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Evolutionary snowdrift game with an additional strategy in fully connected networks and regular lattices

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Abstract

The effects of an additional strategy or character in the evolutionary snowdrift game (SG) are studied in a well-mixed population or fully connected network and in a square lattice. The SG, which is a possible alternative to the prisoner's dilemma game in studying cooperative phenomena in competing populations, consists of two types of strategies, C (cooperators) and D (defectors). The additional L-strategy amounts to a strongly persuasive character that a fixed payoff is given to each player when a L-player is involved, regardless of the character of the opponent. In a fully connected network, it is found that either C lives with D or the L-players take over the whole population. In a square lattice, three possible situations are found: a uniform C-population, C lives with D, and the coexistence of all three characters. The presence of L-players is found to enhance cooperation in a square lattice by enhancing the payoff of cooperators. The results are discussed in terms of the effects in restricting a player to compete only with his nearest neighbors in a square lattice, as opposed to competing with all players in a fully connected network.
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1. Introduction

The evolutionary prisoner's dilemma game (PDG) [1–3] and the snowdrift game (SG) [4] have become standard paradigms for studying the possible emergence of cooperative phenomena in a competitive setting. Physicists find such emergent phenomena fascinating, as similar cooperative effects are also found in interacting systems in physics that can be described by some minimal models, e.g. models of interacting spin systems. These games are also essential in the understanding of coexistence of (and competition between) egoistic and altruistic behavior that appear in many complex systems in biology, sociology and economics. The basic PDG [5,6] consists of two players deciding simultaneously whether to cooperate (C) or to defect (D).

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If one plays C and the other plays D, the cooperator pays a cost of $S = -c$ while the defector receives the highest payoff $T = b$ ($b > c > 0$). If both play C, each player receives a payoff of $R = b - c > 0$. If both play D, the payoff is $P = 0$. Thus, the PDG is characterized by the ordering of the four payoffs $T > R > P > S$, with $2R > T + S$. In a single round of the game, defection is a better action in a fully connected (well-mixed) population, regardless of the opponents' decisions. Modifications on the basic PDG are, therefore, proposed in order to induce cooperations and to explain the wide-spread cooperative behavior observed in the real world. These modifications include, for example, the iterated PDG [1,2], spatially extended PDG [7–10] and games with a third strategy [11–14].

The SG, which is equivalent to the hawk-dove or chicken game [4,15], is a model that somewhat favors cooperation. It is best introduced using the following scenario [16]. Consider two drivers hurrying home in opposite directions on a road blocked by a snowdrift. Each driver has two possible actions—to shovel the snowdrift (cooperate (C)) or not to do anything (not-to-cooperate or "defect" (D), following the PDG notations). If they cooperate, they could be back home earlier and each will get a reward of b'. Shovelling is a laborious job with a total cost of c'. Thus, each driver gets a net reward of $R = b' - c'/2$. If both drivers take action D, they both get stuck, and each gets a reward of $P = 0$. If only one driver takes action C and shovels the snowdrift, then both drivers can also go home. The driver taking action D (not to shovel) gets home without doing anything and hence gets a payoff $T = b'$, while the driver taking action C gets a "sucker" payoff of $S = b' - c'$. The SG refers to the case when $b' > c' > 0$, leading to the ranking of the payoffs $T > R > S > P$. This ordering of the payoffs *defines* the SG. Therefore, both the PDG and SG are defined by a payoff matrix of the form

$$\begin{array}{cc} & \begin{array}{cc} C & D \end{array} \\ \begin{array}{c} C \\ D \end{array} & \begin{pmatrix} R & S \\ T & P \end{pmatrix}, \end{array} \tag{1}$$

and they differ only in the ordering of P and S. It is this difference that makes cooperators persist more easily in the SG than in the PDG. In a well-mixed population, cooperators and defectors coexist. Due to the difficulty in measuring payoffs and the ordering of the payoffs accurately in real-world situations where game theory is applicable [17,18], the SD has been taken to be a possible alternative to the PDG in studying emerging cooperative phenomena [16].

The present work will focus on two aspects of current interest. In many circumstances, the connections in a competing population are better modelled by some networks providing limited interactions than a fully connected network. Previous studies showed that different spatial structures might lead to different behaviors [7,8,19–22]. For example, it has been demonstrated that spatial structures would promote cooperation in the PDG [7,8], but would suppress cooperation in the SG [16]. There are other variations on the SG that resulted in improved cooperation [23,24]. Here, we explore the effects of an underlying network on the evolutionary SG in a population in which there exists an additional type of players. The latter is related to the fact that real-world systems usually consist of people who would adopt a strategy other than just C and D. For example, there may be people who do not like to participate in the competition and would rather take a smaller but fixed payoff that is risk-free and fair. In the context of PDG, such persons are called loners. Hauert et al. studied the effects of the presence of such loners [11,12] in a generalization of the PDG called the public goods game (PGG). Motivated by the works of Hauert et al. [11,12,16], we study the effects of a third strategy characterized by risk averse players in the evolutionary SG. These players are referred to as L-players, following the notations in PDG. In our model, evolution or adaptation is built in by allowing players to replace his strategy or character by that of a better-performing connected neighbor. We focus on both the steady state and the dynamics, and study how an underlying network structure affects the emergence of cooperation. It is found that in a fully connected network, the C-players and D-players *cannot* coexist with the L-players. In a square lattice, however, cooperators are easier to survive. Depending on the payoffs, there are situations in which C-players, D-players and L-players can coexist.

