Two-phase phenomena, minority games, and herding models

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The recently discovered two-phase phenomenon in financial markets [Nature **421**, 130 (2003)] is examined with the German financial index DAX, minority games, and dynamic herding models. It is observed that the two-phase phenomenon is an important characteristic of financial dynamics, independent of volatility clustering. An interacting herding model correctly produces the two-phase phenomenon.

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I. INTRODUCTION

In recent years, much attention of physicists has been drawn to dynamics of financial markets [1–14]. From the view of many-body systems, interactions among agents and producers may generate long-range temporal correlations in financial dynamics, and therefore result in the so-called dynamic scaling behavior. To clearly observe the scaling behavior, one needs to carefully investigate the dynamics at the "microscopic" level.

By analyzing the time series $y(t')$ of a financial index in *minutes or seconds*, Mantegna and Stanley discovered the scaling behavior of the probability distribution $P(Z,t)$ of the return $Z=y(t'+t)-y(t')$ in a *shorter* time t [1]. $P(Z,t)$ exhibits a characteristic fat tail in regime of larger returns. The exponent of the power-law tail is close to 4 [15,16]. Another feature of $P(Z,t)$ is the power law decay of $P(Z=0,t)$ [1]. It is interesting that in contrast to *absence* of the two-point correlation of the returns, the magnitude of the returns is long-range correlated [14,16]. This phenomenon is called volatility clustering, and considered to be the physical origin of the temporal scaling behavior in financial dynamics.

Very recently, by examining the fluctuation of the volume imbalance, Plerou, Gopikrishnan, and Stanley discovered that a *two-phase* behavior exists in financial markets [17]. Introducing a parameter r describing the fluctuation in a time t, the conditional probability distribution $P(\Omega,t,r)$ of the volume imbalance Ω is found to be with a single peak for a small value of r and double peaks for a big value of r. At a critical value r_c, the transition from a single peak to double peaks occurs. This phenomenon is similar to the order-disorder phase transition in traditional physics.

On the other hand, different models and theoretical approaches have been developed to describe financial markets [9,14,18–27]. Among them, important examples are minority games and percolation models and their variants.

The static percolation model successfully reproduces the fat-tail distribution of the returns and some other stylized facts of the financial index, but clusters of agents are not generated intrinsically in dynamics. Recently, a dynamic version of the static percolation model, the so-called EZ herding model, has been proposed [22]. Compared with the static percolation model, clusters are naturally formed in the dynamic process. If a *linear* relation is assumed between the volatility $|Z|$ and the size of the acting cluster, the power-law tail of $P(Z,t)$ of a shorter time t is observed at least in a certain range of Z. The exponent is approximately 1.5. However, the magnitude of the returns in the EZ herding model is *not* long-range correlated.

To achieve the long-range temporal correlation, a dynamic interaction has been introduced [28]. The interacting EZ herding model nicely explains stylized facts of financial markets, at least qualitatively.

The naive minority game [18] is relatively far from real financial markets. An essential improvement is introducing an inactive strategy for all agents [24,29]. Then the size of a cluster fluctuates and it induces the long-range temporal correlation.

The purpose of this paper is to examine the two-phase phenomenon with the financial indices, taking the German DAX as an example, and more importantly, to investigate the relevant behavior in minority games and herding models. Special attention is put on the possible relation between volatility clustering and the two-phase phenomenon.

In the next section, the two-phase phenomenon will be analyzed with the German DAX. In Sec. III, relevant behavior will be investigated in minority games. In Secs. IV and V, two-phase phenomena will be revealed in the EZ herding model and the interacting EZ herding model. Finally, Sec. VI contains the conclusions.

II. TWO-PHASE PHENOMENON

Phase transitions are important phenomena in traditional physics. For a order-disorder phase transition in magnetic systems, for example, the probability distribution of the order parameter for a finite system shows a single peak in the disordered phase, and double peaks in the ordered phase. The disordered and ordered states exhibit essentially different behaviors.

