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A generalized dynamic herding model with feed-back interactions

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Abstract

A dynamic feed-back interaction is introduced to the Eguiluz–Zimmermann model (Phys. Rev. Lett. 85 (2000) 5659). In application to financial dynamics, transmission of information at time t' is supposed to depend on the variation of the financial index at $t'-1$. The generated time series is strongly correlated in time at criticality. Both static and dynamic behavior are investigated.

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1. Introduction

In the past years, there has been a growing interest in application of physical concepts and methods to financial markets [1–11]. Based on records of a financial index $y(t')$ in *minutes or seconds*, the fine structure of the dynamics has been carefully analyzed. It is discovered by Mantegna and Stanley [1], that the probability distribution $P(Z,t)$ of the return $Z = y(t'+t) - y(t')$ shows universal scaling

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behavior. A characteristic behavior of $P(Z, t)$ is the fat tail in regime of larger returns in a shorter time t. Another feature is the power-law decay of $P(Z = 0, t)$. Interestingly, in spite of the *absence* of the two-point correlation of the returns, the magnitude of the returns are long-range correlated. This *volatility clustering* is considered to be the physical origin of the temporal scaling behavior.

Different models have been developed for financial markets [7,9,12–20]. An example is the static percolation model [5,13,14,21]. It could reproduce most of the stylized facts of the financial index, but clusters of agents are not generated intrinsically in dynamics. Recently, a dynamic version of the static percolation model, the so-called EZ model, has been proposed [15]. A pronounced feature of this herding model is its *simplicity*. Now, clusters are naturally formed in the dynamic process. The power-law tail of $P(Z, t)$ is observed, and the exponent for the tail is found to be rather robust [22–25]. However, it is not clear if this simple model could produce volatility clustering and the related dynamic behavior, since the model basically does not assume interaction between transmission of information and the variation of the financial index.

In the literature, volatility clustering has been intensively investigated [9,13,17–21,26,27]. Some dynamic systems, such as the Minority Game-based models [17,9] and a generalized Lotka–Voltera formalism [18], can generate a power-law volatility correlation. Some others, such as the Lux–Marchesi model [26,7] and the Ising-spin-based models [19,27], however, create only an exponential-law volatility correlation. It is still highly desired to further understand the mechanism of volatility clustering, and also recently discussed two-phase phenomenon [28,11] as well as dynamic behavior *non-local* in time [29,10,30].

In this paper, we introduce a dynamic feed-back interaction to the EZ model [15]. Through the interaction, volatility clustering is established during the time evolution. In application to financial dynamics, both the relatively known and the newly discussed stylized facts are examined. This dynamic herding model keeps the attractive feature of simplicity and exhibits dynamic behavior close to real markets.

2. The interacting herding model

The EZ model [15] consists of N agents, which form clusters during dynamic evolution. Initially, each agent is a cluster.

(1) At a time t', an agent i (and thus its cluster) is selected at random.
(2) With probability a, i becomes active and decides an action. Then all agents in the cluster follow. After that, this cluster is broken into a state where each agent is a separate cluster. The size of this cluster is recorded as $s(t')$.
(3) With probability $1 - a$, i remains inactive. Then select another agent j randomly, combine the i and j clusters into a bigger one.

Here a is a constant controlling the dynamic evolution. In application to financial dynamics, the action with probability a is buying or selling at random. All agents in a cluster share the same information and therefore act in the same way. Step (3) represents transmission of information. Considering the time between two actions as the time unit, $1/a$ is the rate of transmission of information. If a is close to 0, transmission of information is fast, and agents tend to form larger clusters and act collectively. Numerical simulations [15] show that for a certain value of a, the probability distribution $P(s)$ obeys a power law. The exponent is close to 1.5. Assuming $s \sim |Z|^{\beta}$, one can compare 1.5β with the value 4 in real markets [31].

In real markets, the rate of transmission of information should not be a constant. When stock markets are fluctuating, agents are sensitive to the related information, and the public media report it intensively. Therefore, a should be smaller. When the stock markets remain stable, a should be bigger. Based on such an observation, we suggest that a at the time t' would depend on $s(t'-1)$, in a form like

$$a = b + cs^{-\delta}\,. \tag{1}$$

Here δ, b and c are all positive constants. Such an interaction may be expected to generate volatility clustering. If a larger cluster acts at $t'-1$, a at t' is smaller and the agents form larger clusters. As a result, a larger cluster would be picked up. If a smaller cluster acts at $t'-1$, a smaller cluster would be picked up at t'. The scientific meaning of $1/b$ is the highest rate of transmission of information restricted by science and technology in our times. If b is comparable to $cs^{-\delta}$, the system is not much different from the EZ model. Therefore, we are mainly interested in a small b.

