PHYSICAL REVIEW D VOLUME 57, NUMBER 3 1 FEBRUARY 1998

Dynamic SU(2) lattice gauge theory at finite temperature

K. Okano*
University of California–Los Angeles, California 90095-1547

L. Schülke and B. Zheng
Universität – GH Siegen, D-57068 Siegen, Germany

(Received 30 April 1997; published 19 December 1997)

The dynamic relaxation process for the (2+1)-dimensional SU(2) lattice gauge theory at critical temperature is investigated with Monte Carlo methods. The critical initial increase of the Polyakov loop is observed. The dynamic exponents θ and z as well as the static critical exponent β/ν are determined from the power law behavior of the Polyakov loop, the autocorrelation, and the second moment at the early stage of the time evolution. The universal short-time scaling behavior of the dynamic system is confirmed. The values of the exponents show that the dynamic SU(2) lattice gauge theory is in the same dynamic universality class as the dynamic Ising model. [S0556-2821(98)00703-6]

PACS number(s): 11.15.Ha, 05.70.Jk, 11.10.Wx

In this paper, we report the first numerical simulation of the short-time dynamics for the (2+1)-dimensional SU(2) lattice gauge theory at critical temperature. Recently great progress has been achieved in critical dynamics for spin systems. For a long time it has been believed that no universal behavior would be present in the short-time regime of critical dynamics. However, for the critical relaxation process starting from an initial state with *very high temperature* and *small magnetization*, it was recently argued by Janssen, Schaub, and Schmittmann [1] with renormalization group methods that there exist universality and scaling even *at macroscopic early times*, which sets in right after a microscopic time scale t_{mic}. Based on the scaling relation it was predicted that at the beginning of the time evolution the magnetization surprisingly undergoes a *critical initial increase*

$$M(t) \sim m_0 t^{\theta}, \tag{1}$$

where θ is a new dynamic exponent. This critical initial increase also shows its fruitful application, together with the short-time scaling behavior of the autocorrelation $A(t)$ and the second moment of magnetization $M^{(2)}$ [2]:

$$A(t) \sim t^{-d/z+\theta}, \tag{2}$$

$$M^{(2)}(t) \sim t^{(d-2\beta/\nu)/z}. \tag{3}$$

Here the important and interesting fact is that the dynamic exponent z and the static exponent β/ν appearing in the above short-time dynamic scaling behavior are exactly those originally defined in the thermal equilibrium limit. The investigation of the universal scaling behavior of the short-time dynamics, therefore, not only enlarges the fundamental knowledge on critical phenomena but also, and more interestingly, provides possible new ways to determine the critical exponents at or near thermal equilibrium. The following strategy may, for example, be emanated from the short-time power law behavior of the observables. We first measure the exponent θ directly from the power law increase of the magnetization (1), then taking it as an input we estimate the exponent z from the autocorrelation (2), and with z in hand we finally obtain the static exponent $2\beta/\nu$ from the second moment (3) [3,4].

Numerical simulations for simple statistical models actually proved the efficiency of this scheme. The critical initial increase of the magnetization in Eq. (1) was observed for the Ising model and the Potts model and the exponent θ was directly measured [5,3]. The scaling relation and universality are confirmed [3–11]. Here, it may be important to note that the first numerical estimate of the exponent θ is from the measurement of $A(t)$ for the Ising model by taking z as an input [6,7]. In this case, however, the error induced by z is relatively big since the dynamic exponent z is usually not known so precisely and the ratio d/z is much bigger than θ. In contrast to this, with the exponent θ obtained from the initial increase of the magnetization, one can get a rather accurate value for the exponent z from the autocorrelation. The traditional measurement of z and β/ν performed in the long-time regime or in thermal equilibrium is known to suffer the notorious critical slowing down. Namely, due to the large time correlation length, for big lattice sizes it becomes very difficult to generate independent configurations. However, in the short-time dynamic approach the measurement is always carried out in the short-time regime, where the system rather rapidly converges to the universal power law behavior as the lattice size increases. Moreover, we do not have the problem of how to generate independent configurations efficiently. This is because, in the dynamic approach, the average is really the sample average rather than the time average based on the ergodicity assumption.