Up to date, financial markets are usually considered to be in a single stationary state. Only very recently has some effort been made to understand whether multiple states exist in financial markets [17]. In Ref. [17], data of the volume imbalance are used for demonstrating the two-phase phenomenon. In another recent article, it is observed that a similar behavior also exists in financial indices [28]. The present paper extends the finding in Ref. [28].

Let us denote the time series of a financial index as $y(t')$. Following the idea in Ref. [17], we introduce the fluctuation $r(t)$ of $y(t')$ within a time interval t

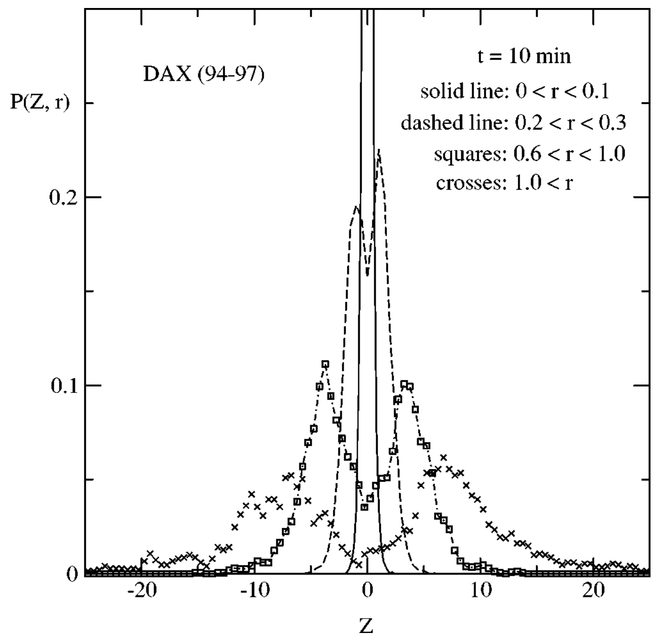

FIG. 1. $P(Z,r)$ of the German DAX from 1994 to 1997 is displayed for $t=10$ min and different values of r.

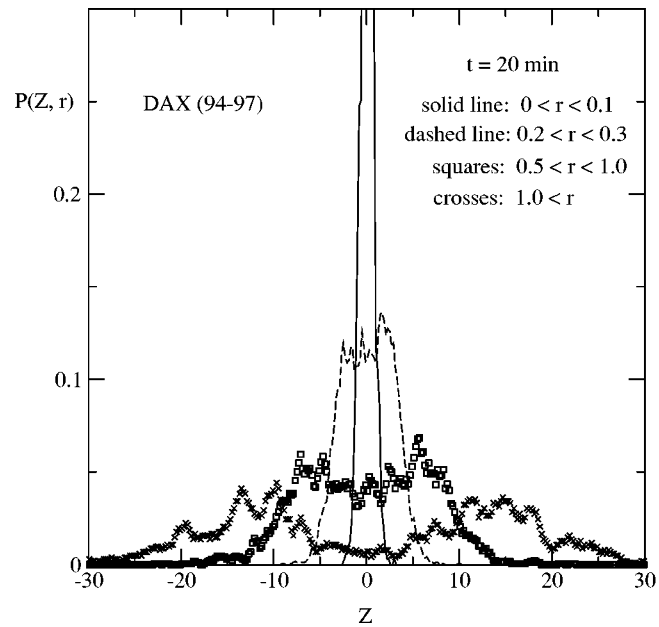

FIG. 2. $P(Z,r)$ of the German DAX from 1994 to 1997 is displayed for $t=20$ min and different values of r.

$$r(t)=\langle|y(t''+1)-y(t'')-\langle y(t''+1)-y(t'')\rangle_t|\rangle_t. \quad (1)$$

Here $\langle\cdots\rangle_t$ is the average over t'' in the time interval $[t',t'+t]$. For a fixed t, we examine the conditional probability distribution $P(Z,r)\equiv P(Z,t,r)$ of the return $Z(t)=y(t+t')-y(t')$ with a specified r. It is observed that the distribution shows a single peak for a small r, while double peaks for a big r. Therefore, there should be a critical value r_c in between. This is similar to a phase transition in traditional physics.

