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Score-dependent payoffs and Minority Games

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Abstract

Payoffs which depend on the scores of the strategies are introduced into the standard Minority Game (MG). The double-periodicity behavior of the standard model is consequently removed, and stylized facts arise, such as long-range volatility correlations and "fat-tails" of the probability distribution of the returns. Furthermore, the score-dependent payoffs could coexist with the inactive strategy in the grand canonical MG, and play an important role in the thermal MG.

Keywords: Minority game; Econophysics

1. Introduction

Modeling the financial markets has attracted much attention of scientists in order to obtain qualitative and quantitative understanding of the trading mechanism. In the past years, different types of models have been proposed [1–11]. Among them, the Minority Game (MG) is one of the important examples, especially because of its simplicity and suitability to an analytic approach. The standard MG [4,12] was initially designed as a simplification of Arthur's famous El Farol's Bar problem [13]. It describes a system in which many heterogeneous agents adaptively compete for a scarce resource, and it captures some key features of a generic market mechanism and the basic interaction between agents and the public information. However, it is a highly simplified model. To make the model more realistic in comparison with the real markets, the micro-economic behavior of the agents should be taken into account.

Consequently, a variety of extensions [8,14–19] of the standard MG have been proposed. For example, an inactive strategy is introduced, which grants the agents with a certain probability of not trading in the market, to capture the characteristics of the real markets. In this case, the number of agents actively trading at each time step varies throughout the game. This type of extension is called the grand canonical MG. Another interesting example is the thermal MG [20] which allows a certain degree of randomness in using the strategies.

However, in the standard MG and many its extensions, the statistical double-periodicity is quite robust and hence is a disadvantage in mimicking real systems. Even for the grand canonical MG, in which the inactive strategy has eliminated the periodicity from some observable, the system still suffers from an intrinsic

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annoyance like other deterministic dynamic systems [11,21]. In this paper, we propose an extension of the MG incorporating score-dependent payoffs. Interestingly, such a dynamic mechanism essentially removes the double-periodicity. Furthermore, stylized facts, such as the long-range volatility correlations and "fat-tails" of the probability distribution of the returns can be also reproduced. The score-dependent payoffs could also coexist with the inactive strategy in the grand canonical MG, and leads to the long-range volatility correlations for the thermal MG.

In most MGs, the payoffs are generally referred to as the rewards to the agents, and it has a simple form. In this paper, payoffs are generalized to be the rewards to the strategies. To look for more reasonable payoffs which could mimic the financial markets is also of great interest for the real social system. There may be thousands of forms for the payoffs of the strategies. However, we may assume that the payoff of each strategy is mainly determined by two factors: the global information of the outcome after all the agents make their own decisions, i.e., the number of the agents in the minority group; and the cumulative performance of the individual strategy, i.e., the score of the strategy itself. Since the resource in the market is limited, it is easy to understand that the payoff increases with decreasing size of the minority group [22]. On the other hand, those strategies with higher scores should be rewarded (punished) more than those with lower scores. This is reasonable, since people are usually more attentive to those outstanding strategies. Therefore, we introduce the so-called score-dependent payoffs in this paper.

In Section 2, the MG with score-dependent payoffs is introduced and the auto-correlation function and return probability distribution are studied. In Section 3, the score-dependent payoffs are introduced to the grand canonical MG and the thermal MG. Section 4 contains the conclusion.

2. Standard MG with score-dependent payoffs

2.1. The model

The MG takes the form of a repeated game with an odd number of agents N who must choose an action independently whether to buy or sell, labeled $a_i(t) = +1$ or -1. Those agents who are in the minority group are winners. Each agent chooses his/her decision at a given time step based on the prediction of a strategy according to the history, namely the m most recent outcomes of the winner side. Since there are a total of 2^m possible history bit-strings of the winner side and there are two possible options (buy or sell) for each history bit-string, there are a total of 2^{2^m} strategies.