In Section 2, the evolutionary SG incorporating an additional character in a population with connections is presented. In Section 3, we present detailed numerical results in fully connected networks and in square

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lattices, and discuss the physics of the observed features. The effects of noise are also discussed. We summarize our results in Section 4.

2. The model

We consider an evolutionary SG in which the competitions between players are characterized by the payoff matrix

$$\begin{array}{cc} & \begin{array}{ccc} C & D & L \end{array} \\ \begin{array}{c} C \\ D \\ L \end{array} & \begin{pmatrix} R & S & Q \\ T & P & Q \\ Q & Q & Q \end{pmatrix} \end{array}. \tag{2}$$

Here, each player takes on one of three possible characters or strategies: to cooperate (C), to defect (D), or take on an action L, which in the PD Game was referred to as the loner [11,12]. The matrix element gives the payoff to a player using a strategy listed in the left-hand column when the opponent uses a strategy in the top row. In the basic SG, it is useful to assign $R = 1$ and to characterize the payoffs by a single parameter $r = c'/2 = c'/(2b' - c')$ that represents the cost-to-reward ratio [16]. In terms of $0 < r < 1$, we have $T = 1 + r$, $R = 1$, $S = 1 - r$, and $P = 0$. When a player takes on the L-character or when a player plays the game with a L-character player, we assume there is no competition going on and both players get a payoff Q. Here, we explore the range of $0 < Q < 1$.

One may ask whether there are scenarios in which the payoff matrix can be applied to. In the PDG with a third character, it was assumed that there are players who do not want to take the risks in investment and instead want to collect a risk-free payoff Q. Since they are inert players, they are called loners. In the SG, a person who takes on the L-character should instead be regarded as an exceptional persuasive person. He is able to persuade whoever he meets to follow his plan. In the snowdrift shovelling scenario, for example, a L-player can be regarded as one who can persuade whoever he meets to hire someone to do the job and share the cost with him, so that both can pass through without doing the job. In this way, any game that involves a L-player results in the same payoff Q to each player, regardless of the character of the opponent of the L-player. Although this character is not that of a loner, we follow the notations in PDG and refer to the players with L-character as L-players. Here, we investigate the range of $0 < Q < 1$. When a C-player and a L-player meet, the L-player convinces the C-player that it is beneficial to get a somewhat smaller payoff ($Q < 1$) than he could possibly get, but without doing the laborious job. As a C-player could either have gotten a payoff of 1 or $1 - r$ in the absence of L-players, it is expected that the L-player would have a bargaining power for $1 - r < Q < 1$. With the evolutionary mechanism to be introduced below, it turns out that indeed the L-character persists in a region in the (Q, r) parameter space where $Q > 1 - r$ and characterized by $r > \sqrt{1 - Q}$ in a fully connected population. When a D-player and a L-player meet, the L-player convinces the D-player that by giving up the hope of getting the highest payoff of $(1 + r)$ and settling with the payoff $Q > 0$, the D-player will be free from getting stuck ($P = 0$ payoff). Our aim is to investigate the fraction of C-character, D-character and L-character in an evolutionary SG in the (Q, r) parameter space.

Spatial networking effects and evolution are incorporated into the SG as follows. At the beginning of the game, the players are arranged onto the nodes of a network and the character $s(i)$ of each player is assigned randomly among the choices of C, D, and L. Our discussion will be mainly on fully connected graphs and regular lattices. In a fully connected network, every player is connected to all other players. In a square lattice, a player is linked only to his four nearest neighbors. Numerical studies are carried out using Monte Carlo simulations as reported in the work of Szabó et al. [13] (see also Refs. [12,14]). The evolution of the character of the players is governed by the following dynamics. At any time during the game, each player competes with all the players that he is linked to and hence has a payoff. A randomly chosen player i reassesses his own strategy by comparing his payoff $P(i)$ with the payoff $P(j)$ of a randomly chosen connected neighbor j. With probability

$$W[s(i), s(j)] = \frac{1}{1 + \exp\big([P(i) - P(j)]/K\big)}, \tag{3}$$

the player i adopts the strategy of player j. Otherwise, the strategy of player i remains unchanged. Here K is a noise parameter [12–14] that determines the likelihood that player i replaces his strategy when he meets someone with a higher payoff. For $K \approx 0$, a player i is almost certain to replace (not to replace) his strategy when he meets someone with a better (worse) payoff. For large K, a player has a probability of $\frac{1}{2}$ to replace his strategy, regardless of whether $P(j)$ is better or worse than $P(i)$. In a fully connected network, a player's character may be replaced by any player in the system. In a square lattice, a player's character can only be replaced by one of his four connected neighbors. As the game evolves, the fractions of players with the three characters also evolve. These fractions are referred to as frequencies. Depending on the parameters r and Q, the C-player frequency f_C, D-player frequency f_D, and L-player frequency f_L take on different values in the long time limit.

