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Corrections to scaling for the two-dimensional dynamic XY model

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With large-scale Monte Carlo simulations, we confirm that for the two-dimensional XY model, there is a logarithmic correction to scaling in the dynamic relaxation starting from a completely disordered state, while only an inverse power law correction in the case of starting from an ordered state. The dynamic exponent z is $z=2.04(1)$.

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In recent years, much attention has been drawn to nonequilibrium short-time behavior of critical dynamics. Traditionally, it is believed that universal dynamic scaling behavior only exists in the long-time regime of the dynamic evolution. In 1989, however, with renormalization group methods Janssen, Schaub, and Schmittmann derived a dynamic scaling form for the $O(N)$ vector model, which is valid up to the *macroscopic* short-time regime [1]. The dynamic process they considered is that the system initially at a very high temperature state with small or zero magnetization, is suddenly quenched to the critical temperature, then released to dynamic evolution of model A. Important is that a new independent critical exponent must be introduced to describe the scaling behavior of the initial magnetization. Afterwards, some evidence for the short-time dynamic scaling was also observed in Monte Carlo simulations [2,3]. On the other hand, it was found that the power law decay of the magnetization starting from a completely ordered state shows up from rather early times, e.g., see [4,5], and it can be used to estimate the dynamic exponent z. Inspired by these works, in the last several years nonequilibrium short-time critical dynamics has been systematically investigated with Monte Carlo methods [6–10]. Simulations have extended from regular classical spin models [9,11–14], to statistical systems with quenched disorder [15–17], quantum spin systems and lattice gauge theories [18–20], dynamic systems without detailed balance [21–23], the hard-disk model [24,25], and fluid systems [26]. References given here are only a part of recent ones and not complete. A relatively complete list of the relevant references before 1998 can be found in Ref. [9]. All numerical and analytical results confirm the existence of a rather general dynamic scaling form in critical dynamic systems at early times.

The short-time dynamic scaling has not been systematically explored in experiments. But the dynamic scaling behavior around a spin-glass transition [2,27–29,16] is very similar to that around a standard critical point. For example, the experimental measurements of the remanent magnetization in spin glasses support not only the power law scaling behavior but also the scaling relations between the exponents [30,16].

The short-time dynamic scaling form not only is conceptually interesting, but also provides new techniques for the measurements of both dynamic and static critical exponents as well as the critical temperature [4,8,28,31]; for a review, see Ref. [9]. Since now the measurements are carried out in the short-time regime, the dynamic approach does not suffer from critical slowing down. Compared with those methods developed in equilibrium, e.g., the nonlocal cluster algorithms, the dynamic approach does study the original local dynamics and can be applied to disorder systems. Furthermore, to solve numerically dynamic equations with a continuous time to the long-time regime is very difficult, but the short-time dynamic approach works well [10]. Such a method should be also very interesting in experiments.

To understand the universal short-time behavior, one should distinguish the macroscopic and microscopic time scales. The dynamic scaling emerges only after a time scale t_{mic} which is sufficiently large in microscopic sense but still very small in macroscopic sense. In Monte Carlo simulations, for example, if a sweep over all spins on a lattice is considered to be a microscopic time unit, t_{mic} is usually from a few to 100 Monte Carlo time steps [9]. Therefore, performing simulations up to some hundred or thousand time steps is usually sufficient to obtain rather good values for critical exponents. However, in the recent study of the two-dimensional XY model (with a Kosterlitz-Thouless phase transition) and the random-bond Ising model [12,32–34], one observes somewhat unexpected phenomena. The dynamic exponent z estimated from a dynamic processes starting from a disordered state is bigger (10 to 15 % for the XY model and 5 to 10 % for the random-bond Ising model) than that from an ordered state. Puzzling is that the resulting static exponents are correct within statistical errors. This behavior should have its origin in the existence of the free vortices or the metastable states. Similar concern for the XY model with different boundary conditions and dynamics can be found also in Ref. [13]. If such a kind of phenomena are not clarified, further applications of the short-time dynamic scaling becomes complicated and difficult.

In a recent paper [12] (see also Ref. [33]), Bray, Briant, and Jervis argue theoretically that there is a logarithmic correction for the two-dimensional XY model in the dynamic process starting from a disordered state. However, the presented numerical data cannot distinguish the two ansatzes, a possible bigger z, or a logarithmic correction. On the other hand, there has been some controversy over the value of the dynamic exponent z (see, e.g., Ref. [13] and references