For fixed b and δ, one can achieve a power-law behavior for the tail of $P(s)$

$$P(s) \sim s^{-\alpha} \tag{2}$$

by adjusting the parameter c. This is a kind of critical point. In Fig. 1, the probability distribution $P(s)$ is displayed on a log–log scale. For the number of agents $N = 10000$, 10^8 samples for average are collected, after 10^6 time steps for thermalization. Searching for the best power-law behavior of $P(s)$, one determines the critical value of c. In Fig. 1, it shows that for $\delta = 1.0$, $b = 0.001$ and $c = 0.6$, $P(s)$ decays by a power law at least up to $s = 1000$. The slope of the curve is $\alpha = 1.67(1)$. Similarly, a critical point can be located for any $\delta < 1.0$, and the exponent α varies with δ. Our simulations show that the critical value of c for $\delta > 1.0$ tends to be 1.0. The simulations are extremely difficult.

Considering the finite size effect, we find that the thermal dynamic limit should be taken by keeping bN fixed. The critical value of c decreases as bN increases. $P(s)$ approaches the thermal dynamic limit in a similar way as in standard statistical systems, and does show a kind of critical behavior.

The volatility auto-correlation is defined as

$$A(t) = [\langle s(t')s(t+t')\rangle - (\langle s(t')\rangle)^2]/\sigma. \tag{3}$$

Here $\langle \cdots \rangle$ is the average over t', $\sigma = \langle s(t')^2\rangle - (\langle s(t')\rangle)^2$. In Fig. 2, $-A(t)$ for $\delta = 0.0$, $A(t)$ for $\delta = 0.8$ and 1.0 are plotted in log–log scale. Thinking a while, one will not be

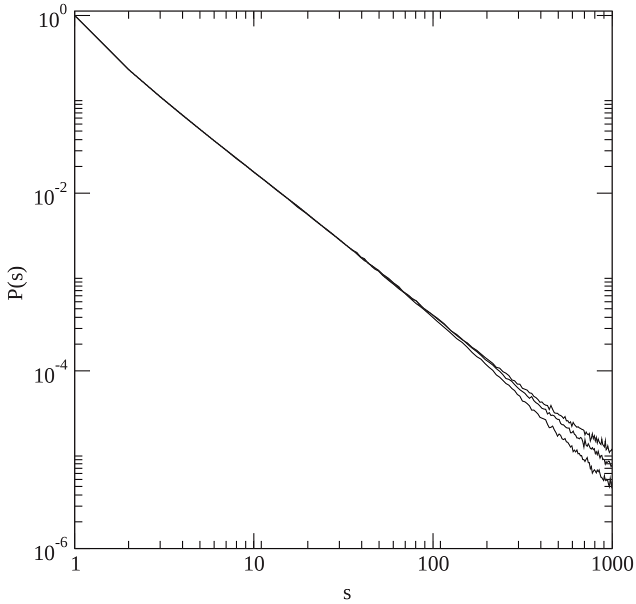

Fig. 1. Probability distribution $P(s)$ displayed in log–log scale. Solid lines are for $\delta = 1.0$, $b = 0.001$ and $c = 0.5$, 0.6 and 0.7, respectively (from above). The number of agents is $N = 10\,000$.

surprised that $s(t')$ for $\delta = 0.0$ is *anti-correlated* in time direction, since $s(t')$ will take small values after a bigger cluster acts at $t' - 1$ and is then dissolved. For $0.0 < \delta < 1.0$, one observes a short-range time correlation. At $\delta = 1.0$, however, $A(t)$ shows a *power-law* behavior, indicating a long-range time correlation. In other words, only $\delta = 1.0$ provides a real critical point, and it describes the feature of financial dynamics. The slope 0.90(10) should be compared with 0.3 to 0.6 in real markets [9].