In Fig. 1, $P(Z,r)$ of the German DAX from 1994 to 1997 is displayed for $t=10$ min. A single peak for $r<0.1$ and double peaks for $r>0.2$ are clearly seen. The shapes of the curves look very similar to those in Ref. [17]. The critical value is estimated to be $r_c\approx 0.15$.

According to Ref. [17], the two-phase phenomenon can be observed up to a time t of half a day (four hours), for the correlating time of financial dynamics is very long. Since the total time length of our data is not very long, the curves become quite fluctuating for t longer than half an hour. In Fig. 2, $P(Z,r)$ is plotted for $t=20$ min. The curves are similar to those in Fig. 1 and indicate $r_c\approx 0.3$. Therefore, the critical value r_c is roughly proportional to t. By definition of $r(t)$, this result should be reasonable.

The physical implication of the two-phase phenomenon is simple and clear. When the time series $y(t')$ is stable, the return $Z(t)$ stays mostly around zero. When the time series $y(t')$ is fluctuating, $Z(t)$ jumps between two peaks. Even though this behavior seems natural, it cannot be produced in some popular models. We are especially interested in whether the two-phase phenomenon has something to do with volatility clustering.

Two-phase phenomenon should be important both theoretically and practically. It indicates that there may be two possible states in financial markets, a more stable state and a more fluctuating state. For example, one may benefit or lose more from trading in the state with double peaks than in the state with a single peak.

III. MINORITY GAMES

The minority game was first introduced to econophysics by Challet and Zhang *et al.* in 1997 [18]. The model looks simple and seems to capture some features of real financial markets. However, it does not offer volatility clustering. To achieve the long-range temporal correlation of the magnitude of the returns, an essential improvement is to introduce an inactive strategy for all agents [24,29]. Then, the size of a cluster fluctuates during dynamic evolution, which results in the long-range temporal correlation.

A representative minority game consists of N_a agents and N_p producers.

(1) At a time t', the agents look at the history of the financial index $y(t')$ up to m time steps before, pay attention to the sign of $y(t''+1)-y(t'')$, $t''=t'-1,\ldots,t'-m$.

(2) Characterized by the sign of $y(t''+1)-y(t'')$, there are 2^m possible states of history. A standard strategy is a rule that one decides to buy or sell for every state of history. Therefore, there are 2^{2^m} standard strategies in total. A special strategy is the ‘‘inactive’’ strategy, with which an agent remains inactive for any states of history.

(3) At the beginning of the game, each agent definitely takes the inactive strategy, randomly chooses s standard strategies, and keeps all these strategies forever. During dynamic evolution, each strategy will be given a dynamic score. The agent decides to buy, sell, or remain inactive following the strategy with the highest score.

(4) A producer is considered as an agent with only one standard strategy and without the inactive strategy.

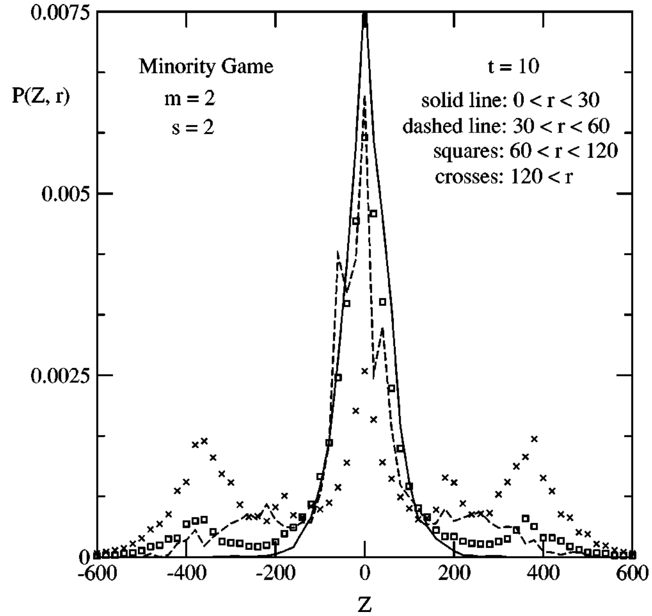

FIG. 3. $P(Z,r)$ of the minority game with $N_a=501$ and $N_p=1000$ is displayed for $t=10$ and different values of r.