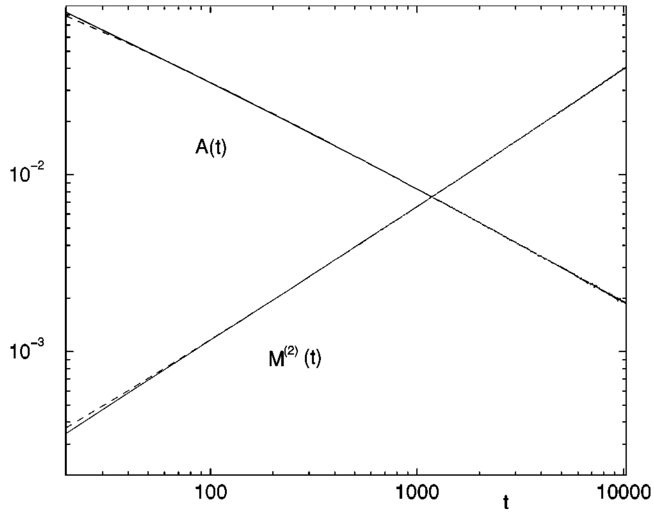

FIG. 1. Time evolution of the second moment and autocorrelation starting from a disordered state in log-log scale. Dashed lines are the fitted curves with a logarithmic correction.

therein), and here it is especially interesting whether z is exactly 2 [12]. In this Rapid Communication, we report our large-scale Monte Carlo simulations for the two-dimensional XY model, examine possible corrections to scaling in dynamic processes starting from both ordered and disordered states, and determine relevant critical exponents accurately.

In the simulations, the system in a macroscopic initial state is suddenly quenched to the transition temperature T_{KT} or slightly below, then released to dynamic evolution of model *A*. In the literature, T_{KT} is reported to be between 0.89 and 0.90. In this paper, we take the temperature $T=0.89$. Following Ref. [12], we adopt the 'heat-bath' algorithm in which a trial move is accepted with probability $1/[1+\exp(\Delta E/T)]$, where ΔE is the energy change associated with the move. All results are presented with a lattice size $L=256$. Simulations with other lattice sizes confirm that the finite size effect for $L=256$ has been completely invisible for our updating times.

Denoting a spin $S_i(t)$, as usual, we define the magnetization, its second moment, and the autocorrelation as $\vec{M}(t) \equiv \langle \Sigma_i \vec{S}_i(t)\rangle/L^2$, $M^{(2)}(t) \equiv \langle [\Sigma_i \vec{S}_i(t)]^2\rangle/L^4$, and $A(t) \equiv \langle \Sigma_i \vec{S}_i(0)\cdot \vec{S}_i(t)\rangle/L^2$, respectively.

In Fig. 1, time evolution of the second moment and the autocorrelation for a *disordered* initial state are displayed with solid lines in log-log scale. In order to detect any corrections to scaling, we have performed the simulations up to $t=10\,240$ Monte Carlo time steps. Samples of the initial configurations (also random numbers) for averaging are 20 000. To estimate the errors, samples are divided into four subsamples. Assuming that there is a logarithmic correction for the nonequilibrium spatial correlation length, according to general scaling analysis, the second moment should behave like [12]

$$M^{(2)}(t) = b_2[t/(1+c_2\ \ln(t))]^{(2-\eta)/z}. \tag{1}$$

Here η is the usual static exponent, z is the dynamic exponent, and b_2 and c_2 are constants. Similarly, the autocorrelation

$$A(t) = b_a[t/(1+c_a\ \ln(t))]^{\theta - d/z}. \tag{2}$$

Here $d=2$ is the spatial dimension. If c_2 and c_a are zeros, the standard power law scaling behavior is recovered. Looking at Fig. 1, $A(t)$ bends obviously downwards, consistent with the logarithmic correction. However, the behavior of $M^{(2)}(t)$ is somewhat complicated and the correction is also less strong than that for $A(t)$. It bends slightly downwards at early times, and changes to upwards only after about 100 time steps. The first behavior is not universal behavior but microscopic-detail dependent. Anyway, if the simulation is performed only up to $t=2000$ or 3000 [12,32], it would be difficult to conclude whether and how the power law is corrected. Now, we fit the two solid lines in Fig. 1 to the *Ansätze* in Eqs. (1) and (2) in a time interval $[100,10\,240]$. The fitted curves are shown with dashed lines in the figure. The quality of the fitting is good, and the resulting exponents are $(2-\eta)/z = 0.866(3)$ and $d/z-\theta = 0.730(1)$.

Here it is very important to address that if directly measuring the slope, e.g., for $M^{(2)}(t)$ in Fig. 1, *in any time intervals* we obtain $(2-\eta)/z$ around 0.77 to 0.78. These values differ from 0.866(3) by more than 10 %. Do $M^{(2)}(t)$ and $A(t)$ fit *uniquely* to the *Ansätze* in Eqs. (1) and (2)? We have tried inverse power law corrections, e.g., for $M^{(2)}(t)$,

$$M^{(2)}(t) \sim t^{(2-\eta)/z}(1+c/t^b). \tag{3}$$

For both $M^{(2)}(t)$ and $A(t)$, the quality of the fitting is even slightly better than with a logarithmic correction. However, the correction exponent b is small, $b=0.211$ and 0.0474 for $M^{(2)}(t)$ and $A(t)$ respectively, while the exponent $d/z-\theta$ remains the same and $(2-\eta)/z$ differs only by 1 or 2%. This strongly indicates that a logarithmic correction is indeed correct. It is believed that the logarithmic corrections are related to the vortex pair annihilation, and do not disappear within a time scale t_{mic} [12,33].