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3. Results and discussions

3.1. Fully connected network

We performed detailed numerical studies on our model. The number of players in the system is taken to be $N = 10^4$. In getting the fraction of players of different characters in the long time limit, we typically average over the results of 10^3 Monte Carlo time steps per site (MCS), after allowing 5×10^3 MCS for the system to reach the long time limit. Averages are also taken over 10 initial configurations of the same set of parameters. Fig. 1 shows the results for fully connected networks. A value of $K = 0.1$ is taken. The C-player frequency f_C, D-player frequency f_D, and L-player frequency f_L are obtained as a function of the cost-to-benefit ratio r for three different values of the L-player's payoff $Q = 0.3$, 0.5, and 0.7. In the absence of L-players [16], $f_C(r) = 1 - r$ and $f_D = r$ in a fully connected network. From Fig. 1, the L-players extinct for a range of values of $r < r_L(Q)$ in which the behavior is identical to the basic SG. For $r > r_L(Q)$, the L-players invade the whole population and both cooperators and defectors disappear. This is similar to the results in the PDG [13] and in the PGG [11]. In a fully connected network, the three characters *cannot* coexist. This is in sharp contrast to the rock–scissors–paper game [25–27] on a fully connected network in which the three strategies coexist. We obtained $r_L(Q)$ numerically. The result is shown in Fig. 1(d) as a curve in the (Q, r) parameter space. It is found that $r_L(Q)$ follows the functional form $\sqrt{1-Q}$, which will be explained later. The curve $r_L(Q)$ represents a phase boundary that separates the (Q, r) parameter space into two regions. The region below (above) the curve corresponds to a phase in which C-players and D-players (only L-players) coexist (exist).

We also studied the temporal evolution in both phases, i.e., for $r < r_L(Q)$ and $r > r_L(Q)$. Taking $Q = 0.5$, for example, $r_L = \sqrt{1/2} = 0.707$. Fig. 2 shows $f_C(t)$, $f_D(t)$ and $f_L(t)$ in the first 10^3 MCS. The initial frequencies are $\frac{1}{3}$ for all three characters. For values of r deep into either phase (see Fig. 2), the transient behavior dies off rapidly and the extinct character typically vanishes after $\sim 10^2$ MCS. In the phase where C and D coexist, f_C and f_D oscillate slightly with time in the long time limit, due to the dynamical nature of the game. It is noted that for $r \approx r_L$, the strategies compete for a long while and the transient behavior lasts for a long time. This slowing down behavior is typical of that near a transition.

The behavior of $r_L(Q) = \sqrt{1-Q}$ follows from the rule of character evolution. In a fully connected network, all C-players have the *same* payoff $P(C)$ and all D-players have the *same* payoff $P(D)$. These payoffs depend on f_C, f_D, and f_L at each time step. The payoff for a L-player is NQ at all time, in a system with $N \gg 1$. For small K, f_L decays exponentially with time if $P(C)$ and $P(D)$ are both greater than NQ. In addition, the phase with only non-vanishing f_C and f_D is achieved by having $P(C) = P(D)$. For this phase in the long time limit, $P(C) = N(f_C + f_D(1-r))$ and $P(D) = Nf_C(1+r)$. Together with $f_C + f_D = 1$ (since $f_L = 0$ in the phase under consideration), the condition $P(C) = P(D)$ implies $f_C = 1 - r$ and $f_D = r$. These results are identical to the basic SG (without L-players) in a fully connected network. The validity of this solution requires $P(C) > NQ$ (and hence $P(D) > NQ$), which is equivalent to $r < \sqrt{1-Q}$. This is exactly the phase boundary shown in Fig. 1(d).