At the beginning of the game, the agents randomly pick S strategies from the full strategy space with repetitions allowed. Each agent i keeps track of the cumulative performance of his/her trading strategy $s, s = 1, ..., S$ by assigning a score $U_{i,s}$ to it. The initial scores for the strategies are set to be zero. At each time step, each agent i adopts the strategy with the highest score $s_i(t)$, and the action will then be $a_i(t) = \sigma^{\mu(t)}_{i,s_i(t)}$, according to the history $\mu(t)$ which is the common information shared by all the agents. $\sigma^{\mu(t)}_{i,s_i(t)} = +1, -1$ denotes buying and selling. When a tie results from the highest score strategies, one of them is picked randomly. The excess demand is then defined as $A(t) = \sum_{i=1}^{N} a_i(t)$. The scores of the strategies are updated as

$$U_{i,s}(t+1) = U_{i,s}(t) + g_{i,s}(t), \tag{1}$$

$$g_{i,s}(t) = -\sigma^{\mu(t)}_{i,s} A(t) b \frac{\exp(aU_{i,s})}{\sum_{s=1}^{S} \exp(aU_{i,s})}. \tag{2}$$

a and b are real parameters, taken to be positive. For the parameter $a = 0$, the standard MG is recovered. In most MGs, the payoff of a strategy s for an agent i at a given time t is generally defined as $g_{i,s}(t) = -\sigma^{\mu(t)}_{i,s} A(t)$. Those strategies giving the prediction consequently proved to be in the minority are rewarded, and the rewards increase with decreasing size of the minority group. Furthermore, as it is ubiquitous in the markets that different people trade with different weights, we assume the payoffs for different strategies should be different according to the cumulative performance of themselves.

The payoffs in our model are defined as Eq. (2). Those strategies with higher scores are rewarded (punished) more than those with lower scores. This is reasonable, since people are usually more attentive to those outstanding strategies. Once a high-score strategy gives a prediction which is subsequently proved to be in the majority, it will be restrained from the market for a long time. While it gives a prediction, which is subsequently proved to be in the minority, it will be likely adopted at the next turn for making more profits. For a low-score strategy, it is not so much affected by the outcome of the game. In addition, the payoffs in Eq. (2) are also agent-dependent, for the denominator $\sum_{s=1}^{S}\exp(aU_{i,s})$ is agent-dependent. More and deeper understanding of this kind will be reported elsewhere [30].

Following Refs. [8,15,23–25], a simple price dynamics of the return $r(t)$ in terms of the excess demand $A(t)$ is defined as

$$r(t) = A(t). \tag{3}$$

2.2. Auto-correlation function and cumulative distribution function

In the real stock markets, there are a huge number of investors making their decisions simultaneously, and the memory size m for normal people are short. So, may be the paradigm of the real markets is similar to that of the MG with small S and m. For the application of the model to the real markets, we mainly report the results of the model for $S = 2$ and $m = 2$. However, for the standard MG with small S and m, a crowd of agents choose a particular strategy and thus produce large fluctuations of the returns [26]. The auto-correlation function of the magnitude of the returns exhibits a double-periodicity behavior with a period $2 * 2^m$ (here $m = 2$) [27]. After we introduce the score-dependent payoffs, the dynamic evolution of the model is then modified due to the change of the ranking of the strategies. One expects the agents will cite those strategies which would have had lower scores in the standard MG, thus the crowd effect will be removed and then the double-periodicity will disappear.

The long-range temporal correlations of the volatility is a well-known stylized fact. The auto-correlation function of the volatility decays by a power law, typically as

$$c(\tau) \sim \tau^{-\lambda} \tag{4}$$

and the exponent λ is estimated to be 0.3 in real financial markets [28,29]. In this paper, We simply define the volatility as $|A|$ in our model, and then the auto-correlation function is written as

$$C(\tau) \equiv \frac{\langle |A(t)||A(t+\tau)|\rangle - (\langle |A(t)|\rangle)^2}{\langle |A(t)|^2\rangle - (\langle |A(t)|\rangle)^2}, \tag{5}$$

where $\langle \cdots \rangle$ represents the average over the time t. We have performed extensive numerical simulations of our model. To avoid the limit size effect, we use a big N. For $N = 5001, S = 2, m = 2$, the results mainly depend on the parameters b and a.

In Fig. 1, the auto-correlation function for different values of the parameter a are displayed for $b = 10.0$. For each value of a, the result averages over 100 runs with 10^5 iterations per run. Before collecting the data, 5000 iterations have been performed for equilibration. The figure shows that as a changes from 0.0 to 1.0, a crossover behavior occurs. According to Eq. (2), the payoff for $a = 0.0$ and $b = 10.0$ equals to $-5\sigma_{i,s}^{\mu(t)}A(t)$ (here $S = 2$), the half of that of the standard MG. As it is shown in Fig. 1(a) the curve for $a = 0.0$ displays a strong periodic behavior with a period $2 * 2^2$, with a shape not exactly the same as but very similar to that of the standard MG. In Fig. 1(b), the auto-correlation functions for $a \neq 0.0$ are plotted on a log–log scale. The curve for $a = 1.0$ is shifted slightly upward for clarity. We observe that as a increases, the auto-correlation function tends to show a power-law behavior, though for a relatively small $a < 0.001$ the periodic behavior still remains. The exponent λ changes from -0.66 to -0.25 for $a \in [0.001, 0.1]$ and approaches a constant for $a > 0.1$. Since the payoffs are the exponential functions of the strategies' scores which are of the order of magnitude of 10^3, a should not be too large.