At this stage, it is natural to ask whether such an investigation may be generalized to field theoretical models. A straightforward extension of the dynamics in statistical models to field theory is to replace the Hamiltonian of the statistical system by a Euclidean action $S[\phi]$, where $\phi=\phi[x]$ represents the field variable and x is the Euclidean spacetime. Introducing an extra time t which is different from the

*On leave of absence from Tokuyama University, Tokuyama-shi, Yamaguchi, 745 Japan.

Euclidean real time x_0, we set up a relaxation dynamics of model A [12] which reaches the thermal equilibrium distribution $P[\phi]\sim e^{-S[\phi]}$ in the limit of $t\rightarrow\infty$. Such relaxation dynamics can be realized by different ways, e.g., by the Langevin equation or by such other Monte Carlo algorithms as Metropolis or heat bath. This relaxation dynamics is well known in high energy physics and has been utilized, e.g., in the numerical simulation of lattice field theory or in stochastic quantization. Of course, the dynamic evolution introduced by Monte Carlo algorithms cannot generally be related to the real one described by the equation of motion. But in some important cases in condensed matter physics it does. A typical example is the diffusion of the interstitial atoms in an interstitial alloy. It is an open question whether the relaxation dynamics discussed here for the field theory has something to do with real physical dynamic processes. However, this kind of dynamic system for the field theory is still of physical interest. Through the short-time dynamic scaling the relaxation dynamics has acquired an important physical role, e.g., from the scaling behavior (1)–(3) one may extract information with real and important physical interest in or near thermal equilibrium. The precise understanding of the short-time dynamic scaling of *dynamic field theory*, therefore, will be extremely important for *numerical simulations of lattice field theory*. On the other hand, the two-dimensional Ising and three-state Potts model are known to be the simplest models presenting critical phenomena. They are known to have quite clean behavior in many respects. One may wonder whether the nice universal short-time behavior is special for these simple systems. For example, it could be that the microscopic time scale t_{mic} for more complicated systems is so big that it is comparable to the macroscopic time scale. If this is the case, we will not be able to observe any universal behavior in the macroscopic short-time regime. To investigate this point based on a more realistic model is also a purpose of this work.

As a first approach to the *dynamic* gauge theory, we choose the (2+1)-dimensional SU(2) lattice gauge theory at finite temperature with a dynamics of model A. The motivation to choose this model is that the deconfining phase transition observed in this model is of the second order. Also it is known that in equilibrium the (2+1)-dimensional SU(2) lattice gauge theory is in the same universality class as the two-dimensional Ising model [13], which has also been numerically established [14,15]. However, it is not *a priori* clear whether the dynamic SU(2) lattice gauge theory is also in the *same dynamic universality class* as the dynamic Ising model. Neither theoretical nor numerical evidence so far exists. Therefore an investigation with Monte Carlo simulations is very interesting and important.

The (2+1)-dimensional SU(2) lattice gauge theory is described by the action

$$S[U]=-\frac{4}{g^2}\sum_P U_P, \tag{4}$$

where the index P indicates the sum over all the fundamental plaquettes

$$U_P=U_\mu(x)U_\nu(x+\hat{\mu})U_\mu^\dagger(x+\hat{\mu}+\hat{\nu})U_\nu^\dagger(x+\hat{\nu}) \tag{5}$$

with μ, ν denoting the directions in the (2+1)-dimensional space. As *a finite temperature theory*, the lattice size should be taken to be $N^2\times N_0$. Here N_0 corresponds to the inverse temperature. Since the theory is superrenormalizable, the continuum scaling law is of the simple form $N_0T\sim g^{-2}$. For this SU(2) lattice gauge theory in equilibrium there exist already rather good numerical results. For a lattice with $N=64$ and $N_0=2$, Christensen and Damgaard obtained the critical point $4/g_c^2=3.39$ and the exponent $\beta=0.120(8)$ [14], while Teper got the critical point $4/g_c^2=3.47$ and the exponent $\nu=0.98(4)$ [15]. Compared with the exact values $\beta=0.125$ and $\nu=1.0$ for the two-dimensional Ising model, the numerical results of the critical exponents for the SU(2) lattice gauge theory support that both models are in the same universality class. Values for the critical point from the two groups of authors show some small difference. The precise determination of the critical point is at the present stage still difficult. For simplicity, in this paper we will take the simple average $4/g_c^2=3.43$ of the above given two values, and will not discuss an error that may be caused by the uncertainty of this value.

