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DYNAMIC RELAXATION OF FINANCIAL INDICES

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The dynamic relaxation of the German DAX both before and after a large price-change is investigated. The dynamic behavior is characterized by a power law. At the minutely time scale, the exponent p governing the power-law behavior takes a same value before and after the large price change, while at the daily time scale, it is different. Numerical simulations of an interacting EZ herding model are performed for comparison.

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In recent years, a large amount of data has been piled up in financial markets, and this allows quantitative analysis of the fine structure of the financial dynamics.[1–7] Although the return of a financial index is short-range correlated in time, the volatility exhibits a long-range temporal correlation.[2,3] The long-range temporal correlation of the volatility leads to the so-called dynamic scaling behavior of the financial time series, and has induced many activities along this direction. Recent progress in the analysis of a financial index includes, for example, the measurements of the return-volatility correlation[4,6] and the dynamic behavior non-local in time[8–12] etc. Based on the empirical study, different models and theoretical approaches have been developed.[13–23]

On the one hand, one typically assumes that the financial market already reaches the stationary state, and investigates the correlations and fluctuations of price returns. On the other hand, it is also very interesting to study the statistical properties of the financial dynamics when the market goes into a *non-stationary* state, such as a financial crash.[24–27] In Ref. 28, it is observed that the dynamic relaxation of a financial index *after* a financial crash is governed by a power law with certain corrections in shorter times. The observable is just the number of times that the magnitude of the return exceeds a given threshold at the time t after a financial

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crash. In Ref. 29, a similar analysis of this kind has been performed. The dynamic relaxation of the volatility is measured after a financial crash, and a power-law behavior is detected.

Stimulated by these works, in this paper we study the dynamic relaxation of a financial index both *before and after* a large price-change, and for both the daily data and minutely data. The "large price-change" here is so selected that the price return is sufficiently large compared with the average one. This implies that the financial dynamics may come into a non-stationary state before and after the large price-change. For the daily data, an *extremely* large price-change with a *negative* price return could be related to the so-called financial crash. The dynamic behavior *after a financial crash* is concerned in Refs. 28 and 29. For the minutely data, the large price-change may generally not be relevant for a real financial crash.

Therefore, the purpose of this paper is multi-fold. We are interested in the dynamic behavior of a financial index in a non-stationary state, but perhaps not certainly in a financial crash. The symmetric or asymmetric dynamic behavior before and after the large price-change is particularly of interest. The analysis of both the daily data and minutely data may reveal the fine structure of the financial dynamics. In addition, there are much more events with a large price-change than those of real financial crashes, and one may obtain better accuracy in computations. Finally, one may also study the symmetric or asymmetric dynamic behavior for the large price-changes with a positive price return and with a negative price return.[4,6,30] However, results of this kind will be presented elsewhere.

Let us denote the financial index at a certain time t as $y(t)$, and $Z(t)$ as the magnitude of the price return in *one time step*, i.e. $Z(t) \equiv |y(t+1) - y(t)|$. (The results are similar if one defines $Z(t) \equiv |\ln y(t+1) - \ln y(t)|$.) In this paper, we also call $Z(t)$ the volatility. Naturally, the dynamic behavior of $Z(t)$ may depend on the time scale, e.g. on whether the time unit is one minute or one day. We define $m_{\pm}(t)$ as the *remanent* and *anti-remanent* volatility

$$m_{\pm}(t) = \frac{\langle Z(t' \pm t)\rangle_c - \sigma}{\langle Z(t')\rangle_c - \sigma} \,. \tag{1}$$

Here $\sigma \equiv \langle Z(t')\rangle$ is the average volatility in the time interval we consider, and $\langle \cdots \rangle_c$ represents the average over those t' which satisfy the condition that $Z(t')$ is larger than a given threshold ζ, i.e. $Z(t') > \zeta$. In this paper, ζ is typically chosen to be 2σ, 3σ, 5σ, 7σ and 10σ. $m_+(t)$ describes how the system relaxes from a large price-change to the stationary state, while $m_-(t)$ depicts how the system approaches a non-stationary state.