We also calculate the auto-correlation function for a relatively small b. The results indicate that as a varies from 0.0 to 1.0, the crossover behavior from periodically fluctuated to long-range correlated still remains. However, the critical value of the parameter a, above which the auto-correlation function shows a power-law behavior, decreases as the parameter b increases.

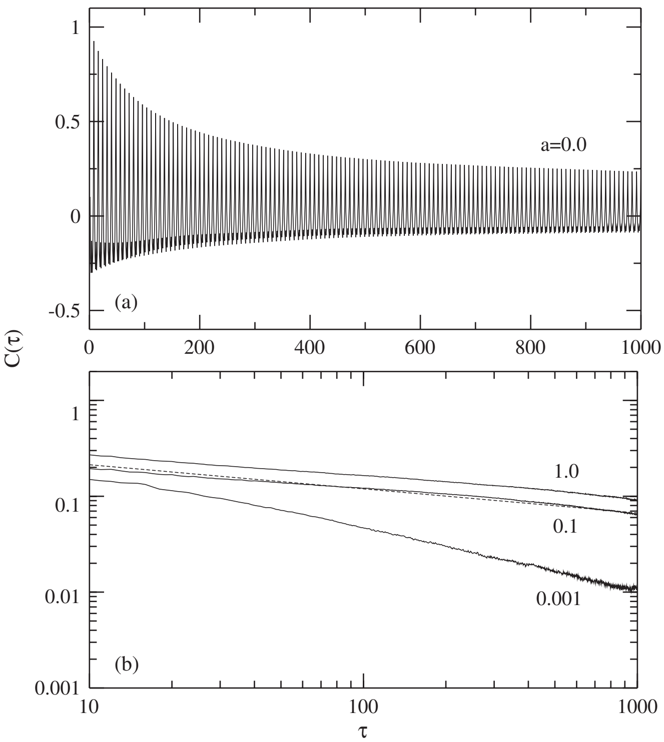

Fig. 1. Auto-correlation function of the magnitude of the returns for different values of the parameter a: (a) $a = 0.0$, (b) $a = 0.001, 0.1, 1.0$, for the standard MG with score-dependent payoffs at $b = 10.0, N = 5001, S = 2, m = 2$. The dashed line is with a slope of -0.25.

Briefly speaking, for a larger value of the parameter a, the double-periodicity behavior of the auto-correlation function disappears, and the power-law behavior emerges. It confirms our conjecture that the score-dependent payoffs could remove the crowd effect and thus improve the market efficiency. To show this more explicitly, one can compute the standard deviation σ of the price fluctuations, which is a convenient reciprocal measure of how effective the system is at distributing resources. Indeed, we observe that the score-dependent payoffs reduce the standard deviation σ. To further understand the power-law behavior of the auto-correlation function, we also investigate the average frequency of the strategies cited continuously in a time t for each agent. We find that the average frequency obeys a power-law behavior with respect to the time t. This indicates that the strategies still have a non-negligible probability of being cited continuously in a long period of time. It seems that the agents are more cautious of their choices of the strategies after the score-dependent payoffs are introduced. This leads to the long-range volatility correlations [30].

In real markets, the cumulative probability distribution function (CDF) of the returns is known to have fat tails like

$$P(|A|) \sim |A|^{-\mu} \tag{6}$$

with an exponent $\mu \sim 3.0$ on average [29]. In this paper, the CDF of our model is also carefully investigated. Since the CDF for positive and negative returns are symmetric, we investigate the CDF of the magnitude of the returns.

The CDF also shows a crossover behavior similar to that of the auto-correlation function. To compare with the results of the auto-correlation function, the CDF for different values of the parameter a at a fixed $b = 10.0$ are plotted on a log–log scale in Fig. 2. For $a = 0.0$, the CDF decays rapidly to zero displaying a Gaussian-like shape. As the parameter a increases, a crossover behavior occurs. For a big value of a, a power-law tail of the CDF is observed. The exponent μ equals 4.0 for $a = 0.1$ and remains almost the same for $a > 0.1$.