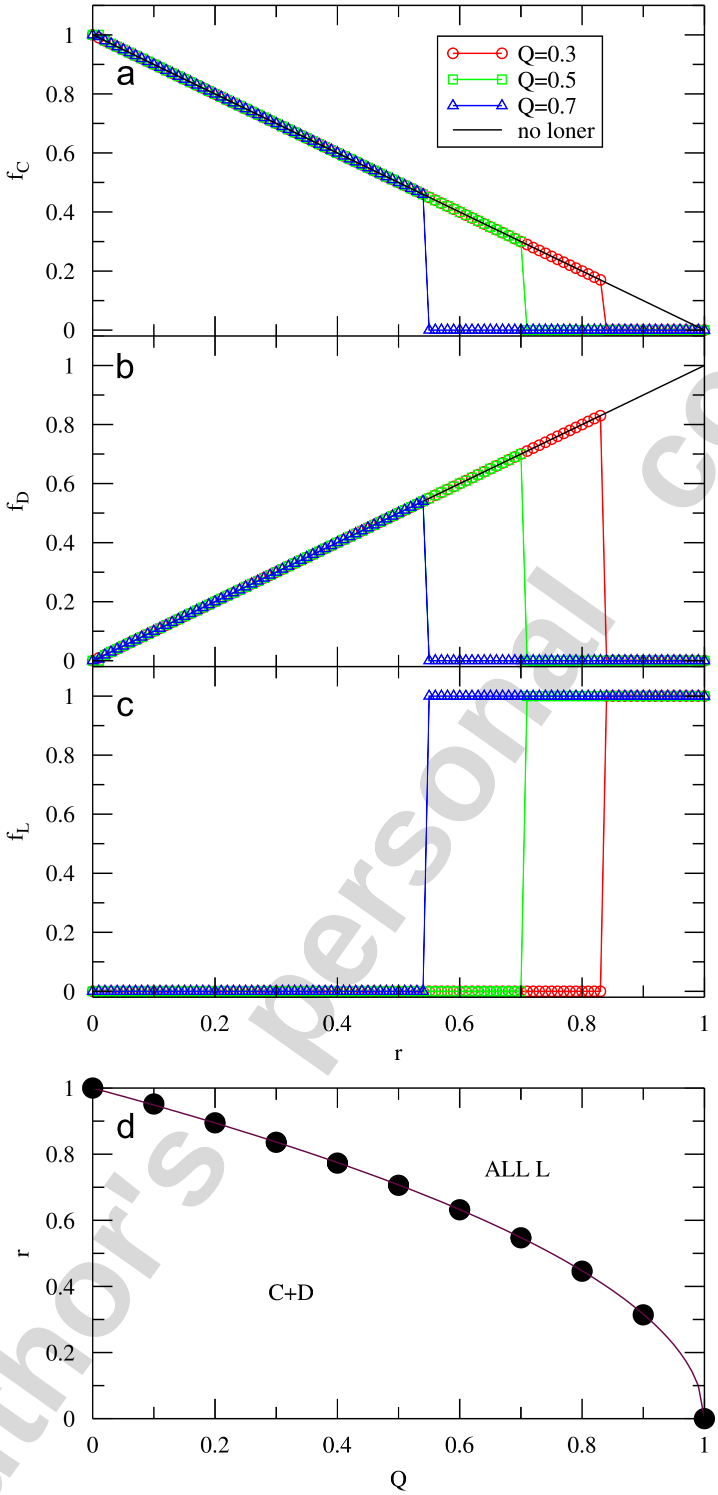

Fig. 1. (Color online) (a) C-player frequency f_C, (b) D-player frequency f_D, (c) L-player frequency f_L as a function of r for three different values of L-player payoff $Q = 0.3$, 0.5, and 0.7 in a fully connected network. The results of the Snowdrift game without L-players are also included for comparison (solid lines), and (d) phase diagram showing the two phases separated by $r_L(Q)$ in the $r-Q$ parameter space. The symbols show the numerical results of $r_L(Q)$ and the line gives the functional form $\sqrt{1-Q}$.

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3.2. Square lattice

The behavior of the game in a square lattice is expected to be quite different, due to the restriction that a player can only compete with his connected neighbors. We carried out simulations on 100×100 square

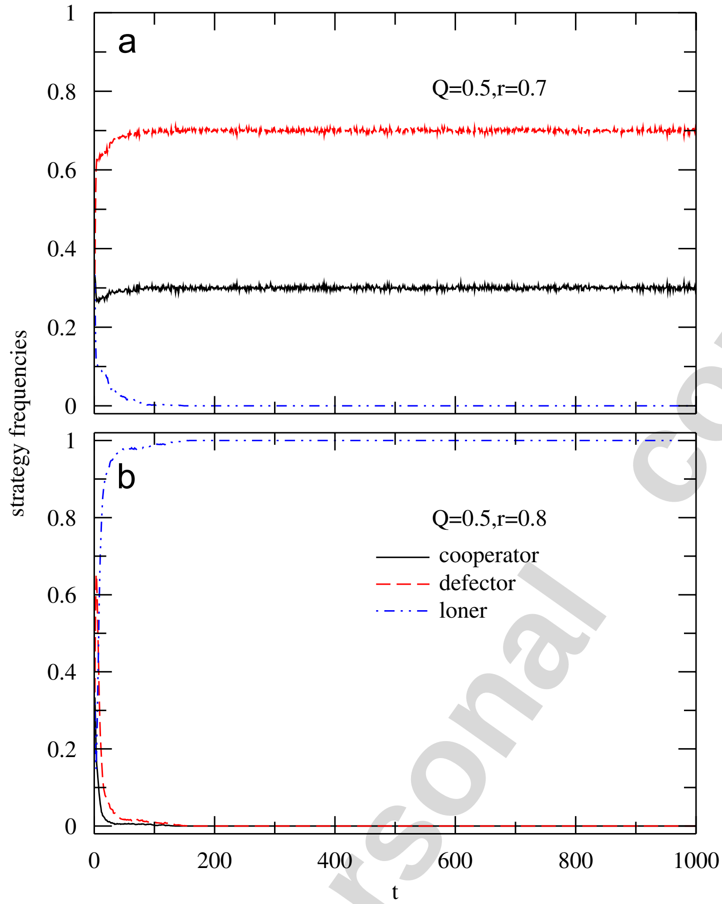

Fig. 2. (Color online) Typical short-time behavior of the C-player, D-player, and L-player frequencies in a fully connected network for: (a) the phase that C-players and D-players coexist ($Q = 0.5$ and $r = 0.7$); and (b) the phase that L-players take out the whole system ($Q = 0.5$ and $r = 0.8$).

