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Generalized persistence probability in a dynamic economic index ☆

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Abstract

A generalized persistence probability distribution is introduced in an economic index, describing dynamic behavior nonlocal in time. It concerns the time series of the magnitude of the variation of the index. Universal scaling behavior is revealed by analyzing the 'experimental' data of the German DAX. Results are compared with those of a random walk and a dynamic herding model.

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Keywords: Nonequilibrium kinetics; Financial dynamics; Critical dynamics

In the past years, it has been piled up large amount of data in financial markets, especially those records of economic indices in *minutes or seconds*. This allows relatively accurate analysis of the fine structure of financial dynamics. In 1995, Mantegna and Stanley had carefully analyzed the data of a stock market index in US—the Standard and Poor 500 [1]. The probability distribution $P(Z,t)$ of the variation Z in a *shorter* time t (denoted as Δt in Ref. [1]) obeys a *truncated* Lévy distribution. Temporal scaling behavior is observed. An exponent α has been introduced to describe the scaling behavior and is estimated to be $\alpha = 1.40(5)$ from the power law decay of $P(0,t)$. In a longer time t, e.g., above one month, $P(Z,t)$ crosses over slowly to a Gaussian distribution.

Stimulated by the work of Mantegna and Stanley [1], many activities have been devoted to the study of financial markets [2–12]. More systematic investigation of the tails of $P(Z,t)$ and the volatility of price fluctuations in the Standard and Poor 500 has been presented [13,14]. In spite of *absence* of the two-point correlation of the variations, the volatility is long-range correlated. This is believed to be the physical origin of the universal scaling behavior in financial dynamics. Different models and theoretical approaches have been proposed to describe financial markets [9,10,15–19]. A class of these models is constructed based on the percolation theory [6,10,16,20].

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Up to date, stationary properties of financial indices and dynamic behavior local in time are mainly concerned. For full understanding of financial markets, however, dynamic behavior *nonlocal* in time should be also very important in theory and practice. An interesting example is the so-called persistence probability, which has been systematically investigated in nonequilibrium dynamics such as phase ordering dynamics and critical dynamics. As time evolves, the persistence probability decays by a power law characterized by a persistence exponent. It is shown that the persistence exponent is in general *independent of* other known critical exponents [21]. The persistence exponent plays an important role in a variety of systems and has been directly measured in experiments [22–26].

In a very recent work [27], the persistence probability has been introduced in financial dynamics. Eliminating properly the effect of the background, power law scaling behavior is observed in the data of the German DAX. The persistence exponent θ_p is estimated to be 0.49(2), very close to 1/2, which is a value for a random walk. It is somewhat puzzling why financial dynamics offers a persistence exponent similar to that of a random walk, even though the probability distribution $P(Z,t)$ of the financial index is significantly different from Gaussian. A possible explanation is: since the variation Z of the index is almost uncorrelated in time, the persistence probability defined with the index $y(t')$ [27] is not so sensitive to the distribution $P(Z,t)$.

The persistence probability is not only important in the fundamental theory of nonequilibrium dynamics, but also helpful in thorough understanding of financial markets. It needs further study. In this Letter, we investigate the persistence probability defined with the *magnitude* of the variation Z of the index, using the data of the German DAX. Since the magnitude of the variation Z is long-range correlated, we expect a persistence exponent different from that of a random walk. Results will be also compared with those of a dynamic herding model.

Let us denote the value of the DAX at a certain time t' as $y(t')$, and the magnitude of the variation in a time interval Δt as $Z_a(t',\Delta t) \equiv |y(t'+\Delta t) - y(t')|$. We define the persistence probability $P_+(t)$ ($P_-(t)$) as the probability that $Z_a(t'+\tilde{t},\Delta t)$ has always been above (below) $Z_a(t',\Delta t)$ in time t, i.e., $Z_a(t'+\tilde{t},\Delta t) > Z_a(t',\Delta t)$ ($Z_a(t'+\tilde{t},\Delta t) < Z_a(t',\Delta t)$) for all $\tilde{t} < t$. The average is taken over the time variable t'.