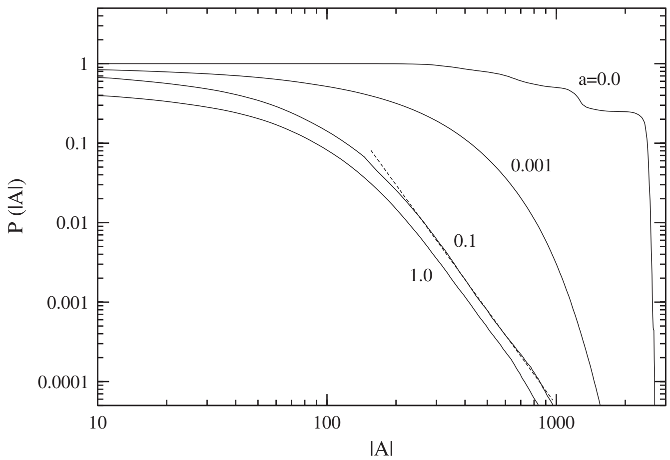

Fig. 2. CDF of the magnitude of the returns for different values of the parameter a: (a) $a = 0.0$, (b) $a = 0.001, 0.1, 1.0$, for the standard MG with score-dependent payoffs at $b = 10.0, N = 5001, S = 2, m = 2$. The dashed line is with a slope of -4.0.

In general, the main features of our model are dominated by the parameter a, which is a measure of the intensity of the rewards and punishment. The larger a is, the more the high-score strategies are rewarded (punished). The parameter b is a linear coefficient, and it does not affect the qualitative results. For $a = 0.0$, the strategies which give the same prediction but with different scores are ended up with the same payoffs and the model behaves essentially the same as the standard MG. As a increase from 0.0 to a big value, the crossover behavior is observed for both auto-correlation function and CDF. The critical value of the parameter a, above which the dynamic system shows power-law behaviors, decreases with the increase of the parameter b. Finally, we find that for $b = 10.0$ and $a = 0.1$, our model reproduces the main stylized facts of the real markets. As it is shown in Figs. 1 and 2, the auto-correlation function and CDF show a power-law behavior, and the exponents are estimated to be $\lambda = 0.25$ and $\mu = 4.0$, close to those of the real markets.

3. Grand canonical and thermal MGs

3.1. Grand canonical MG with score-dependent payoffs

The grand canonical MG, in which an inactive strategy is included, has been proposed to mimic the real markets. In fact, the grand canonical MG reproduces most stylized facts [8,14–19], except for certain dynamic properties, e.g., the time correlation non-local in time, etc. [30]. In the preceding section, we assume that the score-dependent payoff is an essential ingredient of the MGs, which also leads to the long-range temporal correlations. One expects that the score-dependent payoffs could coexist with the inactive strategy in the MGs and the main results would remain. Therefore, we introduce the score-dependent payoffs to the grand canonical MG [8].

The model consists of two types of agents: speculators and producers. Speculators are assigned a number $S + 1$ of strategies, among which S are active strategies the same as those of the standard MG and the other is an inactive strategy with $\sigma_{i,0}^{\mu(t)} = 0$. Each speculator keeps track of the cumulative performance of his/her strategies, and adopts the strategy with the highest score. The score of the active strategies takes the same updated form as Eqs. (1) and (2). In the grand canonical MG, the score of the inactive strategy is usually set to be zero, since it does not predict any action. Such a scheme is too simple, especially after the score-dependent payoffs are introduced. We need to rewards or punish every strategy reasonably.

Therefore, we assume that the score of the inactive strategy could be evaluated according to the performance of the highest score active strategy. Suppose a speculator's highest score strategy is the