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lattices with periodic boundary conditions. Fig. 3(a)–(c) shows $f_C(r)$, $f_D(r)$ and $f_L(r)$ for three different values of the L-player payoff Q. The results for the spatial SG (without L-players) on a square lattice [16] is also shown (solid lines in Fig. 3(a) and (b)) for comparison. A value $K = 0.1$ is used. Several features should be noted. For $r < r_L^{(SL)}(Q)$, the L-players eventually vanish with f_C and f_D take on the mean values corresponding to those in the spatial SG without L-players. This behavior is similar to that in fully connected networks. For $r > r_L^{(SL)}(Q)$, however, the behavior is different from that in fully connected networks. Here, C, D, and L characters coexist. Above $r_L^{(SL)}$, f_D drops with r to a finite value, leaving rooms for f_L to increase with r. The cooperator frequency remains finite above $r_L^{(SL)}$. Therefore, the cooperator frequency or the cooperative level in the system as a whole is significantly *improved* by the presence of L-players. For $r > r_L^{(SL)}$, increasing the payoff Q of L-players leads to a higher cooperator frequency and lower defector frequency. Reading out $r_L^{(SL)}$ for different values of Q, we get the phase boundary as shown in Fig. 3(d) that separates a region characterized by the coexistence of three characters and a region in which only C and D coexist. The results indicate that, due to the restriction imposed by the spatial geometry that a player can only interact with his four nearest neighbors, it takes a certain non-vanishing value of r for L-players to survive even in the limit of $Q \to 1$. The behavior is therefore different from that in a fully connected network for which the boundary is given by $\sqrt{1-Q}$. Note that there exists a region of small values of r in which the steady state consists of a uniform population of C strategy (see Fig. 3(a) and (d)). For small Q, L-players are easier to survive, when compared with the fully connected case. Putting these results together, the phase diagram (see Fig. 3(d)) for a square lattice, therefore, shows three different phases. The most striking effect of the spatial structure is that cooperators now exist in every phase.