In order to simulate a critical relaxation, one should first prepare an initial state. The magnetization in the SU(2) lattice gauge theory is defined as the globally averaged Polyakov loop

$$M(t)=\frac{1}{N^2}\sum_i \langle W_i(t)\rangle, \tag{6}$$

where W_i is the Polyakov loop at site i which locates in the two-dimensional lattice with lattice size N. The average $\langle\cdots\rangle$ is over the random forces and the independent initial configurations. Compared with the Ising model the Polyakov loop W_i plays the role of the Ising spin S_i. Following the idea of Janssen, Schaub, and Schmittmann [1], the initial state should have zero spatial correlation length and small initial magnetization. To get initial configurations with zero spatial correlation, we just remember that for fixed N_0, the center symmetry Z_2 of the SU(2) theory is broken for large $4/g^2$, while it remains unbroken for small $4/g^2$. Therefore, for the initial state we take the coupling $4/g^2=0$. A nonzero initial magnetization can be achieved in many ways. A natural one is to introduce an initial external magnetic field h. Summarizing, the initial configurations can be generated by the initial action $S_0=h\Sigma_iW_i$. The final procedure is to adjust the configuration so generated as to give the initial magnetization sharply. Different methods for this sharp preparation can be found in the literature [5,3,4,11,16,17].

After an initial configuration is prepared, the system is suddenly quenched to the critical temperature with the action (4) and then released to evolve with the dynamics of model A. In this paper we adopt the *heat-bath algorithm* for the dynamic evolution. We stop the update of the dynamic system at a reasonable time, which is typically some hundreds Monte Carlo time steps, and repeat the process. The average is taken over different independent initial configurations and random numbers. The total sample for the lattice size up to $N=64$ is 9600 while for $N=128$ it is 8000. Errors are estimated by dividing the total sample into three or five groups. It may be important to note that those errors are pure statis-

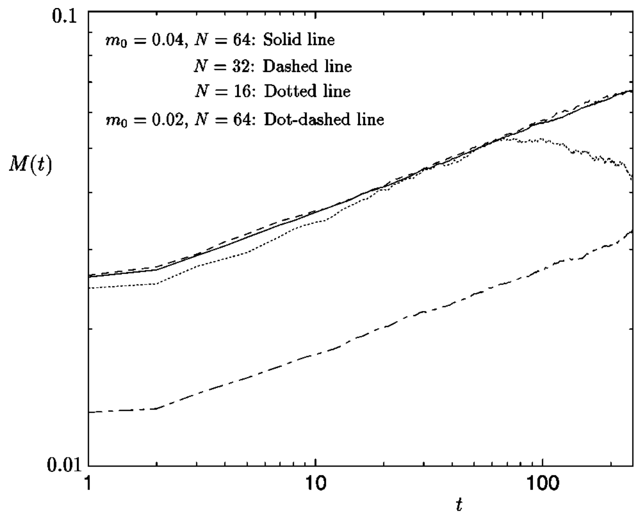

FIG. 1. The time evolution of the magnetization for different lattice sizes N and initial magnetization m_0 with the heat-bath algorithm is plotted on a double-log scale.

tical ones which are estimated following exactly the same way as in [4]. The uncertainty of the critical temperature g_c and the microscopic time t_{mic}, the finite size effect and so on can be additional sources of errors, which are not taken into account in our results. Systematic errors from the random numbers and the correction to the scaling laws have also not been considered. The real errors, therefore, can be bigger than those given in this paper.