After introducing the interaction in Eq. (1), there are three parameters bN, c and δ in our model in the thermal dynamic limit. To achieve the power-law tail of $P(|Z|)$ and volatility clustering in financial dynamics, we should set c to its critical value and $\delta = 1$. The dependence of the dynamic behavior on bN is somewhat weak. In principle, other forms of the interaction may also be assumed. However, if a decays slower than a power law as s increases, a long-range temporal correlation cannot be generated. If a decays faster than a power law, simulations are complicated as the case of a power-law interaction with $\delta > 1$.

In Fig. 3, the probability of zero returns $P(Z = 0, t)$ is displayed in log–log scale. One observes a power-law behavior for $\delta = 1.0$ at the critical value of c, and the slope of the curve is 0.86(4), close to 0.71(3) in real markets [1]. Within a certain time interval, $P(Z = 0, t)$ for $\delta = 0.8$ exhibits also a power-law-like behavior. However, one could not find such a behavior for $\delta = 0.0$ since $s(t')$ is anti-correlated.

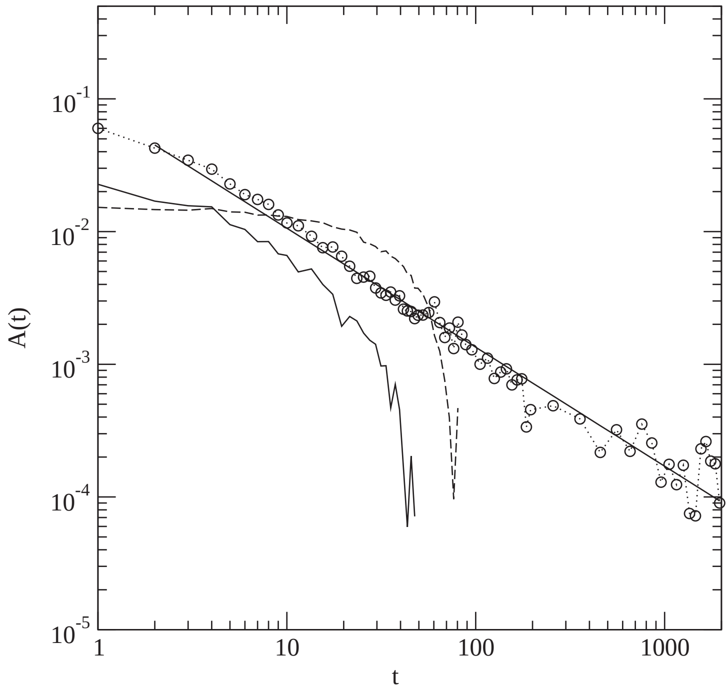

Fig. 2. Volatility auto-correlation in log–log scale. The dashed line is $-A(t)$ for $\delta = 0$, $c = 0$, $b = 0.05$ and $N = 10\,000$, the lower solid line is $A(t)$ for $\delta = 0.8$, $b = 0.0005$, $c = 0.3$ and $N = 20\,000$, and the circled line is $A(t)$ for $\delta = 1.0$, $b = 0.00025$, $c = 0.6$ and $N = 40\,000$. The slope of the solid line fitted to the circles is 0.90(10).

Very recently, by examining the fluctuation of the volume imbalance, Plerou, Gopikrishnan and Stanley found that there exists a two-phase behavior in financial markets [28]. Here we observe that such a phenomenon can be also seen in the financial index. Following Ref. [28], we introduce the fluctuation $r(t)$ of the financial index $y(t')$ within a time interval t

$$r(t) = \langle |y(t''+1) - y(t'') - \langle y(t''+1) - y(t'')\rangle_t|\rangle_t. \tag{4}$$

Here $\langle \cdots \rangle_t$ is the average over t'' in the time interval $[t', t'+t]$. For a fixed t, we examine the conditional distribution $P(Z, r)$, where Z is the return $Z(t) = y(t+t') - y(t')$ with a specified r. The distribution has a single peak for a small r, while double peaks for a big r. In between them is a critical value r_c.