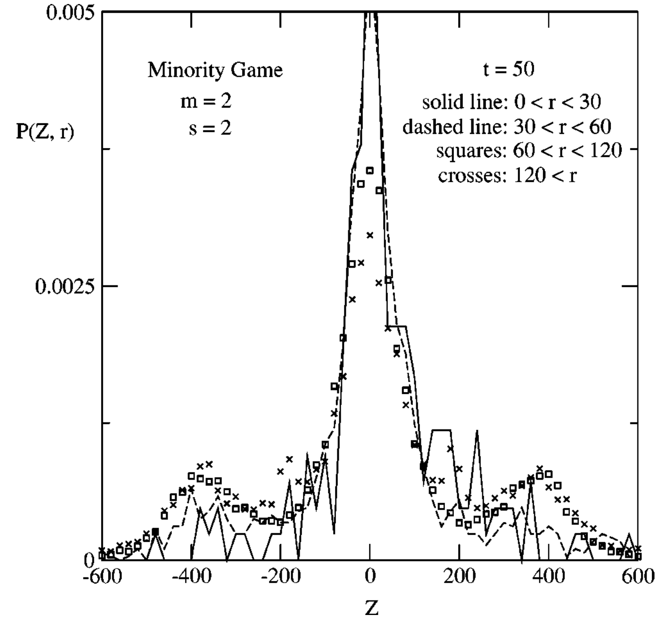

FIG. 4. $P(Z,r)$ of the minority game with $N_a=501$ and $N_p=1000$ is displayed for $t=50$ and different values of r.

(5) If there are N_+ buyers and N_- sellers at a time t', $y(t')-y(t'-1)=N_+-N_-$.

(6) If a standard strategy offers the decision of buying (selling) at a time t', while the financial index decreases (increases), this strategy gains $|y(t')-y(t'-1)|$ points; otherwise it loses $|y(t')-y(t'-1)|$ points. The score of the inactive strategy always remains zero.

With the above dynamic rules, the initial history of $y(t')$, and initial scores for all strategies, a time series $y(t')$ will be generated. The above model can be modified in several ways. For example, a nonzero score can be given to the inactive strategy. This means that the agents would buy or sell only in the case where they would benefit from the trading. Further, *dynamic scoring* for the inactive strategy can also be introduced. All these versions of minority games yield similar results to those we consider in this paper.

In our simulations, we take $N_a=501$, $N_p=1000$, $m=2$, and $s=2$. The average is taken over 64 runs with different initial conditions. The length of each run is 40 000 time steps. The average over initial conditions is important since the dynamics of the minority game is *deterministic*.

In Fig. 3, the conditional probability distribution $P(Z,r)$ has been displayed for $t=10$. For smaller values of r, the curves obviously show a central single peak (solid and dashed lines). As r gets bigger, double peaks indeed appear (squares and crosses). However, the central single peak remains. This is in *disagreement* with real financial markets. For a longer time t, the situation is even worse. The single peak is dominating for any r. This is shown in Fig. 4.

It is known that there is a long-range temporal correlation of the magnitude of the returns in this improved minority game. However, our simulations show that the two-phase phenomenon is absent. The physical origin of such a failure can be traced back to the *periodic* character of minority games. Even though the inactive strategy has eliminated the periodicity from some observables, it seems that periodicity is intrinsic in the minority games we consider in this paper.

IV. THE EZ HERDING MODEL

The EZ herding model, first introduced by Eguiluz and Zimmermann [22], is a dynamic extension of the static percolation models. It consists of N agents, which form clusters during dynamic evolution. Initially, each agent is a cluster. The dynamics evolves in the following way.

(1) At a time step t', an agent i (and thus its cluster) is selected at random.

(2) With probability a, i becomes active and indicates whether to buy or sell. Then all agents in the cluster follow. After that, this cluster is broken into a state that each agent is a separate cluster. The size of this cluster is recorded as $s(t')$.