In Fig. 1, time evolution of the magnetization is plotted in double-log scale for different initial magnetization m_0 and lattice sizes N. The solid line above is the magnetization profile for $m_0 = 0.04$ and $N = 64$, while the dotted and dashed line are those for $N = 16$ and 32, respectively. The dot-dashed line below represents the time dependence of the magnetization with $m_0 = 0.02$ and $N = 64$. From the figure one can realize that in the *first* Monte Carlo time step the magnetization drops. For example, for $M(t=0) = m_0 = 0.04$ with lattice size $N = 64$ it drops to $M(t=1) = 0.026$. Such a drop within the microscopic time scale t_{mic} is a typical non-universal behavior which essentially depends on microscopic details. Similar phenomena have also been observed for the two-dimensional Potts model [4] and the XY model [11]. With the Metropolis algorithm for the Potts model, the magnetization decreases continuously even up to around 10 Monte Carlo time steps. From Fig. 1, we clearly see that after one Monte Carlo time step, the magnetization indeed increases and for big enough lattice sizes it is quickly stabilized to the universal power law behavior given in Eq. (1). The microscopic time scale is $t_{\text{mic}} \sim 20$. For $m_0 = 0.04$, the magnetization profiles for $N = 32$ and 64 do not show a big difference. The finite size effect for $N = 64$ is already quite small. From the slope of the curves in Fig. 1, one can measure the critical exponent θ. For $m_0 = 0.04$ and $N = 64$, from a time interval [20,250] we obtain the exponent $\theta = 0.192(2)$. This value is consistent with that for the two-dimensional Ising model [4,9]. The comparison can be done in Table I. In Table I, the value of the exponent β/ν for the Ising model is exact while those of the exponents θ and z are taken from the literature [4]. Rigorously speaking, the critical exponent θ is defined in the limit $m_0 = 0$. A numerical

TABLE I. The exponents θ, z, and $2\beta/\nu$ measured from the short-time dynamics with the heat-bath algorithm. The values for the Ising model have been taken from Ref. [4].

	θ	z	β/ν
SU(2)	0.192(02)	2.135(27)	0.120(18)
Ising	0.191(01)	2.155(03)	0.125

measurement in this limit is practically not possible. Therefore, a linear extrapolation to the limit $m_0 = 0$ from finite m_0 should in principle be carried out [3,4]. For this reason, we have also performed the simulation for $m_0 = 0.02$ and $N = 64$. Since m_0 now is smaller, the fluctuations become bigger. The measured exponent is $\theta = 0.186(12)$. Within the errors we cannot distinguish the results for $m_0 = 0.02$ and $m_0 = 0.04$. Therefore in this paper a linear extrapolation will not be performed.

Now we proceed to measure the autocorrelation $A(t)$ and the second moment $M^{(2)}(t)$:

$$A(t) \equiv \frac{1}{N^4} \sum_i \langle W_i(t) W_i(0) \rangle, \tag{7}$$

$$M^{(2)}(t) \equiv \frac{1}{N^4} \left\langle \left(\sum_i W_i(t) \right)^2 \right\rangle. \tag{8}$$

To measure the autocorrelation $A(t)$ and the second moment $M^{(2)}(t)$, we have performed the simulation of $m_0 = 0$ with a lattice size $N = 128$. In Fig. 2, $A(t)$ and $M^{(2)}(t)$ are plotted in double-log scale with the solid line and dotted line, respectively. After a microscopic time scale $t_{\text{mic}} \sim 50$, a nice power law behavior is observed. In the numerical simulation for the Ising model and Potts model, it is known that the microscopic time scale for the second moment is somehow longer than that for the magnetization [4]. This is also the case for the SU(2) lattice gauge theory. We have performed a power

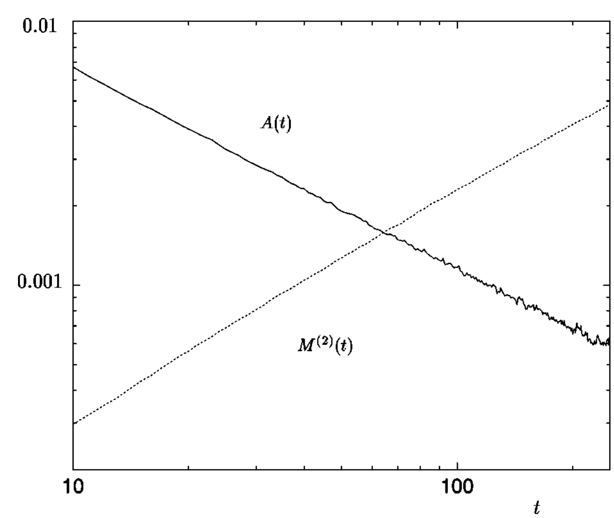

FIG. 2. The autocorrelation $A(t)$ (solid line) and the second moment $M^{(2)}(t)$ (dotted line) calculated in the heat-bath algorithm with $m_0 = 0$ is plotted in double-log scale. The lattice size has been taken as $N = 128$.