We now assume that both $m_+(t)$ and $m_-(t)$ obey a power law,

$$m(t) = K(t+\tau)^{-p} \,, \tag{2}$$

with K and τ being positive constants. For reducing the fluctuations in computations, one may integrate Eq. (2) between 0 and t. In this way, the cumulative

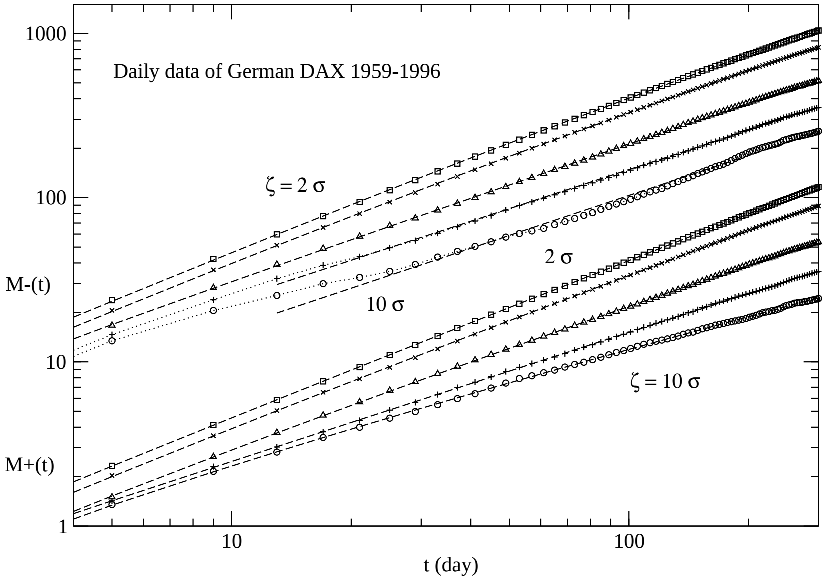

Fig. 1. $M_+(t)$ and $M_-(t)$ calculated with the daily data of the German DAX. From the above, the threshold is $\zeta = 2\sigma$, 3σ, 5σ, 7σ, and 10σ respectively. Dashed lines are the best fits with Eq. (3).

function of $m(t')$ is written as

$$M(t) = K[(t+\tau)^{1-p} - \tau^{1-p}]/(1-p)\,, \tag{3}$$

for $p \neq 1$. The physical origin for the power-law behavior of $m_\pm(t)$ is clear. It is just the long-range temporal correlation of $Z(t)$. Such a power-law dynamic behavior has been well-understood in dynamic critical phenomena, even in the case far from equilibrium.[31,32]

We first analyze the daily data of the German DAX from 1959 to 1996, with a total number of 9318 data points. To search for the large price-change, we set the threshold ζ to be 2σ, 3σ, 5σ, 7σ and 10σ, and the corresponding number of data points with $Z(t') > \zeta$ is counted to be 1227, 657, 193, 60 and 23 respectively. $\zeta = 10\sigma$ is almost the largest threshold, with which one may have reasonable samples for average. In Fig. 1, $M_+(t)$ and $M_-(t)$ have been displayed. A nonlinear dynamic behavior is well-described by Eq. (3), starting from $t \sim 5$ days at least up to $t \sim 300$ days, although for $M_-(t)$ with $\zeta = 7\sigma$ and 10σ, there exist certain deviations from Eq. (3) in early times. From the figure, one finds that the exponent $1 - p_+$ for $M_+(t)$ decreases gradually as the threshold ζ increases, while $1 - p_-$ for $M_-(t)$ remains more or less unchanged.

In the first sector of Table 1, we summarize the exponent p_+ and p_- obtained with the best fittings in Fig. 1. The errors are estimated by dividing the data into two or three subgroups and changing the time windows for the measurements. As ζ

Table 1. p_+ and p_- of the daily data and minutely data of the German DAX in comparison with those of the interacting EZ herding model. τ_+ and τ_- of the interacting EZ herding model take rather small values.

DAX (day)					
ζ	2σ	3σ	5σ	7σ	10σ
p_+	0.061(6)	0.120(12)	0.194(10)	0.218(4)	0.337(17)
τ_+	4.48	8.93	5.25	0.06	1.85
p_-	0.219(15)	0.225(13)	0.180(7)	0.210(13)	0.180(12)
τ_-	40.7	23.2	0.67	0.00	0.00
DAX (min.)					
p_+	0.34(3)	0.38(3)	0.43(3)	0.42(2)	0.45(4)
τ_+	9.8	11.6	12.3	10.9	12.9
p_-	0.30(2)	0.37(2)	0.45(2)	0.46(2)	0.51(3)
τ_-	4.01	5.44	7.64	7.66	11.8
Inter EZ model					
p_+	0.71(3)	0.71(1)	0.72(1)	0.74(2)	0.77(2)
p_-	0.70(1)	0.74(3)	0.81(1)	0.82(2)	0.84(3)

increases, p_+ obviously shows an increasing tendency, from $p_+ = 0.06(1)$ for $\zeta = 2\sigma$ to 0.34(2) for $\zeta = 10\sigma$, while p_- fluctuates around an "average" value 0.20(2). In other words, the dynamic behavior is *asymmetric* before and after a large price-change.