(3) With probability $1-a$, i remains inactive. Another agent j is then selected randomly. If i and j are in different clusters, combine the two clusters into a bigger one.

Here a is a constant, and apparently controls the dynamic evolution. From the view of financial markets, all agents in a cluster share the same information and therefore act in the same way. The step (3) represents transmission of information. Let us consider the time between two actions as the time unit. If a is close to 1, transmission of information is slow. Agents act almost independently. If a is close to 0, transmission of information is fast. Agents tend to form larger clusters and act collectively. In other words, $1/a$ is the rate of transmission of information.

Numerical simulations [22] show that for a "critical" value of a, the probability distribution $P(s)$ obeys a power law, at least in a certain range of s. The exponent is close to 1.5. If a linear relation between s and volatility $|Z|$ is assumed, one can compare it with the value 4 in real markets [15].

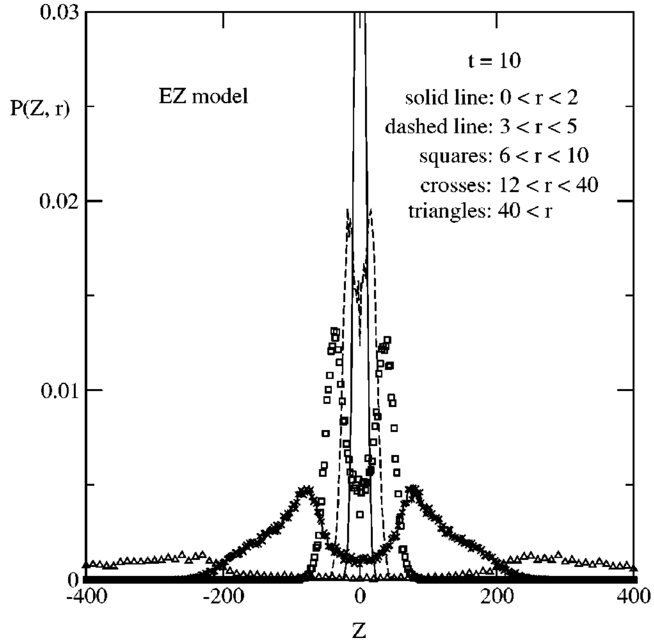

FIG. 5. $P(Z,r)$ of the EZ model with $N=10\,000$ and $a=0.05$ is displayed for $t=10$ and different values of r.

In Fig. 5, $P(Z,r)$ of the EZ herding model has been displayed for $t=10$. The total number of the agents is $N=10\,000$, and the constant a is set to 0.05, close to the possible "critical" value [22]. Interestingly, a double-peak structure is observed, even though there is only a short-range temporal anticorrelation of the magnitude of the returns in this model [28]. However, the shapes of the double peaks look somewhat different from those of the German DAX. Especially, the peak tends to be step-function-like as r becomes big.

Because of *absence* of the long-range correlation of the magnitude of the returns in this naive herding model, $P(Z,r)$ of a longer t behaves rather differently from that of a shorter t. In Fig. 6, $P(Z,r)$ of $t=100$ is plotted. The double peaks become very narrow, and there are no data for a bigger r.

To summarize, the minority game with an inactive strategy offers volatility clustering but does not produce the two-phase phenomenon, while the EZ herding model is just opposite. Therefore, volatility clustering and the two-phase phenomenon are two independent characteristics of financial markets and the relevant dynamic models.

V. THE INTERACTING EZ HERDING MODEL

Similar to the naive minority game, the EZ herding model is also rather far from real financial markets. Most importantly, it lacks volatility clustering. The origin should be the *constant a*. In real markets, the rate of transmission of information should *not* be a constant. When stock markets are very fluctuating, agents are sensitive to the related information, and the public media report it intensively. Therefore, a should be smaller. When the stock markets remain stable, everyone is not so interested in it and a should be bigger.