We first perform the measurements using the minute-to-minute data from December of 1993 to July of 1997. The records in minutes of this period are about 350 000. Following the idea in Ref. [27], the background evolution of a form $y_b(t') = c_1 + c_2 e^{\lambda t'}$ is subtracted from $y(t')$. The background affects the behavior of $P_+(t)$, but very little of $P_-(t)$, for $y_b(t')$ is a slowly increasing function. On the other hand, in the records in minutes, there are about 15 percent of 'zeros', i.e., $Z_a(t', \Delta t = 1\ \text{min}) = 0$. Because of these 'zeros', the behavior of $P_+(t)$ is biased to the background we introduce. This seems nonphysical. Therefore, we always skip these 'zeros' in the measurements in this Letter.

In Fig. 1, it is clearly seen that $P_-(t)$ (upper solid line) obeys a power law in four orders of magnitude,

$$P_-(t) \sim t^{-\theta_p}, \tag{1}$$

with θ_p being the so-called persistence exponent, while $P_+(t)$ (lower solid line) decays much too faster. This is different from the persistence probabilities defined with $y(t')$ in Ref. [27], where both $P_+(t)$ and $P_-(t)$ do show power law behavior after subtraction of the background. By refining the constants c_1, c_2 and λ, or introducing other forms of the background like $y_b(t') = c_1 + c_2 t'^{\lambda}$, we confirm that the behavior of $P_-(t)$ and $P_+(t)$ is rather robust to the background. Therefore, we conclude that $P_+(t)$ *indeed behaves differently from* $P_-(t)$.

Looking at the curve of $P_-(t)$ in Fig. 1, we observe quasi-periodic dropping in the first 2000 minutes. The period is roughly one working day, i.e., about 420 minutes. This behavior should be traced back to the disconnection of the index between two successive days. $Z_a(t', \Delta t = 1\ \text{min})$ at the last minute of a day is much bigger than those during the day. In general, this fact results in somewhat faster decreasing of $P_-(t)$ in the first periods of time, because the 'length' of a working day changes from 330 to 480 minutes in those years. When one measures the slope in a time interval [500, 40 000], the persistence exponent is $\theta_p = 0.925(20)$. The real value of θ_p should be even a little smaller.

To push forward our investigation, we have also performed the measurements with the daily data of the German DAX from 1957 to 1997. In Fig. 2, $P_-(t)$

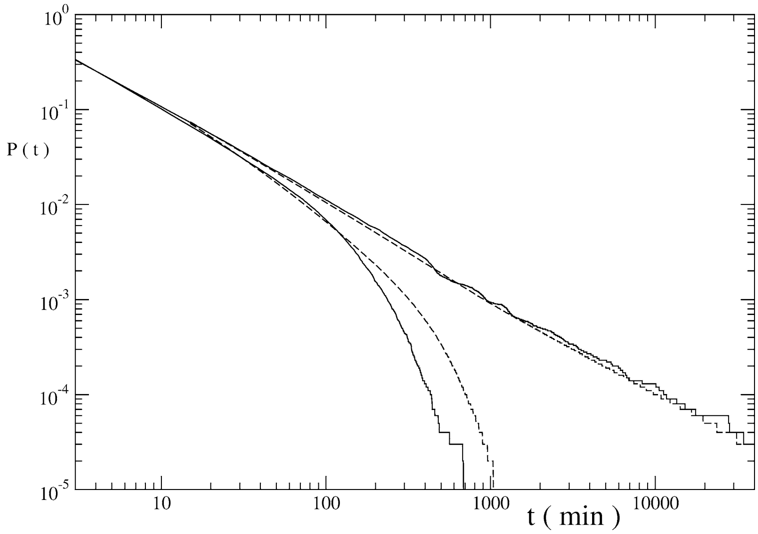

Fig. 1. Persistence probabilities in log–log scale. Lower and upper solid lines are $P_+(t)$ and $P_-(t)$ from the minute-to-minute records of the German DAX respectively, while lower and upper dashed lines are those of the dynamic herding model.