In fitting the data to Eq. (3), the constant τ shows a weak irregular dependence on the threshold ζ, for the curves fluctuate somewhat irregularly in the early times. However, the exponent p is not too sensitive to the constant τ, because τ only affects the fitting in early times. For example, we may put $\tau_+ = \tau_- = c$, with c being zero or a fixed constant smaller than 10, and the asymmetric behavior of p_+ and p_- remains.

Here, we should emphasize that the average over different events, which satisfy the condition $Z(t') > \zeta$, is important. Without the average, an individual curve is rather fluctuating. It is difficult to estimate the exponent p_+ and p_-, and to draw a reliable conclusion. In Fig. 2, for example, three individual events are shown for the daily data of the German DAX. Every single curve shows an increasing trend with a qualitative power law, but it is hard to have a good fit to Eq. (3) and to detect the difference between p_+ and p_-.

To further understand the non-equilibrium dynamic properties of the financial dynamics, we have also performed the measurements with the minutely data of the German DAX from December 1993 to December 1996. The total records in minutes of this period are about 360 000 for the German DAX. We should keep in mind that $Z(t) = |y(t+1) - y(t)|$ now is calculated in the unit of one minute rather than one day. Therefore, even if the threshold ζ takes a large value, it may not mean a real *macroscopic* crash, but only brings the dynamic system to a microscopic non-stationary state. For example, the average price-change $\sigma = 0.317$ for the minutely

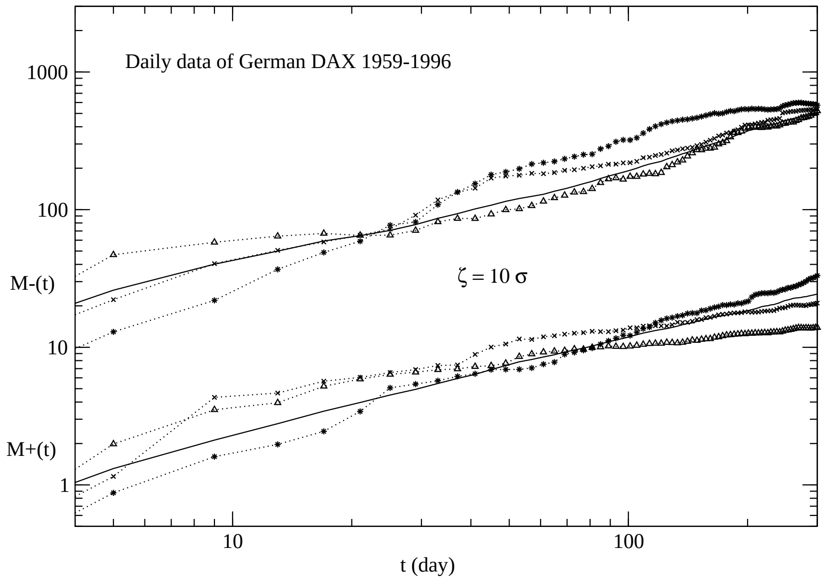

Fig. 2. $M_+(t)$ and $M_-(t)$ calculated with the daily data of the German DAX. The threshold is $\zeta = 10\sigma$. The solid lines are the average curve, while stars, triangles and crosses represent three individual events.

data of the German DAX is not comparable with $\sigma = 6.72$ for the daily data of the German DAX.

In Fig. 3, the dynamic behavior of $M_+(t)$ and $M_-(t)$ calculated with the minutely data of the German DAX has been displayed. The shape of the curves look very similar for both $M_+(t)$ and $M_-(t)$, and also does not change so much with the threshold ζ. Obviously, the curves can be nicely fitted to Eq. (3), starting from $t \sim 5$ minutes. Again, τ does not significantly affect the exponent p. Even if one puts $\tau = 0$, for example, it does not change the exponent p so much. Measurements of the exponent p are summarized in the second sector of Table 1. Both p_+ and p_- slightly increase as the threshold ζ increases. Especially, within errors p_+ and p_- of the minutely data remain more or less the same for every ζ, and this is significantly different from the case of the the daily data. Another difference between the daily data and minutely data is, that both τ_+ and τ_- of the daily data drop to zero quickly as the threshold ζ increases, while those of the minutely data increase slightly with ζ. From Table 1, we conclude that the dynamic behavior at the microscopic time scale, typically in minutes, is *symmetric* before and after a large price change.