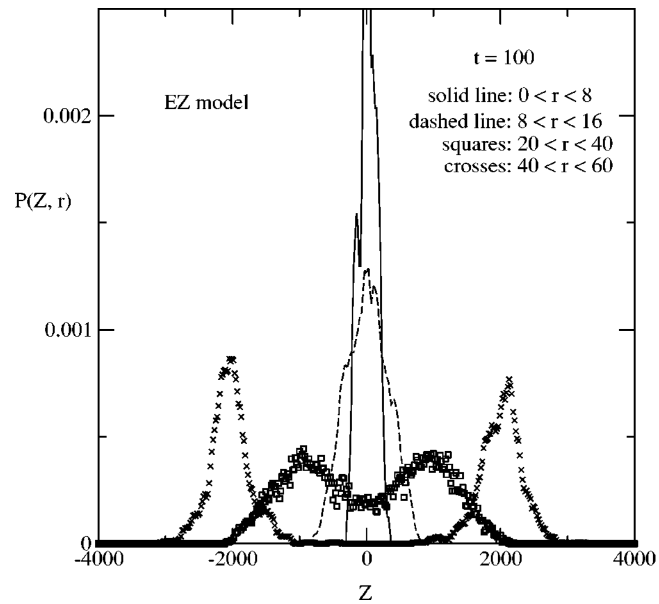

FIG. 6. $P(Z,r)$ of the EZ model with $N=10\,000$ and $a=0.05$ is displayed for $t=100$ and different values of r.

Based on such an observation, it is suggested [28] that a at the time t' should depend on $s(t'-1)$, in the form

$$a=b+cs^{-\delta}. \tag{2}$$

Here δ, b, and c are all positive constants. Such an interaction may generate a long-range temporal correlation of volatility. If a larger cluster acts at $t'-1$, a at time t' is smaller and the agents form larger clusters. As a result, larger clusters would be picked up. If a smaller cluster acts at $t'-1$, a at time t' is bigger and the agents do not form larger clusters. As a result, smaller clusters would be picked up.

$1/b$ represents the highest rate of transmission of information restricted by science and technology in our times. If b is comparable to $cs^{-\delta}$, the system is not so different than the EZ herding model. Therefore, we are mainly interested in a small b.

For any fixed b and δ, one can achieve a power law behavior for the tail of $P(s)$ by adjusting the parameter c

$$P(s)\sim s^{-\alpha}. \tag{3}$$

For simplicity, we consider only the case of $\delta=1.0$. Then the critical value of c is about 0.6 [28]. Keeping in mind the assumption that the volatility $|Z|$ is proportional to the size s of the acting cluster, the interacting EZ herding model reproduces all the stylized facts of financial markets examined in Ref. [28], at least qualitatively. Some quantities are even quantitatively in agreement with those of real financial markets.

For a shorter time t, $P(Z,r)$ of the interacting EZ herding model exhibits almost the same behavior as that of the EZ herding model. This further indicates that the two-phase phenomenon itself is not much related to the long-range correlation of the magnitude of the returns.

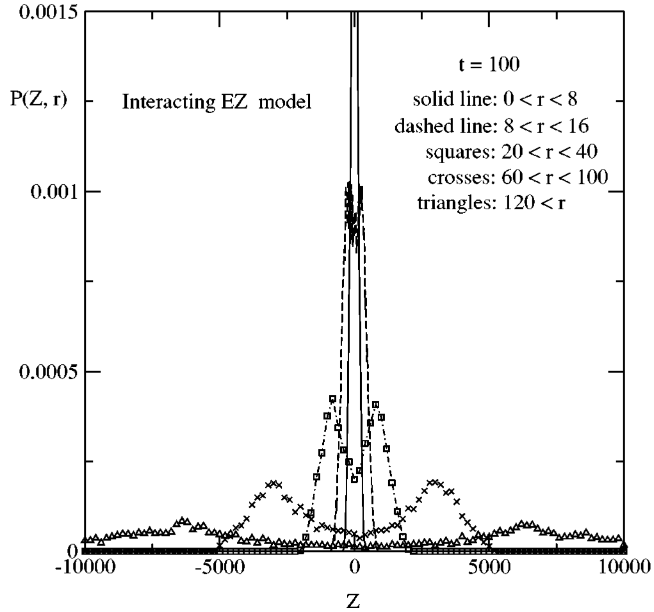

FIG. 7. $P(Z,r)$ of the interacting EZ model with $N=10\,000$, $bN=10$, $\delta=1$ and $c=0.6$ is displayed for $t=100$ and different values of r.