In Fig. 4, $P(Z, r)$ of the German DAX from 1994 to 1997 has been displayed for $t = 10$ minutes. The single peak for $r < 0.1$ and double peaks for $r > 0.5$ are clearly observed. In the same figure, similar curves are plotted with our simulation data for $\delta = 1.0$. The coincidence between the real market and our model is nearly perfect. Here one may wonder whether the two-peak behavior comes from the condition

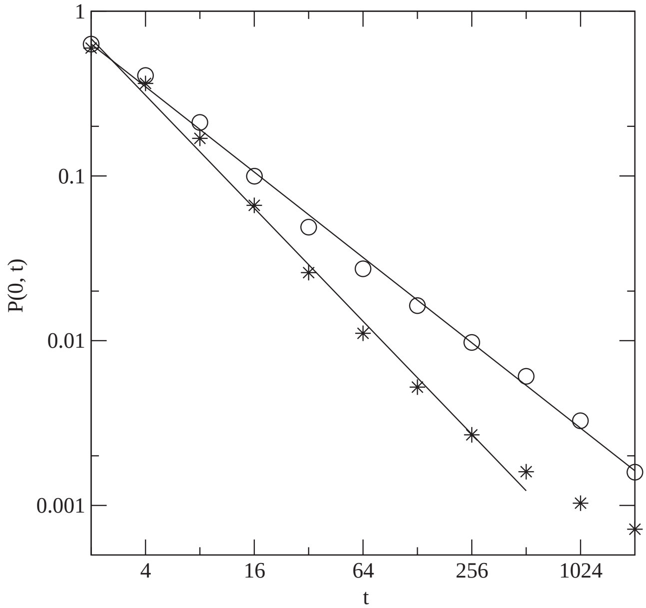

Fig. 3. Probability of the zero returns $P(Z = 0, t)$ in log–log scale. Circles are for $\delta = 1.0$, $b = 0.00025$, $c = 0.6$ and $N = 40\,000$, while stars are for $\delta = 0.8$, $b = 0.0005$, $c = 0.3$ and $N = 20\,000$. Slopes of the solid lines fitted to circles and stars are 0.86(4) and 1.14(6), respectively.

imposed by r, since a bigger r tends to pick up returns with larger fluctuations. Indeed, the naive two-peak behavior itself has nothing to do with volatility clustering. However, if we increase t, e.g., up to half a day for the market data [28], or up to hundreds of Monte Carlo time steps in our simulations, the two-peak behavior would disappear if volatility clustering is absent. This can be confirmed in the EZ model or other time series without long-range correlations. In addition, it is non-trivial to model the two-phase phenomenon. For example, Minority Games with an inactive strategy may generate volatility clustering but fail to produce the two-phase behavior. Typically, the peak around $Z = 0$ remains even for a bigger r. Details of this kind will be reported elsewhere [11].

In recent years, much attention has been drawn to dynamic behavior *non-local* in time in non-equilibrium physics. An example is the persistence probability distribution [10,29,32]. Starting from a time t' and $s(t')$, the persistence probability $P_+(t)$ ($P_-(t)$) is defined as the probability that $s(t' + \tilde{t})$ has always been above (below) $s(t')$ in time t, i.e., $s(t' + \tilde{t}) > s(t')$ ($s(t' + \tilde{t}) < s(t')$) for all $\tilde{t} < t$. The average is taken over t'. In general, the persistence probability provides additional information to the auto-correlation.

In Fig. 5, the persistence probability distribution is plotted in log–log scale. The first observation is that $P_+(t)$ for $\delta = 1.0$ decays much faster than $P_-(t)$ does. This is consistent with that of financial markets, and should be a reflection of the high–low