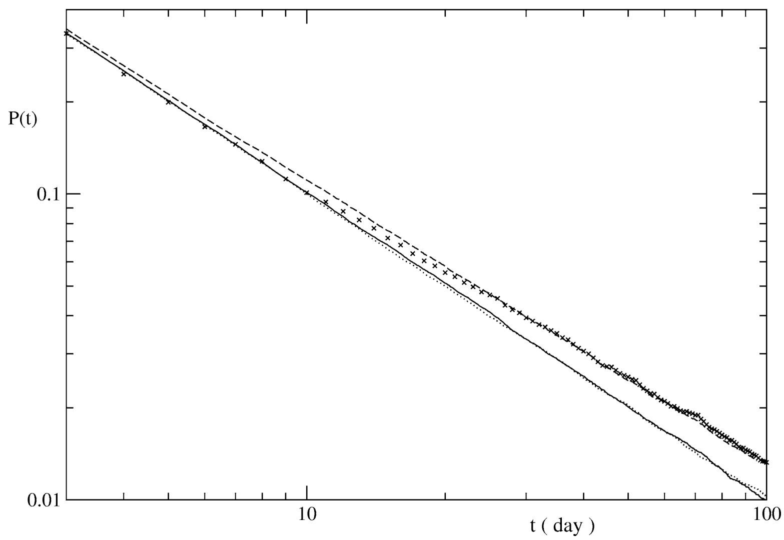

Fig. 2. Persistence probabilities in log–log scale. The crossed line is $P_-(t)$ from the daily data of the German DAX, and the dashed line is that of the dynamic herding model. Dotted and solid lines are $P_+(t)$ and $P_-(t)$ of a random walk.

(crossed line) is plotted on a log–log scale. After 100 day (comparable with 40 000 minutes in Fig. 1), the data fluctuate much. We observe that at least up to 100 day, one observes power law behavior. The persistence exponent is $\theta_p = 0.90(2)$, in agreement with 0.925(20) estimated from the minute-to-minute data. Since the measurements with the daily data do not suffer from the disconnection of the index between two successive days, the value $\theta_p = 0.90(2)$ should be closer to the real value of θ_p.

For comparison, we perform simulations for a dynamic herding model. Starting from the herding model of Eguiluz and Zimmermann [10], we introduce a dynamic interaction: the rate $1/a$ of transmission of information at time t' depends on the size of the acting cluster s at time $t' - 1$ in a form like $a =$

$b + cs^{-1}$. Volatility in this dynamic herding model is long-range correlated [28]. In Fig. 1, $P_+(t)$ and $P_-(t)$ (lower and upper dashed lines) of this model are compared with those of the German DAX. Apparently, the model reproduces rather nicely the behavior of the real market, for both $P_+(t)$ and $P_-(t)$.

In Fig. 2, $P_+(t)$ and $P_-(t)$ (dotted and solid lines) of a random walk are also plotted, to be compared with $P_-(t)$ of the German DAX and the dynamic herding model (crossed and dashed lines). The random walk is clearly different from financial dynamics and the dynamic herding model. For the random walk, both $P_-(t)$ and $P_+(t)$ show power law behavior with a same exponent. The persistence exponent for the random walk is $\theta_p = 1.0$, while that measured from $P_-(t)$ is $\theta_p = 0.90(2)$ and 0.934(10) for the German DAX and the dynamic herding model, respectively.

In conclusions, we have investigated the persistence probabilities $P_\pm(t)$ defined with the magnitude of the variation of the index, using data of the German DAX from 1959 to 1997. Power law scaling behavior for $P_-(t)$ is found up to some months. It is rather robust. The persistence exponent is estimated to be $\theta_p = 0.90(2)$. $P_+(t)$ decreases faster and does not show universal scaling behavior. This is very different from the persistence probability defined with the index $y(t')$. Persistence probabilities of financial dynamics behave rather differently from those of a random walk, where both $P_+(t)$ and $P_-(t)$ show a power law behavior with a persistence exponent $\theta_p = 1.0$. A dynamic herding model with long-range volatility correlation can simulate the behavior of $P_\pm(t)$ of financial dynamics. The persistence exponent from $P_-(t)$ is $\theta_p = 0.934(10)$.

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