original and decoupled daily data of the German DAX and Shanghai Index. All curves of $P_-(t)$ exhibit a nice power law behavior. After eliminating the leverage or anti-leverage effect, the exponent θ_p remains the same. The fact $0.5 < \theta_p < 1$ indicates that the volatility is long-range correlated in time.

From our survey above, we conclude that either the leverage effect or anti-leverage effect does not originate from the asymmetric probability distribution of returns and long-range time correlation of volatilities. The return-volatility correlation is a rather independent property of the financial dynamics. In fact, with the retarded volatility model in eq. (4), one can generate the leverage effect or anti-leverage effect in a time series, independent of the probability distribution of returns and time correlation of volatilities.

For example, we may take $K(t) = m\exp(-t/\tau)$ [4,7,37]. A positive $K(t)$ induces a leverage effect, while a negative $K(t)$ leads to an anti-leverage effect [4,7]. $\sigma(t')$ in eq. (4) is the reference volatility we need to input, and it can be generated, *e.g.*, by a Gaussian random process, the EZ model or a dynamic interacting herding model [13,19,20]. All three models produce symmetric probability distributions of returns with different functional forms. The EZ model yields a power law tail in the probability distribution of returns, but without the long-range time correlation of volatilities. The dynamic interacting herding model

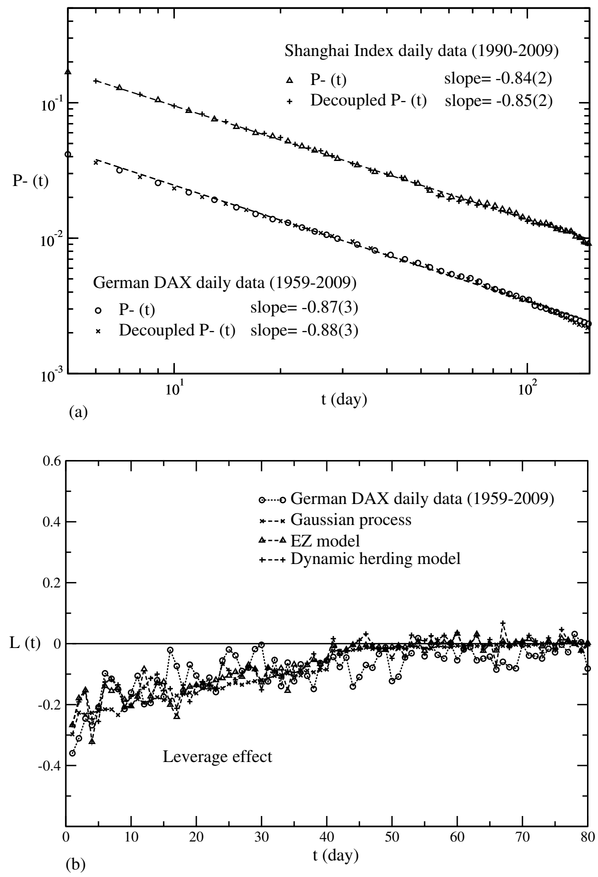

Fig. 3: (a) The persistence probability $P_-(t)$ of volatilities is plotted for both the original and decoupled daily data of the German DAX and Shanghai Index. (b) The leverage effects generated by the retarded volatility model in eq. (4) with different kinds of reference volatilities, *i.e.*, the Gaussian random process, EZ model and dynamic interacting herding model. All three simulations reproduce the leverage effects similar to that of the German DAX.

exhibits both a power law tail in the probability distribution of returns and the long-range time correlation of volatilities. In fig. 3(b), the leverage effects from the simulations with eq. (4) are presented, in comparison with that of the German DAX. Here we have chosen $m = 0.1$ and $\tau = 40$. The anti-leverage effect can be produce similarly with a negative m.