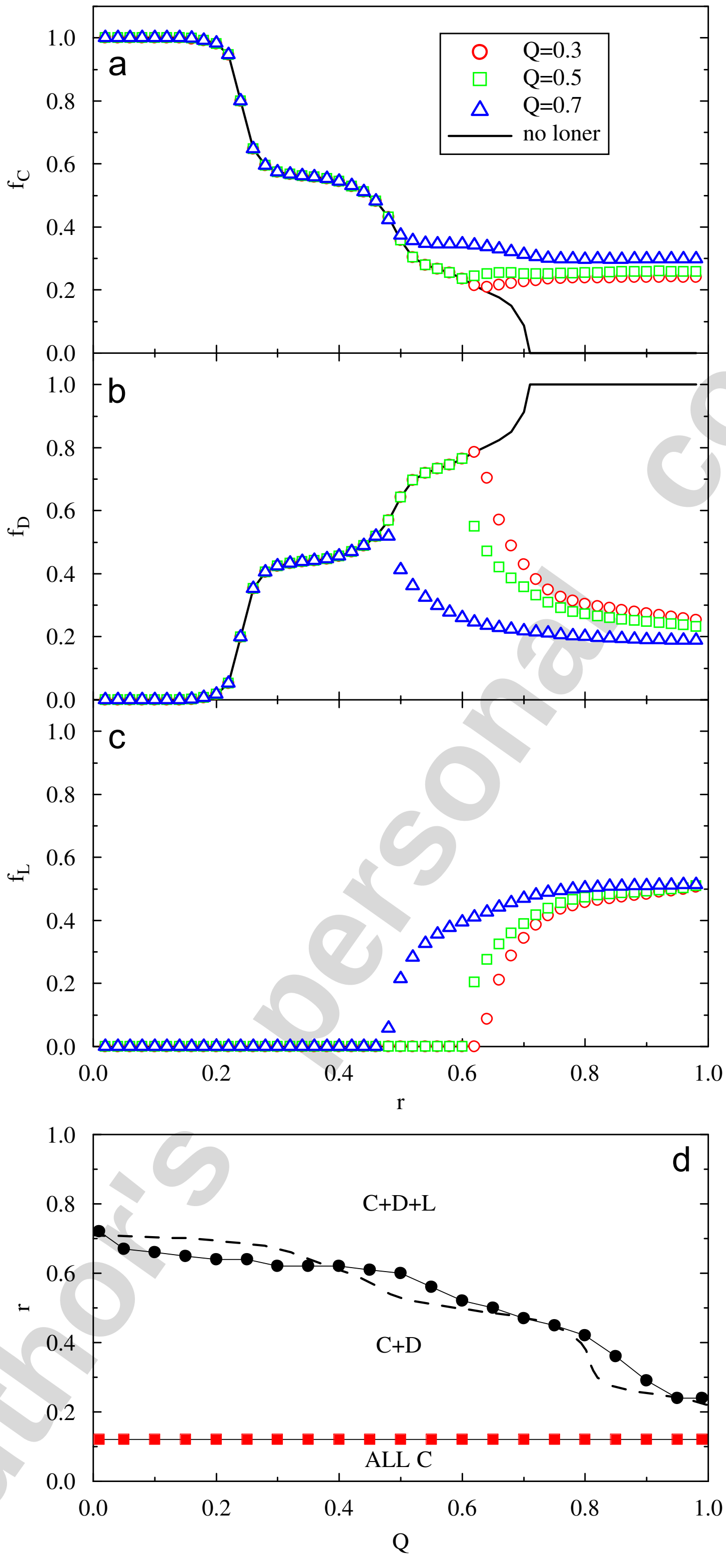

Fig. 3. (Color online) (a) C-player frequency f_C, (b) D-player frequency f_D, (c) L-player frequency f_L as a function of r for three different values of the L-player payoff $Q = 0.3$, 0.5, and 0.7 in a square lattice. The results of the snowdrift game without L-players in a square lattice are also included for comparison (solid lines), and (d) phase diagram showing the different phases in the $r-Q$ parameter space. The dashed line shows the phase boundary obtained by an approximation as discussed in the text.

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Interestingly, we found that the phase boundary $r_L^{(SL)}(Q)$ in Fig. 3(d) could be described quantitatively as follows. We *assume* that the survival of L-players is related to the cooperator frequency. In particular, L-player survival requires the cooperator frequency to drop below a certain level $\overline{f}(Q)$ and that this value is the same in a square lattice as in a fully connected network. That is to say, we assume that L-players could survive, for a given value of Q and K, only when $f_C < \overline{f}(Q) = 1 - \sqrt{1-Q}$. Numerical results also indicate that when all L-players extinct, f_C and f_D follow the results in a spatial SG without L-players. This is shown as the solid line in Fig. 3(a). Therefore, for a given value of Q, we can simply read out the value of r such that $f_C = \overline{f}(Q)$ from the results in the spatial SG in a square lattice. For different values of Q, this procedure results in the dashed line shown in Fig. 3(d) which describes the phase boundary quite well.

Fig. 4 shows the temporal dependence of f_C, f_D, and f_L in a square lattice for two values of r at $Q = 0.5$. For $r = 0.55$ (Fig. 4(a)), which corresponds to a case in which only cooperators and defectors coexist, the number of L-players decay rapidly in time, typically within 100 MCS. After the transient behavior, the cooperator and defector frequencies only oscillate slightly about their mean values. This behavior is similar to that in the C and D coexistence phase in Fig. 1(d) for fully connected networks. For $r = 0.65$ (Fig. 4(b)), which corresponds to a case with the three characters coexist, the long time behavior of f_C, f_D and f_L is oscillatory. Similar behavior has been found in the rock–scissors–paper game [25–27] and in the voluntary PDG [14]. Due to the dynamical nature of character evolution, there are continuous replacements of one character by another and this oscillatory behavior is expected.

The major difference between a square lattice and a fully connected network is that in a fully connected network, each player competes with all other players. As a result, there are only three payoffs in the system—one for each type of player, at each time step. The L-players, for example, have a constant payoff of NQ, while

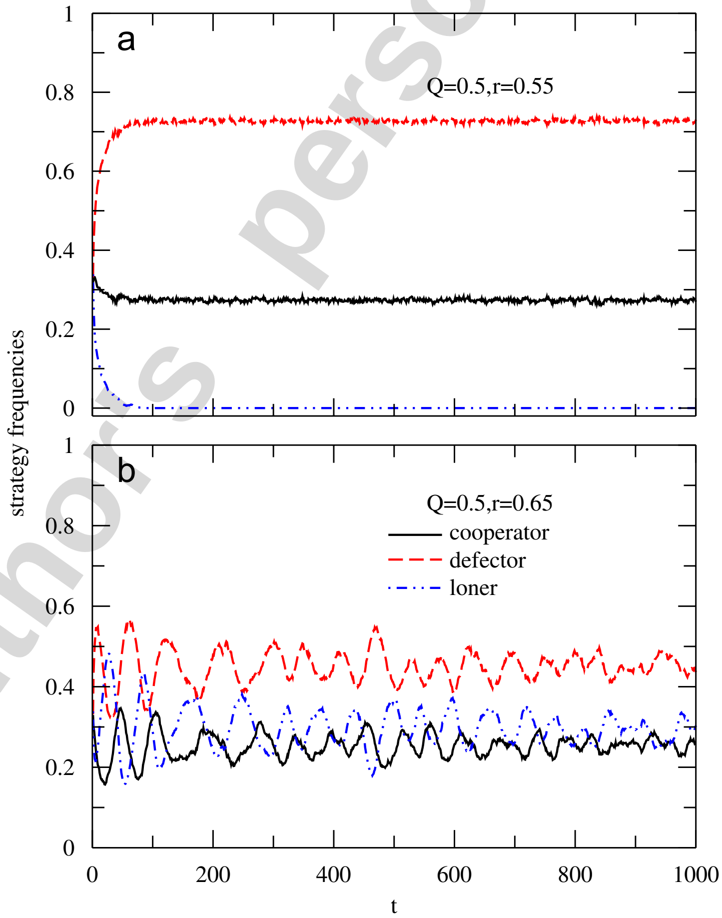

Fig. 4. (Color online) Typical short-time behavior of the C-player, D-player, and L-player frequencies in a square lattice for: (a) the phase in which C-players and D-players coexist ($Q = 0.5$ and $r = 0.55$); and (b) the phase in which C-players, D-players, and L-players coexist ($Q = 0.5$ and $r = 0.65$).