For a dynamic process starting from an ordered state, i.e., $\vec{M}(0) = (1,0)$, no logarithmic corrections are claimed theoretically, since no free vortices exist. It is interesting to confirm this numerically and obtain independently the dynamic exponent z and the static exponent η for comparison. In this dynamic process, the magnetization $\vec{M}(t) = [M(t),0]$ is subject to the power law scaling behavior [9]

$$M(t) \sim t^{-\eta/2z}. \tag{4}$$

In order to determine the dynamic exponent z independently, we introduce a time-dependent Binder cumulant, $U \equiv M^{(2)}/M^2 - 1$, which behaves like

$$U(t) \sim t^{d/z}. \tag{5}$$

In Figs. 2 and Fig. 3, $M(t)$ and $U(t)$ are displayed with solid lines in log-log scale. Samples (now only respect to random

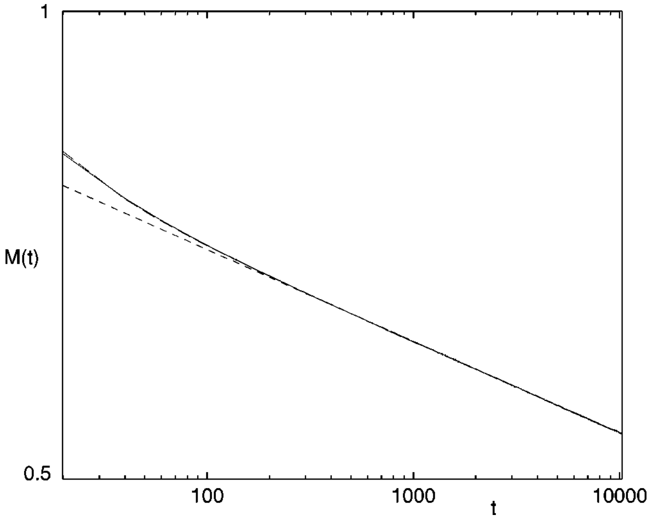

FIG. 2. Time evolution of the magnetization starting from an ordered state in log-log scale. The dashed line is for a power law fit and the long dashed line is with an inverse power law correction.

numbers) for averaging are 10 000. Both curves show deviation from power law up to $t \sim 200$ or 300. However, a logarithmic correction does not fit to the curves. Therefore, we should either accept a relatively bigger t_{mic}, or consider inverse power law corrections. With an *Ansätz* similar to Eq. (3), in a time interval $[100,10\,240]$ we obtain $\eta/2z = 0.0588(3)$ and $d/z = 0.982(10)$. The fitted curves are shown with long-dashed lines in Fig. 2 and Fig. 3. They overlap nicely with the numerical data (solid lines). Without considering corrections to scaling, the estimated exponents differ about 1% (with relatively bigger t_{mic}). The corresponding curves are shown with dashed lines in Figs. 2 and 3.

Finally, to complete our investigation we study a dynamic

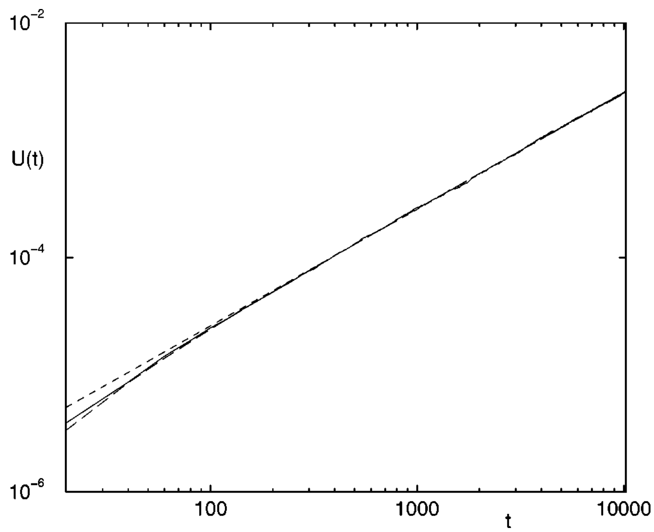

FIG. 3. Time evolution of the Binder cumulant starting from an ordered state in log-log scale. The dashed line is for a power law fit and the long dashed line is with an inverse power law correction.