Is the leverage or anti-leverage effect uniformly distributed in the time series of the financial dynamics, or essentially dominated by large volatilities? This is important for deeper understanding of the leverage and anti-leverage effects. In fig. 4, the return-volatility correlation calculated under the conditions $|r(t')| < 8\sigma$, $|r(t')| < 2\sigma$, $|r(t')| > 2\sigma$, and with all $r(t')$, are displayed for the daily data of the German DAX and Chinese indices. Here $\sigma = 1$ is the standard deviation of the normalized return $r(t')$. To obtain the curve of $|r(t')| < 2\sigma$, we skip the data whenever $|r(t')|$ or $|r(t'+t)| > 2\sigma$ in calculating $L(t)$ with eq. (3). With those data skipped by the curve of $|r(t')| < 2\sigma$, we plot the

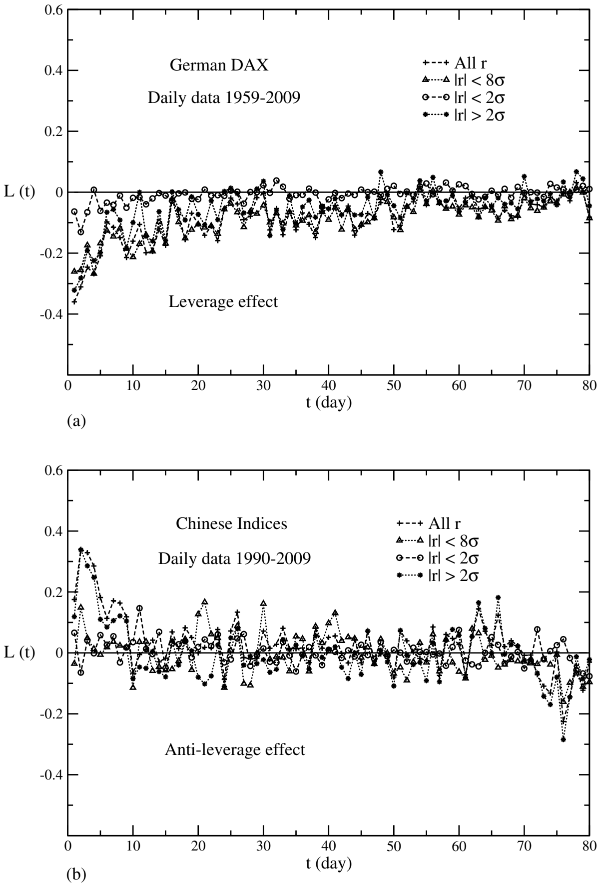

Fig. 4: The return-volatility correlation of the daily data of the German DAX (a) and Chinese indices (b), calculated under the conditions $|r(t')| < 8\sigma$, $|r(t')| < 2\sigma$, $|r(t')| > 2\sigma$, and with all $r(t')$. $\sigma = 1$ is the standard deviation of the normalized return $r(t')$.

curve of $|r(t')| > 2\sigma$. For the German DAX, the leverage effect becomes very weak for $|r(t')| < 2\sigma$. In other words, the leverage effect is mainly contributed by the volatilities $|r(t')| > 2\sigma$. This is confirmed by the curve of $|r(t')| > 2\sigma$. Since there are only 7 volatilities larger than 8σ, the curve of $|r(t')| < 8\sigma$ is not too different from that of all $r(t')$. For the Chinese indices, the anti-leverage effect is essentially dominated by the volatilities $|r(t')| > 8\sigma$, because the curve of $|r| < 8\sigma$ already fluctuates around 0. Briefly speaking, either the leverage effect of the German DAX or the anti-leverage effect of the Chinese indices is dominated by the large volatilities, at least for the daily data. Since the Chinese financial market is more volatile, such a phenomenon looks more prominent. For the minutely data, it is somewhat complicated, and small volatilities may also contribute in some cases. Further, we could investigate the different contributions of the large volatilities induced by external events and generated intrinsically by the dynamics itself. Details of this kind will be presented elsewhere.

In summary, with the retarded volatility model, we can eliminate the leverage effect of the German DAX and anti-leverage effect of the Chinese indices on both daily and minutely time scales, while other characteristics of the time series, such as the probability distribution of returns, time correlation and persistence probability of returns and volatilities etc, remain essentially unchanged. In addition, the probability distribution of returns of the German DAX and Chinese indices are not asymmetric before and after eliminating the leverage or anti-leverage effect. These results suggest that at least for the German DAX and Chinese indices, the leverage or anti-leverage effect in financial markets arises from a kind of feedback return-volatility interactions, rather than the long-range time correlation of volatilities and asymmetric probability distribution of returns. Finally, we show that the leverage effect of the German DAX and anti-leverage effect of the Chinese indices are dominated by large volatilities. For the data set analyzed in ref. [45] (not reachable for us), one may follow the approach in this paper to clarify how the leverage effect originates.

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