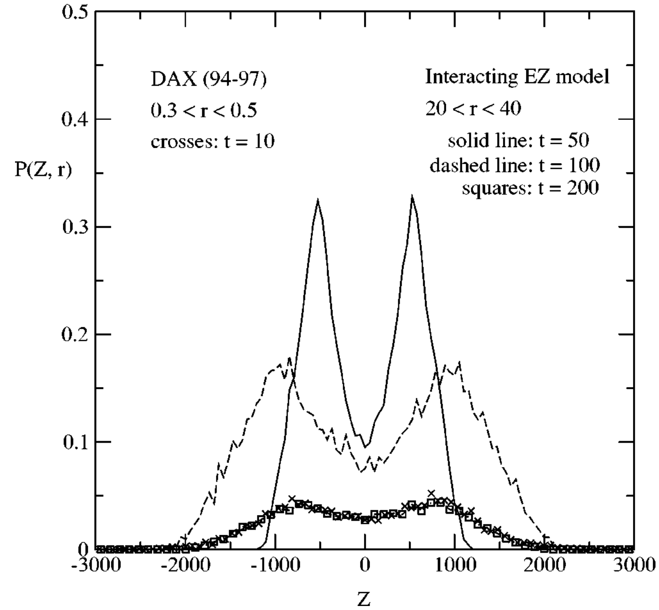

FIG. 8. $P(Z,r)$ of the interacting EZ model with $N=10\,000$, $bN=10$, $\delta=1$ and $c=0.6$ is displayed for $20<r<40$ and different t, and is compared with that of the German DAX for $0.3<r<0.5$ and $t=10$ min (crosses). For comparison, curves have been suitably rescaled by constant factors.

For a longer time t, *without* the long-range correlation, the peaks would shrink to narrow ones, and $P(Z,r)$ would decay to zero rapidly as r increases. This is shown for the EZ herding model in the last section. For the interacting EZ herding model, however, the magnitude of the returns is long-range correlated. The behavior of $P(Z,r)$ is different. In Fig. 7, $P(Z,r)$ of $t=100$ is plotted. The shapes of the double peaks look very similar to those of the German DAX, and different from those of the EZ herding model.

To further understand the behavior of $P(Z,r)$, $P(Z,r)$ with *fixed* $20<r<40$ is displayed for different t in Fig. 8. Apparently, the double peaks tend to be less prominent as t increases. Therefore, r_c increases with t. This is consistent with the discovery in real financial markets in Sec. II. Finally, we compare $P(Z,r)$ of the interacting EZ herding model for $20<r<40$ and $t=200$ (squares) with that of the German DAX for $0.3<r<0.5$ and $t=10$ (crosses) in Fig. 8. The two curves overlap nicely. Therefore, one minute in the German DAX roughly corresponds to 10 or 20 time steps in the interacting EZ herding model.

VI. CONCLUSIONS

We have investigated the two-phase phenomenon in financial markets with the time series of the German DAX from 1994 to 1997. The conditional probability distribution $P(Z,r)\equiv P(Z,t,r)$ shows a single peak for a small value of r and double peaks for a big value of r. The transition value r_c is roughly proportional to t. All these results are consistent with those obtained with the data of volume imbalance in Ref. [17].

Even though the minority games with an inactive strategy offer a long-range temporal correlation of the magnitude of the returns, it does not produce the two-phase phenomenon. In contrast to this, in spite of the absence of the long-range correlation in the EZ herding model, the two-phase phenomenon is observed. Therefore, the long-range correlation and two-phase phenomenon are two independent characteristics of financial dynamics and the relevant dynamic models.

The shape of the double peaks of $P(Z,r)$ for the EZ herding model looks different from that of financial dynamics, especially for a longer t. By adding an interaction to the EZ herding model, the long-range temporal correlation is generated. $P(Z,r)$ with a big value of r keeps the double peaks, and the shape is in good agreement with that of the German DAX.

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