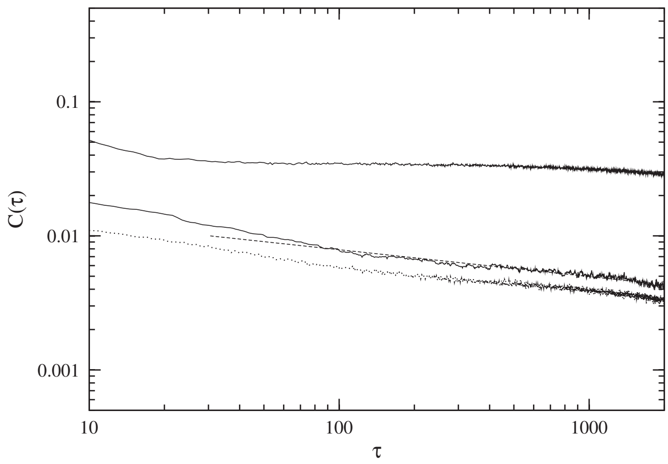

Fig. 3. Auto-correlation function of the magnitude of the returns for the grand canonical MG with score-dependent payoffs for $N_s = 500$ and $N_p = 1001$. Solid lines are for the models with the inactive strategy evaluated by the active strategy with the highest score: the upper solid line is for $b = 0.01, a = 0.01, S = 2, m = 2$, and the lower solid line is for $b = 0.01, a = 0.01, S = 3, m = 6$. The dashed line is with a slope of -0.2. The dotted line is for the model with the inactive strategy which has a score close to 0 at $b = 10.0, a = 0.1, S = 4, m = 7$.

inactive strategy. If the second-highest score strategy, namely the highest score active strategy, makes a prediction which is consequently proved to be in the minority group, the score of the inactive strategy should be deducted as

$$g_{i,0}(t) = -|A(t)|b\frac{\exp(aU_{i,0})}{\sum_{s=1}^{S+1}\exp(aU_{i,s})}, \quad (7)$$

because it would be profitable if he/she would have adopted this active strategy. If the highest score active strategy makes a prediction which is consequently proved to be in the majority group, the score of the inactive strategy should be increased as

$$g_{i,0}(t) = +|A(t)|b\frac{\exp(aU_{i,0})}{\sum_{s=1}^{S+1}\exp(aU_{i,s})}, \quad (8)$$

because it avoids the loss if the active strategy would have been adopted. In the case that the highest score strategy is not the inactive strategy, the situation is similar. The new scoring mechanism for the inactive strategy looks quite reasonable and does not lead to any oddity.

The other type of agents are producers. They only have one active strategy randomly picked at the beginning of the game. Let N_s and N_p be the number of speculators and producers, both of them contribute to the outcome of the game, thus the excess demand is then defined as $A(t) = \sum_{i=1}^{N_s+N_p} a_i(t)$. The history is drawn randomly and independently from the possible 2^m states following Ref. [8], and the price dynamic of the return is defined the same as Eq. (3).

Similar to the case in the preceding section, we report the results of the grand canonical MG with score-dependent payoffs for relative small m, S and relative big N_s and N_p. In most cases the payoffs of the active strategies are negative, so a and b should be small in case the inactive strategy dominates the speculators' behavior. Fig. 3 shows that the long-range temporal correlations of the magnitude of the returns also occur in the present model, and the exponent λ depends quantitatively on the values of the parameters. For $b = 0.01$, $a = 0.01$, $N_s = 500$, $N_p = 1001$, $S = 3$ and $m = 6$, the slope of the curve in a time interval [100, 2000] is measured to be -0.2, close to that of the real markets. The slope of the curve for $S = 2$ and $m = 2$ is bigger than that of the standard MG with score-dependent payoffs for the same values of the parameters. The increase of the exponent λ may be due to the double effects from both the inactive strategy and the score-dependent payoffs.

Similar as the S active strategies for each speculator, the scores of the inactive strategy also vary for different speculators. This sounds also reasonable, since some people are more conservative, and others are less. However, one may ask whether the long-range temporal correlations remain if the scoring of the inactive strategy is kept the same for various speculators. For example, we make the simplest possible assumption that the score of the inactive strategy is set to be close to 0, updated according to $U_{i,0}(t+1) = U_{i,0}(t) + \varepsilon$, with ε being a random variable corresponding to the random noise of the social background. This implies that a speculator is willing to use an active strategy only if it is able to give a positive gain after taking into account the random noise. The auto-correlation function in this case is plotted by the dotted line in Fig. 3, and the curve obeys a power-law behavior with an exponent very close to that of the preceding model presented in this subsection.

