Computer Physics Communications 121–122 (1999) 338–340

Computer Physics Communications

www.elsevier.nl/locate/cpc

Monte Carlo simulations of short-time critical dynamics

B. Zheng [1]

FB Physik, Universität Halle, 06099 Halle, Germany

Abstract

Monte Carlo simulations of the short-time critical dynamics will briefly be reviewed in this paper. © 1999 Elsevier Science B.V. All rights reserved.

For a critical dynamic system in the *long-time* regime, it is well known that there exists also a universal dynamic scaling form. For a magnetic system, e.g., the Ising model, the finite size dynamic scaling form for a physical observable O is [2,3]

$$O(t,\tau,L) = b^{-x} O\big(b^{-z}t, b^{1/\nu}\tau, b^{-1}L\big). \qquad (1)$$

Here t is the dynamic time variable, $\tau \sim (T - T_c)/T_c$ is the reduced temperature, L is the lattice size and b is the rescaling factor, while x and ν are static exponents and z is the dynamic exponent. It is important that the dynamic scaling is independent of initial conditions.

There is a variety of dynamic systems (see Ref. [2] for details). In this paper we discuss the dynamics generated by Monte Carlo algorithms, which belongs to model A [2].

Is there any universal behaviour in the macroscopic short-time regime of the dynamic evolution? The traditional answer is no. However, this has been recently changed. Let us consider the following dynamic process: a magnetic system initially at high temperature and with a *small* magnetization is suddenly quenched to the critical temperature (without an external magnetic field), then released to a dynamic evolution of model A. A dynamic scaling form which is valid up to the macroscopic short-time regime, has been derived with an ε-expansion up to two loop order by Janssen, Schaub and Schmittman [4] and its finite size form, e.g., for the kth moment of the magnetization, is written as

$$M^{(k)}(t,\tau,L,m_0) = b^{-k\beta/\nu} M^{(k)}\big(b^{-z}t, b^{1/\nu}\tau, b^{-1}L, b^{x_0}m_0\big). \qquad (2)$$

Here β is the well known static critical exponent while the *new independent* exponent x_0 is the scaling dimension of the initial magnetization m_0.

Numerical simulations support the theoretical prediction for the short-time dynamic scaling [5–8]. Further, the short-time dynamic scaling is found to be very general [9,10,1].

Now we discuss some characteristic properties of the short-time dynamic scaling.

(1) From the dynamic scaling form (2), one can easily deduce that at early time, the magnetization surprisingly undergoes a *critical initial increase* [4,7,8]

$$M(t) \sim m_0 t^{\theta}, \qquad (3)$$

where the exponent θ is related to x_0 by $\theta = (x_0 - \beta/\nu)/z$. This unexpected behaviour was first theoretically predicted [4] and then numerically observed [7,11]. In Fig. 1, the initial increase of the

[1] E-mail: zheng@hera.physik.uni-halle.de.

0010-4655/99/$ – see front matter
PII: S0010-4655(99)00349-5

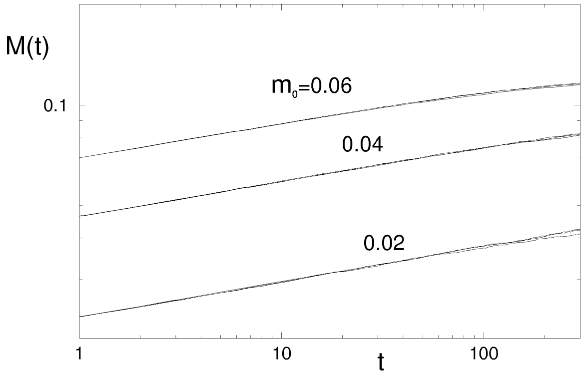

Fig. 1. $M(t)$ for the 3D Ising model.

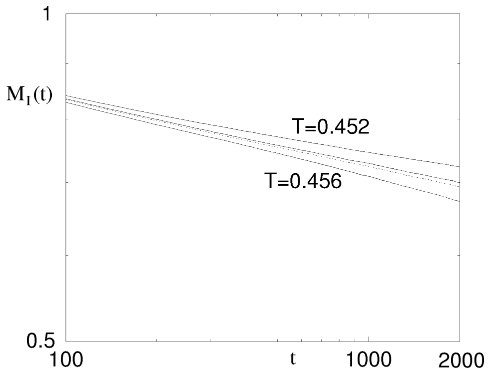

Fig. 2. Determination of T_c and $\beta/\nu z$ for the 2D FFXY model.

magnetization is displayed for the 3D Ising model in log–log scale [12]. A nice power law behaviour is observed at early time. Measuring the slopes of the curves for different m_0 and extrapolating the results to $m_0 = 0$, we obtain $\theta = 0.108(2)$. Another measurement with the technique of damage spreading gives $\theta = 0.104(3)$ [8].

(2) All static exponents and the dynamic exponent z in the short-time dynamic scaling form (2), are the same as those originally defined in equilibrium and in the long-time regime. This leads to new ways for the measurements of the critical exponents as well as the critical temperature [1,13]. Since now the measurements can be carried out already in the beginning of the time evolution, in principle, we do not suffer from how to generate independent configurations. The dynamic approach is free of critical slowing down.

For the measurements of the static exponents an alternative dynamic process starting from an ordered state ($m_0 = 1$) is preferable, since the fluctuation is weaker. The scaling form for this dynamic process is simply

$$M^{(k)}(t, \tau, L) = b^{-k\beta/\nu} M^{(k)}\big(b^{-z}t, b^{1/\nu}\tau, b^{-1}L\big). \quad (4)$$

For sufficiently large lattice size L we can easily deduce the scaling behaviour for the magnetization ($k = 1$)

$$M(t, \tau) = t^{-\beta/\nu z} G\big(t^{1/\nu z}\tau\big). \quad (5)$$

At the critical temperature, $\tau = 0$, the magnetization undergoes a power law decay, but for $\tau \neq 0$ this power law will be modified by the scaling function $G(t^{1/\nu z}\tau)$. Therefore, searching for the best power law behaviour for the magnetization, one can locate the critical temperature T_c. In Fig. 2, the magnetizations at $T = 0.452$, 0.454 and 0.456 are plotted in double-log scale for the chiral degree of freedom in the two-dimensional fully frustrated XY (FFXY) model. $M(t)$ at other temperatures can be estimated by the quadratic interpolation. The dotted curve has the best power law behaviour and gives $T_c = 0.4545(2)$ and $\beta/\nu z = 0.0602(2)$.

To extract the critical exponent $1/\nu z$, we differentiate Eq. (5) and it leads to

$$\partial_\tau \ln M(t, \tau)|_{\tau=0} = t^{1/\nu z} \partial_{\tau'} \ln G(\tau')|_{\tau'=0}. \quad (6)$$

Therefore $\partial_\tau \ln M(t, \tau)|_{\tau=0}$ has also a power law behaviour at early time.

Finally, we introduce a Binder cumulant $U(t, L) = M^{(2)}/M^2 - 1$. Due to the short spatial correlation length in the short-time regime, finite size scaling analysis shows at the critical temperature

$$U(t, L) \sim t^{d/z}. \quad (7)$$

Here we determine the dynamic exponent z independently. Final results for all exponents and discussions can be found in Ref. [13].

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