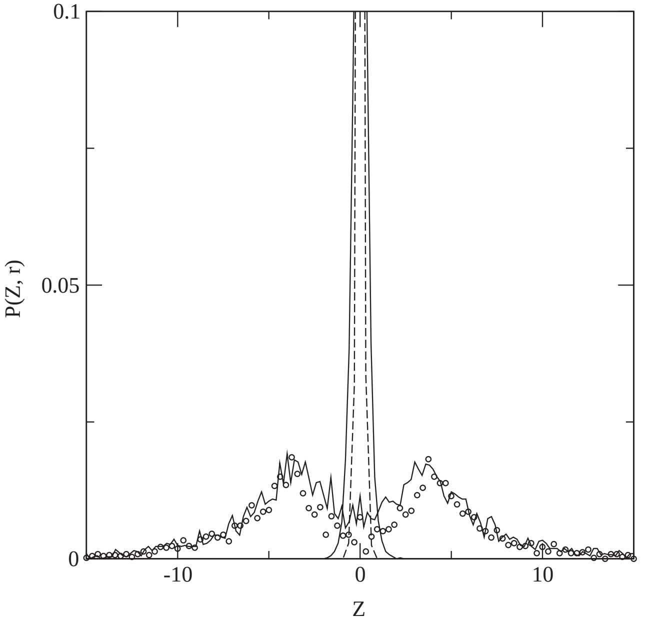

Fig. 4. Solid lines with a single peak and double peaks are $P(Z, r<0.1)$ and $P(Z, r>0.5)$ of the German DAX for $t = 10$ minutes. The dashed line and the circled line are $P(Z, r<24)$ and $P(Z, r>120)$ of our model with $\delta = 1.0$, $b = 0.00025$, $c = 0.6$ and $N = 40\,000$ for $t = 100$. For comparison, Z and $P(Z, r)$ have been rescaled by constant factors.

asymmetry [33]. Secondly, $P_-(t)$ shows a power-law scaling behavior but $P_+(t)$ does not. The persistence exponents measured from the slopes of $P_-(t)$ in Fig. 5 are 0.934(10) and 0.925(20) for our model with $\delta = 1.0$ and the German DAX respectively. For the cases of $\delta = 0.0$ and 0.8 and Minority Games, the high–low asymmetry can also be reproduced, but the persistence exponent is about 1.0, the same as that of a random walk.

3. Concluding remarks

To summarize, we have introduced a feed-back interaction to the EZ model [15]. The rate of transmission of information at t' is assumed to be $1/a = 1/(b + cs^{-\delta})$ with s being the size of the acting cluster at $t' - 1$. By adjusting the parameter c (for fixed δ and bN), one achieves a power-law behavior for the tail of $P(s)$. (i) $\delta = 0.0$, $s(t')$ is short-range anti-correlated. (ii) $0.0 < \delta < 1.0$, $s(t')$ is short-range correlated. (iii) $\delta = 1.0$, $s(t')$ is long-range correlated. The fat tail of $P(s)$, probability of the zero returns $P(Z = 0, t)$, the volatility correlation, and the two-phase phenomenon [28] as well as the persistence probability do describe the stylized facts of real markets. (iv) $\delta > 1.0$, it is complicated and needs further study.

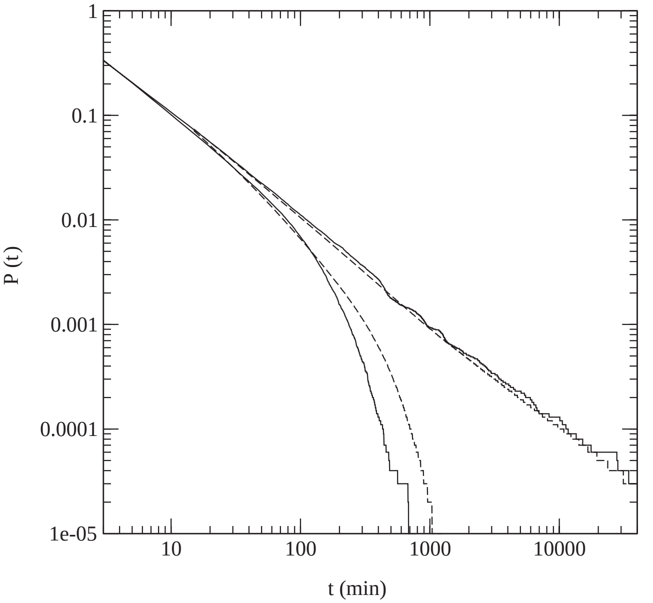

Fig. 5. Persistence probabilities in log–log scale. The lower and upper solid lines are $P_+(t)$ and $P_-(t)$ from the minute-to-minute records of the German DAX respectively, while the lower and upper dashed lines are those of our model with $\delta = 1.0$, $b = 0.00025$, $c = 0.6$ and $N = 40\,000$.

Our generalized EZ model keeps the attractive feature of simplicity but captures important properties of real markets. On the other hand, there are quite rich physical contents in our model. Investigation of other aspects, such as the transitions from $b > 0$ to $b = 0$ and from $\delta = 1.0$ to $\delta > 1.0$, is undergoing.

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