To summarize, the non-equilibrium dynamic relaxation of $M_+(t)$ and $M_-(t)$ of both the daily data and minutely data of the German DAX index is well-described by the power law in Eq. (3). The exponents p_+ and p_- of the daily data behave differently as the threshold ζ increases, while those of the minutely data remain

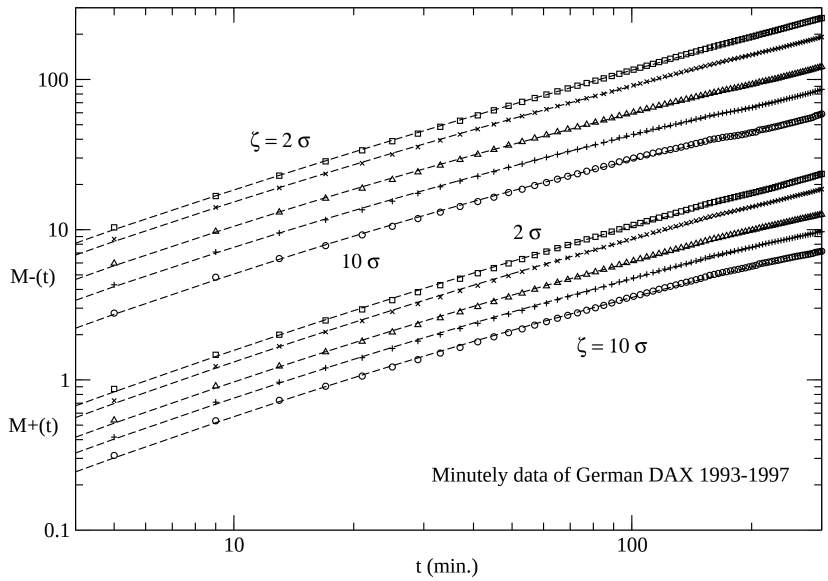

Fig. 3. $M_+(t)$ and $M_-(t)$ calculated with the minutely data of the German DAX. From the above, the threshold is $\zeta = 2\sigma$, 3σ, 5σ, 7σ, and 10σ respectively. Dashed lines are the best fits with Eq. (3).

the same. Actually, such a phenomenon is often seen in traditional physics. For example, microscopic equations of motion reversible in time may result in irreversible macroscopic dynamic processes.[32] Here, the initial conditions should play an essential role.

Now we will describe a multi-agent model at the microscopic level to simulate the dynamic relaxation of $M_+(t)$ and $M_-(t)$. Such kind of models is different from many empirical models which directly describe the movement of a financial index or stock prices around a financial crash. In our model, the price is generated through the interactions between the agents. Let us start from the EZ herding model introduced by Eguiluz and Zimmermann,[16] which is a dynamic extension of the static percolation models. It consists of N agents, which form clusters during dynamic evolution. Initially, each agent is a cluster. The dynamics evolves in the following way.

(i) At a time step t', an agent i (and thus its cluster) is selected at random.
(ii) With probability a, i becomes active and decides buying or selling, and all agents in the cluster follow. After that, this cluster is broken into a state where each agent is a separate cluster. The size of this cluster is then recorded as $s(t')$.
(iii) With probability $1-a$, i remains inactive. Another agent j is selected randomly. If i and j are in different clusters, combine the two clusters into a bigger one.

Here, a is a constant, and apparently controls the dynamic evolution. From the view of financial markets, all agents in a cluster share the same information and therefore act in the same way. Step (iii) represents transmission of information. Numerical simulations show that at a 'critical' value of a, the probability distribution $P(s)$ approximately obeys a power law at least in a certain range of s.[16] The EZ herding model is somewhat attractive due to its simplicity. However, the EZ herding model does not produce the long-range temporal correlation of the volatility,[21] for example, if one assumes $s(t')$ is proportional to the volatility. To confirm this, we have measured the dynamic relaxation of $M_\pm(t)$. It is completely different from that of the financial dynamics. $M_\pm(t)$ takes negative values rather than positive ones since $s(t')$ is anti-correlated in time.