the cooperators and defectors have payoffs that depend on $f_C(t)$ and $f_D(t)$. Once NQ is higher than the payoffs of cooperators and defectors, the number of L-players grows until they take over the whole population. In a square lattice, however, each player has a payoff that depends on his character *and* the detail of his neighborhood, i.e., the characters of his four connected neighbors. This implies that the C-players and D-players in a square lattice may have several different payoffs depending on the characters of his connected neighbors. The L-players have a constant payoff of $4Q$. The non-uniform payoffs among C-players and D-players in a lattice allow some C and D players to coexist with the L players, by evolving to spatial local configurations that favor their survivals.

Since the adaptive rule is related to the payoff of each character, it will be interesting to compare the payoffs in a spatial SG without and with L-players. Fig. 5(a) shows the mean payoffs of cooperators and defectors as a function of r in a spatial SG in a square lattice *without* L-players. The averaged payoff over all players is also shown. For small r, there is a phase with all C players and the payoff is 4 for each of the C players. For large r, there is a phase with all D players and the payoff is zero. For intermediate r where C and D players coexist, the mean payoff drops gradually with r. In a spatial SG *with* L-players (Fig. 5(b)), it is observed that the mean payoffs basically follow that in Fig. 5(a) in the phase where L-players are completely replaced. When L-players can survive, the presence of these L-players increases the payoffs of both the remaining cooperators and defectors. The L-players themselves have a payoff of 2 in a 2D square lattice. The cooperators' payoff is enhanced once L-players survive and the increase follows the same form as the increase in the L-player frequency with r (compare the circles in Fig. 5(b) with the squares in Fig. 3(c) in the range of r when L-players

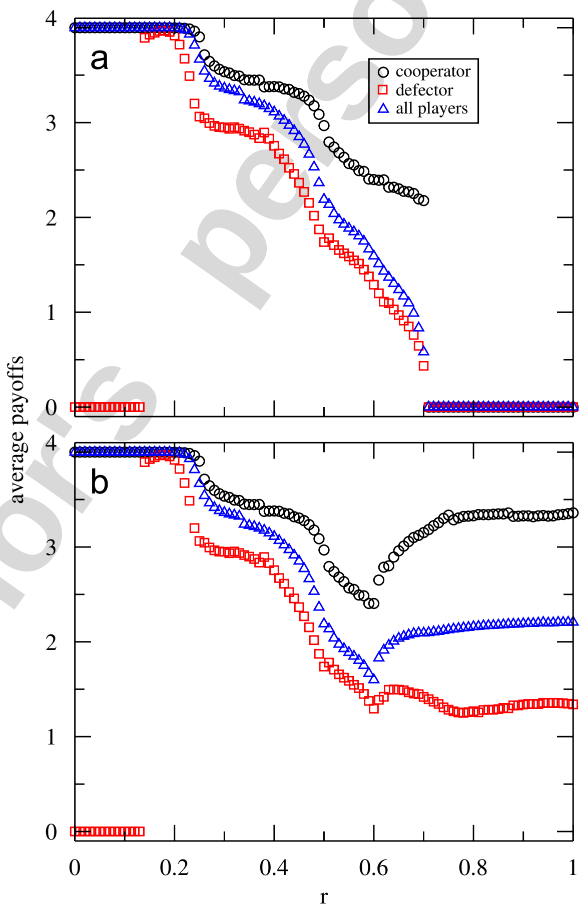

Fig. 5. (Color online) (a) Average payoffs of each character as a function of r in a snowdrift game without L-players on a square lattice. The payoff averaged over all players is also shown. (b) Average payoffs of cooperators and defectors as a function of r in a snowdrift game with L-players. The parameters are $Q = 0.5$ and $K = 0.1$. Note that the L-players, if exist, have a constant payoff of $4Q$. The payoff averaged over all players is also shown.

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survive). When L-players survive, the payoff averaged over all players is significantly *enhanced* due to their presence. This is similar to what was found in the voluntary PDG [14].

