Comment on "Universal Fluctuations in Correlated Systems"

In [1], it is suggested that the probability distribution functions (PDF's) of fluctuating observables for critical systems in different universality classes exhibit the same form. This Comment concerns the PDF's for equilibrium systems.

In [2], it is pointed out that the standard scaling form is a sufficient condition for the data collapse. The suggestion in [1] seems to contradict the idea of standard universality. To clarify it, we have performed calculations for different critical systems. First, the temperature is fixed at the bulk critical value T_c. In Fig. 1, the normalized PDF $P(m)$ for the Ising and XY models are displayed. Here $m = (M - \langle M \rangle)/\sigma$, where M is the magnetization, $\langle M \rangle$ is its mean, and σ is the standard deviation. Data collapse for different lattice sizes L's is observed. However, the differences between the curves for different models are clearly not minor perturbations. The PDF for the Potts model is even not symmetric in M and can hardly be put in the figure.

The critical temperature can be modified in a finite system. Let us take a size-dependent coupling $K(L) \sim 1/T(L)$ such that $\tau \equiv [K(L) - K_c]/K_c = s/L^{1/\nu}$, where ν is the static exponent and s is a constant. Following similar scaling analysis as in [2] (see also [3]), it leads to $P(m, \tau, L) = P(m, L^{1/\nu}\tau) = P(m, s)$. At the critical regime, data for different L's at $T(L)$'s also collapse onto a single curve, but $P(m, s)$ changes continuously with s. If s is small, in other words, if the spatial correlation length $l(L)$ is much larger than the lattice size L, $T(L)$ can be considered approximately as a size-dependent critical temperature. Choosing $s = 2.90$, $P(m, s)$ for the 2D Ising model falls onto that for the 2D XY model at $T = 0.89$. This is shown in Fig. 1. However, $T(L)$ with such a large value of s should not be defined as a size-dependent critical temperature, since in the infinite limit of L, the behavior of the system [not only $P(m)$] remains very different from that at the bulk T_c. To confirm this, we have calculated the spatial correlation function and found that at $s = 2.90$ the correlation length $l(L)$ is much smaller than the lattice size L.

We do not think the *shape* of $P(m)$ for the XY model is a characteristic property at the critical point. The observation is that for systems with a second order transition, the tail of the PDF for negative m at the T_c reaches a nonzero value at $M = 0$ and is roughly power-law-like (before cut at $M = 0$). This reasonably indicates that the system can transit from the positive sector of M to the negative one. Below T_c, symmetry breaking occurs. The (infinite) system cannot transit from one sector to another. The exponential-like tail at a large but not too large s should be a signal of symmetry breaking. (But when s is infinite, the tail crosses over to Gaussian.) For the XY model, the

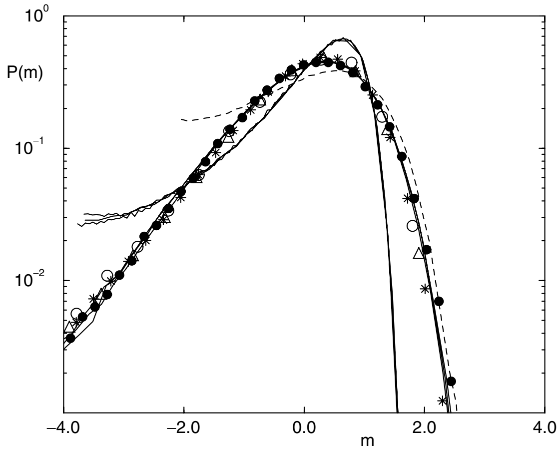

FIG. 1. The three solid lines with a higher peak are for the 2D Ising model at T_c with $L = 32$, 64, and 128. The three solid lines with a lower peak are for the 2D XY model at $T = 0.89$ with $L = 16$, 32, and 64. The dashed curve is for the 3D Ising model at T_c with $L = 32$. The circles, triangles, and stars are for the 2D Ising model at $s = 2.90$ with $L = 32$, 64, and 128. The filled circles are for the 3D Ising model at $s = 2.21$ with $L = 32$.

fluctuations are mainly rotational. The exponential tail of $P(m)$ is only an indication for the energy barrier in small M regime. The conjecture is that the exponential-like tail of the PDF induced typically by energy barriers may be similar to the exponential decay of the correlation functions with a finite correlation length. It can be rather generic and independent of whether the system is with or without a first order, second order, or Kosterlitz-Thouless phase transition. The results for the 1D and 3D XY models in [3] support this statement. A critical point is only a sufficient condition for data collapse for infinite systems.

The complete form of the PDF is, in general, not independent of universality classes. However, the large s regime may be somewhat special and it needs more investigation. For example, as shown in Fig. 1, $P(m, s)$ for the 3D Ising model at $s = 2.21$ fits also to the curve of the XY model at $T = 0.89$.

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