law fit in the time interval [60,250] and obtained the exponent $\theta - d/z = -0.745(12)$ while $(d-2\beta/\nu)/z = 0.825(14)$. These results are consistent with those of the two-dimensional Ising model, $\theta - d/z = -0.737(01)$ and $(d-2\beta/\nu)/z = 0.817(07)$. Taking the exponent θ as an input, we can calculate the values for the dynamic exponent z and the static exponent β/ν. All these results and, for comparison, those of the Ising model are given in Table I. Even though the dynamic exponent z has been known for a long time, for the lattice gauge theory our measurement is the first reliable one. The measured value $\beta/\nu = 0.120(18)$ is also in agreement with $\beta/\nu = 0.120(08)$ obtained in Ref. [14], and with the exact value $\beta/\nu = 0.125$ for the two-dimensional Ising model.

In conclusion, we have investigated the universal short-time behavior of the (2+1)-dimensional dynamic SU(2) lattice gauge theory. The critical initial increase of the Polyakov loop is observed. The dynamic exponents θ and z as well as the static critical exponent β/ν are determined from the power law behavior of different observables at the early stage of the time evolution. The values are consistent within the errors with those of the two-dimensional Ising model. The results strongly support that there exists a universal short-time scaling behavior for the dynamic SU(2) lattice gauge theory, and also suggest that the (2+1)-dimensional dynamic SU(2) lattice gauge theory and the two-dimensional dynamic Ising model are in the same dynamic universality class. An important extension of the present work is its application to the real dynamic process described by the equation of motion.

One of the authors (K.O.) would like to thank Professor Z. Bern for inviting him for a one year stay in the Physics Department, University of California, Los Angeles. He also thanks deeply all the members of the Theoretical Elementary Particle Physics Group in UCLA for their kind hospitality. Work supported in part by the Deutsche Forschungsgemeinschaft; DFG Schu 95/9-1.

[1] H. K. Janssen, B. Schaub, and B. Schmittmann, Z. Phys. B **73**, 539 (1989).

[2] H. K. Janssen, in *From Phase Transition to Chaos*, Topics in Modern Statistical Physics, edited by G. Györgyi, I. Kondor, L. Sasvári, and T. Tél (World Scientific, Singapore, 1992).

[3] L. Schülke and B. Zheng, Phys. Lett. A **204**, 295 (1995).

[4] K. Okano, L. Schülke, K. Yamagishi, and B. Zheng, Nucl. Phys. **B485**, 727 (1997).

[5] Z. B. Li, U. Ritschel, and B. Zheng, J. Phys. A **27**, L837 (1994).

[6] D. A. Huse, Phys. Rev. B **40**, 304 (1989).

[7] K. Humayun and A. J. Bray, J. Phys. A **24**, 1915 (1991).

[8] N. Menyhárd, J. Phys. A **27**, 663 (1994).

[9] P. Grassberger, Physica A **214**, 547 (1995).

[10] P. Czerner and U. Ritschel, Phys. Rev. E **53**, 3333 (1996).

[11] K. Okano, L. Schülke, K. Yamagishi, and B. Zheng, J. Phys. A **30**, 4527 (1997).

[12] P. C. Hohenberg and B. I. Halperin, Rev. Mod. Phys. **49**, 435 (1977).

[13] B. Svetitsky and L. G. Yaffe, Nucl. Phys. **B210**, 423 (1982).

[14] J. Christensen and P. H. Damgaard, Nucl. Phys. **B348**, 226 (1991).

[15] M. Teper, Phys. Lett. B **313**, 417 (1993).

[16] S. N. Majumdar, A. J. Bray, S. Cornell, and C. Sir, Phys. Rev. Lett. **77**, 3704 (1996).

[17] L. Schülke and B. Zheng, Phys. Lett. A **233**, 93 (1997).