3.3. Effects of noise

All the results reported so far are for the case of $K = 0.1$. This corresponds to a case where the player is highly likely to replace his character when he meets a better-performing player. In Fig. 6, we show the effects of the noise parameter K for a fixed $Q = 0.3$. As K increases, the step-like structure in f_C as a function of r becomes less obvious and f_C is gradually suppressed in the $r \to 1$ limit. The most important effect of a 2D square lattice is that each player is restricted to interact with his four neighbors. Take a player of character $s(i)$, he will only encounter a finite number of configurations for which he is competing in. For example, his four neighbors may consist of 4 C-players; 3 C-players and 1 D-player or 1 L-player, etc. Each of these configurations corresponds to a $P(i)$. In a square lattice, therefore, there will be a finite number of payoffs for a C-player, depending on the characters of the neighbors. Similarly, there are finite number of payoffs for a D-player. The L-players always get a payoff of $4Q$. For $K \approx 0$, the adaptive mechanism is strictly governed by the ordering of these payoffs. The distribution of players in a square lattice will then evolve in time according to how the payoffs are ordered. In the long time limit, only a few favorable local configurations will survive and the number of players in each of these favorable configurations is high. As one increases r slightly, the ordering of the finite number of payoffs may not change. Therefore, f_C will not change with r until we reach certain values of r that the ordering of the payoffs is changed. This gives rise to the more sudden changes in f_C as observed at some values of r and it is the reason for having step-like features in f_C and f_D for small values of K. As the noise parameter K increases, the adaptive mechanism is less dependent on the exact ordering of the payoffs. Therefore, the changes in f_C with r becomes more gradual as K increases. Interestingly, less obvious step-like structures in f_C are also observed in the spatial SG without L-players in

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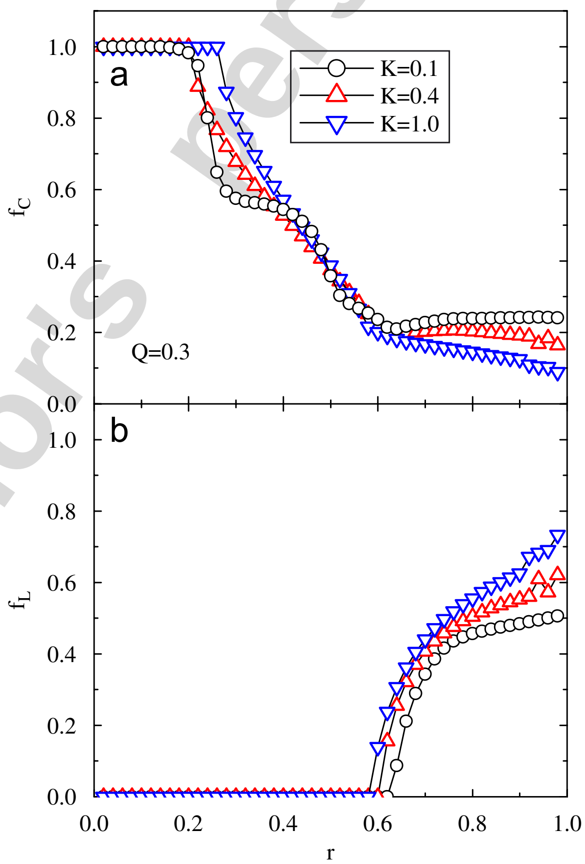

Fig. 6. (Color online) (a) The C-player frequency f_C and (b) the L-player frequency f_L as a function of r for three different values of the noise parameter $K = 0.1$, 0.4 and 1.0.

2D lattices with a larger coordination number [16]. This is also related to the picture we just described. A lattice with more neighbors will give a higher number of neighborhood configurations and hence more values of the payoffs. More configurations also imply the number of players encountering a certain configuration is smaller. Thus, the number of players involved in a change in the ordering of the payoffs as r changes is smaller. This has the effect of making the drop in f_C gradual. Therefore, increasing K for a given fixed coordination number is similar in effect as increasing the coordination number for fixed K.

4. Summary

We studied the effects of the presence of a third character in an evolutionary snowdrift game in fully connected networks and in square lattices. When a L-player is involved in a game, there is a constant payoff Q to both players, regardless of the character of the opponent. In a fully connected network, either cooperators live with defectors or L-players take over the whole population. The condition for L-players to take over is found to be $Q > 1 - r^2$. This result can be understood by following the payoffs of each strategy. In a fully connected network, the strategies' payoffs are particularly simple in that they depend only on the strategy frequencies at the time under consideration, with each type of player having the same payoff.

In a square lattice, the spatial SG with L-players behave quite differently. It is found that the cooperators can survive in the full parameter space covering $0 < r < 1$ and $0 < Q < 1$. Depending on the values of these parameters, there are three possible phases: a uniform C-player population, C-players and D-players coexist, and coexistence of the three characters. The underlying lattice thus makes the survival of cooperators easier. The presence of L-players is also found to promote the presence of cooperators. The average payoff among all players is also found to be enhanced in the presence of L-players. We discussed the influence of a square lattice in terms of the payoffs of the players. In a square lattice, spatial restriction only allows a player to interact only with the four nearest neighbors. This leads to a payoff that depends both on the character and on the local environment in which the player is competing in. The players in the local environment, in turns, are also competing in their own local environment. This will lead to clustering or aggregation of players in the square lattice into configurations that the payoffs favored. The dependence of the frequencies on r in a square lattice then reflects the change in preferred configurations as r is changed.

We also studied the effects of the noise parameter in the adaptive mechanism. It is found that as the noise parameter increases, the change of the frequencies with r becomes more gradual. This is related to the importance of the ordering of the many possible payoff values in the adaptive mechanism. As the noise parameter increases, the exact ordering of the payoffs becomes less important and the change in frequencies becomes more gradual.

In closing, we note that it will be interesting to further investigate the effects of L-players in the snowdrift game in networks of other structures. Among them are the re-wiring of regular lattices into a small-world network or a random network and the scale-free networks [28].

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