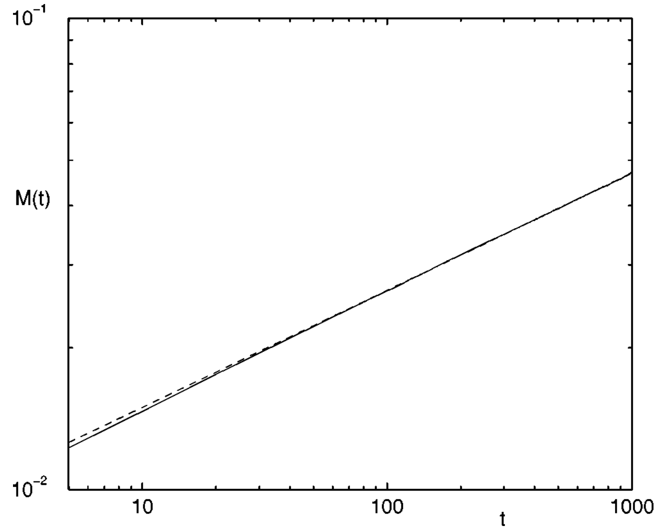

FIG. 4. Time evolution of the magnetization starting from a disordered state but with a small initial value in log-log scale. The dashed line is for a power law fit.

process starting from a disordered state but with a *small* initial magnetization $\vec{M}(0) = (m_0,0)$. If assuming a dynamic scaling form, one can deduce that at the early times, the magnetization $\vec{M}(t) = [M(t),0]$ obeys a power law [9]

$$M(t) \sim t^{\theta}. \tag{6}$$

Here θ is a new independent critical exponent related to the initial condition [1,9]. Since we need a small initial magnetization m_0 and suffer from large fluctuation in longer times, the simulation is only performed up to $t = 1000$. Samples for averaging is 14 000. In Fig. 4, $M(t)$ is displayed with a solid line on log-log scale. From these data, we cannot detect a logarithmic correction. In a time interval $[100,1000]$, direct measurement of the slope yields an exponent $\theta = 0.250(2)$, which is the same as considering an inverse power law correction. The dashed line in Fig. 4 corresponds to a simple power law fit. Of course, we cannot exclude that a logarithmic correction may be detected if we perform simulations up to $t = 10\,000$. But data analysis of the exponents below will show that this will very probably not happen.

In Table I, we summarize all the measured exponents. For the dynamic process starting from an ordered state, through the measured d/z we can obtain independently the dynamic exponent z, denoted as z_1 in the table. Then, with z_1 as input, we calculate the static exponent $\eta = 0.240(3)$ from $\eta/2z$. This value is slightly bigger than $\eta = 0.23$ estimated in simulations in equilibrium [35], but we believe our value is more accurate. With η in hand, from the index $(2-\eta)/z$ in the dynamic process starting from a disordered state, we estimate another value $z_2 = 2.03(1)$ for the dynamic exponent z. Finally, combining the results of θ and $d/z - \theta$ we obtain the third value $z_3 = 2.04(1)$. Three estimates of z from different dynamic processes agree very well. This supports the logarithmic corrections in Eqs. (1) and (2). A remark here is that

TABLE I. Critical exponents measured for different dynamic processes. The dynamic exponent z_1 is estimated from d/z. With z_1 as input, from $\eta/2z$ we obtain η. With η in hand, z_2 is calculated from $(2-\eta)/z$. From θ and $d/z-\theta$ we estimate z_3.

d/z	z_1	$\eta/2z$	η	$(2-\eta)/z$	z_2	θ	$d/z-\theta$	z_3
0.982(10)	2.04(2)	0.0588(3)	0.240(3)	0.866(3)	2.03(1)	0.250(2)	0.730(1)	2.04(1)

even if there might be a logarithmic correction for the magnetization in Eq. (6), it must be rather weak and θ would not be modified so much, otherwise z_3 will deviate from z_1 and z_2. Our impression is that even a small initial magnetization would suppress the effect of the vortex pairs.

Without considering a logarithmic correction, why does one observe a bigger effective dynamic exponent z but a correct static exponent η? Qualitatively, indeed the logarithmic corrections in both $M^{(2)}(t)$ and $A(t)$ effectively result in a bigger z. But it is probably only by chance that a correct η is quantitatively kept.

In conclusions, with Monte Carlo simulations we have investigated the short-time behavior of the dynamic processes starting from both ordered and disordered states for the two-dimensional XY model. The results confirm that there is a logarithmic correction to scaling in case of starting from a disordered state, but an inverse power law correction in case of starting from an ordered state. The dynamic exponent is $z=2.04(1)$, slightly bigger than the theoretical value, $z=2$. We are satisfied with this result, since for many statistical systems z is also different from the ''classical'' value $z=2$.

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