3.2. Thermal MG with score-dependent payoffs

To further study the robust feature of the score-dependent payoffs, we introduce it to another extension of the MG, the thermal MG [20]. The model is defined in the following way. Instead of adopting the strategy with the highest score, each agent is allowed a certain degree of stochasticity in the choice of the strategy at any time step. For each agent i the probability of employing his/her strategy $s = 1, \ldots, 2^{2^m}$ is given by,

$$P_i^S(t) \equiv \frac{\exp(U_{i,s}(t)/T)}{Z_i}, \quad Z_i \equiv \sum_{s=1}^{2^{2^m}} \exp(U_{i,s}(t)/T), \tag{9}$$

where $U_{i,s}(t)$ evolves with Eqs. (1) and (2), and the parameters are fixed to be $b = 10.0$ and $a = 0.1$. $1/T$ is a measure of the power of resolution of the agents [20]: when $T = 0$ they are perfectly able to distinguish which is the best strategy, while for increasing T they are more and more confused, when $T \to \infty$ they choose their strategy completely at random. Here, we use the real history and define the price dynamic

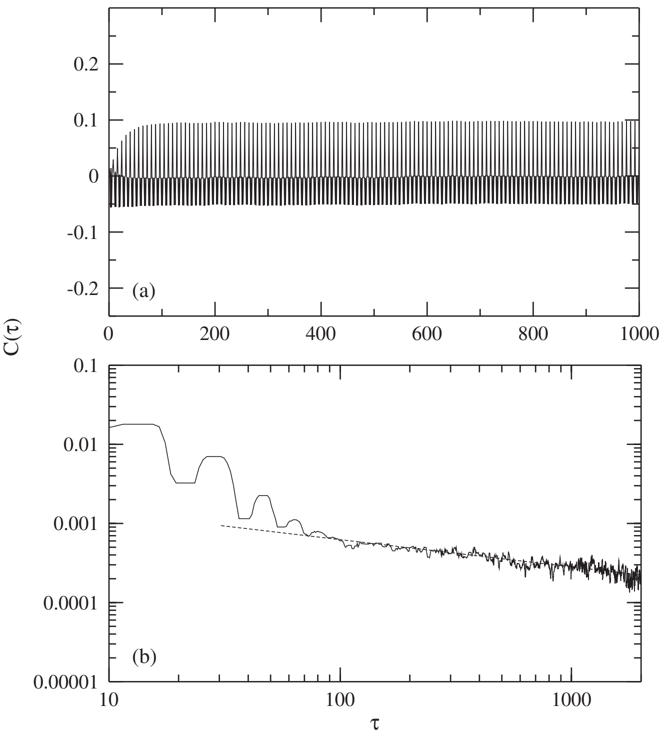

Fig. 4. Auto-correlation function of the magnitude of the returns for the thermal model at $T = 110, N = 501, m = 2$, (a) with original payoffs and (b) with score-dependent payoffs. The solid-dash line is with a slop of -0.34.

the same as in Eq. (3). The present model differs from the original thermal MG just by scoring the strategies. Compared with the standard MG with score-dependent payoffs, each agent takes the strategy from the full strategy space.

We fix the parameters $N = 501$ and $m = 2$, and consider the consequence of having introduced the temperature for the auto-correlation function. For $T = 0$ we do observe that the model behaves very similarly to that of the standard MG, while for $T \to \infty$ we observe a random case in which the agents choose their strategy completely at random. For a intermediate value of T an interesting phenomenon occurs: in Fig. 4(b) we observe that the auto-correlation function obeys a power law, though the curve still has a weak periodic behavior at the early stage of the time and it may be due to the use of the full-strategy space for each agent. The slope of the curve in a time interval [70, 2000] is estimated to be −0.34, very close to that of the real markets.

We assume that the long-range temporal correlations of the present model arise from the score-dependent payoffs. In order to testify it, we revert the payoffs of the present model to the original form $g_{i,s}(t) = -\sigma_{i,s}^{\mu(t)} A(t)$. As it is shown in Fig. 4(a), the auto-correlation function of the model with the original payoffs is periodically fluctuated. In general, the score-dependent payoffs can be introduced to different extensions of the MG, and result in the long-range temporal correlations for all the cases. This is a robust feature of the score-dependent payoffs.

4. Conclusions

In summary, we introduce the score-dependent payoffs to the standard MG and various its extensions. For the standard MG with score-dependent payoffs, we testify that the score-dependent payoffs could be an alternative way of reproducing the stylized facts of the real markets, for example, the long-range volatility correlations and "fat-tails" of the probability distribution of the returns, etc. It is also shown that the score-dependent payoffs could coexist with the inactive strategy in the grand canonical MG. Finally, the score-dependent payoffs are introduced to the thermal MG, and the model consequently tends to be long-range correlated. The results are rather robust, and it indicates that the score-dependent payoffs bring a non-trivial dynamic mechanism to the MGs.

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