In real markets, the rate a of transmission of information should *not* be a constant. When stock markets become fluctuating, agents are sensitive to the related information, and the public media report it intensively. Therefore, a should be smaller. When the stock markets remain stable and everyone is not much interested in it, a should be bigger. Based on such an observation, it is suggested that, a at the time t' should depend on $s(t'-1)$, e.g. in a form like[20,21]

$$a = b + cs^{-1} \,. \tag{4}$$

Here, b and c are positive constants. Such an interaction may generate a long-range temporal correlation of the volatility. If a larger cluster acts at $t'-1$, a at time t' is smaller and the agents form larger clusters. As a result, larger clusters would be picked up. If a smaller cluster acts at $t'-1$, a at time t' is larger and the agents do not form larger clusters. As a result, smaller clusters would be picked up. This model has been successful in describing the volatility clustering and related stylized facts.[20,21]

In this paper, we perform the simulations with $N = 40000$ and $b = 0.00025$. The critical value of c is estimated to be 0.6. In Fig. 4, $M_+(t)$ and $M_-(t)$ of this interacting EZ herding model have been plotted. Obviously, up to $\zeta = 10\sigma$, the curves are well-fitted to Eq. (3). In the third sector of Table 1, p_+ and p_- are summarized. In this case, τ is relatively small and is not given in the table. Compared with the results of the minutely data of the German DAX, p_+ and p_- of the interacting EZ model take somewhat larger values. However, the qualitative behavior of p_+ and p_- of the interacting EZ model is the same as that of the minutely data of the German DAX index. For example, both p_+ and p_- increase slightly with the threshold ζ, while p_+ and p_- take approximately the same value at each value of ζ.

In conclusion, we have investigated the dynamic behavior of a financial index both before and after a large price-change. The dynamic relaxation of $M_+(t)$ and $M_-(t)$ is found to obey a power law described by Eq. (3). From the exponents p_+ and p_- measured with both the daily data and minutely data of the German DAX, it is observed that the dynamic behavior of the daily data before and after a large price-change is asymmetric in time, while that of the minutely data is symmetric.

Numerical simulations of the EZ herding model and an interacting EZ herding model have been performed. The interacting EZ herding model qualitatively

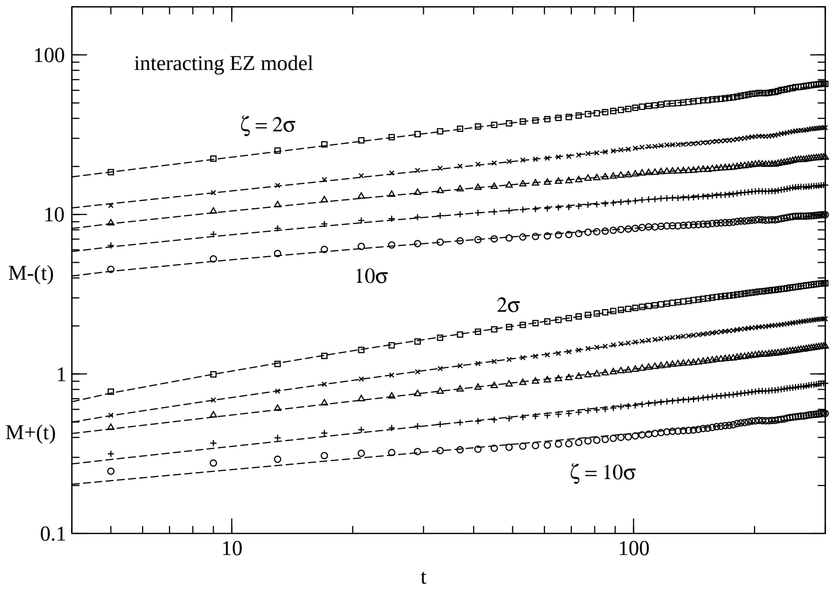

Fig. 4. $M_+(t)$ and $M_-(t)$ simulated with the interacting EZ model. From the above, the threshold is $\zeta = 2\sigma$, 3σ, 5σ, 7σ, and 10σ respectively. Dashed lines are the best fits with Eq. (3).

reproduces the dynamic behavior of the minutely data in real markets. However, it remains a challenge how to model the asymmetric behavior of the daily data of the German DAX.

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