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Temporal correlation functions of dynamic systems in non-stationary states

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PAPER

Temporal correlation functions of dynamic systems in non-stationary states

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**Abstract**

Besides the fluctuation–dissipation theorem around the stationary state, how the fluctuations in non-stationary states correlate with the future motion of the dynamic variables remains a challenging problem. Further, most temporal correlations of the dynamic variables are zero or very weak in a large class of dynamic systems if the dynamic effects of non-stationary states are not considered. We propose novel methods to compute the temporal correlation functions taking into account the dynamic effects of the non-stationary states. In various dynamic systems, we reveal that the past dynamic fluctuations do drive the future motion of the dynamic variables. This dynamic effect of the non-stationary states is a robust, intrinsic and important property of the complex dynamic systems.

1. Introduction

In the stationary (or equilibrium) state of a dynamic system, dynamic behaviors can be described by temporal and spatial correlation functions of the dynamic variables and fluctuations. In non-stationary states, however, how to characterize the dynamic behaviors is a challenging problem. If the dynamic system is in a sort of time-dependent steady states or not too far from the stationary state, one may still expect to compute the temporal and spatial correlations. In a large class of dynamic systems, however, most temporal correlations are zero or very weak if the dynamic effects of the non-stationary states are not considered. Important examples considered in this paper are the social, biological and ecological systems.

Fluctuations play important roles in various dynamic systems. For example, detrended fluctuation analysis (DFA) is widely used to quantify correlations and scaling properties of complex time series in various dynamical systems [1–6]. DFA permits the detection of the auto-correlations, even when non-stationary effects exist [2, 3]. In addition, DFA yields better estimates for the long-range auto-correlations than other techniques, such as the auto-correlation function, especially for short time series [7]. On the other hand, under certain conditions, the fluctuation–dissipation theorem for the dynamic systems in the stationary state provides a theoretical relation between the response to small external perturbations and the spontaneous fluctuations, but no general results are available for the dynamic systems in non-stationary states [8–13]. For dynamic systems, the general problem whether and how the past dynamic fluctuations affect the future motion of the dynamic variables is crucial, and remains challenging, especially in the cases that the equations of motion are unknown. In addition, the correlation between the fluctuations and the dynamic variables is very weak in most cases, if the dynamic effects of the non-stationary states are not taken into account.

Complex dynamic systems may usually be non-stationary, or at least with features of non-stationary states [14–32]. For example, the auto-correlation of the dynamic fluctuations is rather strong in a large class of complex dynamic systems [18–20, 22, 29, 30]. In other words, the dynamic fluctuation averaged even in a large time scale changes with time significantly. This phenomenon actually is a characteristic of the non-stationary states. Computations of the temporal and spacial correlation functions are essentially hindered by the dynamic effects of the non-stationary states, in addition to random noises. Therefore, it is very important to develop

methods to compute the correlation functions taking into account the non-stationary effects. This is the main purpose of this paper, with focus on the temporal correlation between the past fluctuation and the future motion of the dynamic variables.

We do not attempt to construct the deterministic or stochastic models, or to decompose the time series into components, but do compute the correlation functions directly based on the time series of the empirical data. The results are presented for the social, biological and atmospheric systems. The ideology and methodology can be potentially applied to various dynamic systems. Both DFA and our method tackle non-stationary signals. We focus on the fluctuation–variation correlation function in this paper, and our method can be in principle extended to other spatio-temporal correlation functions. DFA mainly considers the auto-correlation of the dynamic variables or their fluctuations. It is very interesting how DFA may be generalized to more general correlation functions.

In section 2, we describe the computations of the temporal correlation functions in the stationary state. In section 3, we propose the method to compute the temporal correlation functions in non-stationary states. In section 4, the method is applied to the atmospheric and biological dynamic systems. The conclusion is given in section 5.

2. Temporal correlation functions of stationary state

Let us first assume that the dynamic system is in the stationary state, and denote the time series of a dynamic system as $Y(t')$. To avoid the impact of long-term trends of $Y(t')$, we define the dynamic variable as the logarithmic difference $R(t') = \ln Y(t') - \ln Y(t' - 1)$. For comparison of different time series, the normalized dynamic variable is introduced,

$$r(t') = [R(t') - \langle R(t') \rangle] / \sigma. \quad (1)$$

Here $\langle \dots \rangle$ stands for the average over time t' , and σ is the standard fluctuation of the time series $R(t')$.

To explore the dynamic behaviors, one may compute the temporal correlation functions. The auto-correlation functions of a time series $r(t')$ and its absolute value $|r(t')|$ are very important. The absolute value $|r(t')|$ is the simplest quantity for describing the fluctuation. For the dynamic systems considered in this paper, the auto-correlation of $r(t')$ is zero or very weak, while that of $|r(t')|$ is strong.

With our notations, the auto-correlation functions of $|r(t')|$ is defined as

$$A(t) = \langle |r(t')||r(t' + t)| \rangle - \langle |r(t')| \rangle^2, \quad (2)$$

where $\langle \dots \rangle$ represents the average over time t' . Naturally, one may also study the auto-correlation of the fluctuations with DFA, by decomposing the time series into the sign series and the magnitude series [1, 2, 4, 33]. Such an approach may provide further understanding associated with nonlinear and multi-fractal properties [34, 35].

The focus of this paper is on the next important temporal correlation functions, for example, those between the variation $r(t')$ and the fluctuation $|r(t')|$. The variation–fluctuation correlation function is defined by

$$L(t) = \langle r(t') \cdot |r(t' + t)|^2 \rangle / Z, \quad (3)$$

with $Z = \langle |r(t')|^2 \rangle^2$ [36]. Here $|r(t' + t)|^2$ can be also replaced by $|r(t' + t)|$. $L(t)$ describes how the variation $r(t')$ affects the future fluctuation. The usual fluctuation–variation correlation function is defined as

$$f(t) = \langle |r(t')| \cdot r(t' + t) \rangle, \quad (4)$$

which characterizes how the fluctuation at t' influences the variation at the future time $t' + t$. Since the equation of motion of the dynamic system is often unknown and the auto-correlation of $r(t')$ is zero or very weak, the fluctuation–variation correlation is particularly crucial, and it provides the information what may drive the motion of the dynamic variables.

As an important class of complex dynamic systems, financial markets have attracted much attention in the past two decades [11, 19, 20, 36–40]. Mobile phones show an extremely high penetration rate across the globe in recent years [41], and human dynamic behaviors such as consumer perceptions from the collection of the mobile-based data become of great interest. The atmospheric system is another important example of complex dynamic systems. For instance, the application of the fluctuation–dissipation theorem to climate science is appealing [8, 10, 42]. Various biological systems are also complex dynamic systems, including the human brain. The human brain spontaneously generates neural oscillations with a large variability in frequency, amplitude, duration, and recurrence. However, little is known about the long-term spatio-temporal structure of the complex patterns of ongoing activities. Recently, the dynamic behaviors of the human brain electroencephalogram (EEG) have been intensively concerned.

In this paper, we have collected the data including, six mobile application (APP) indexes, the stock indexes of the Chinese and British stock markets and the commodity futures markets in China, five climate time series, as well as human brain EEG data of twenty healthy individuals. The six APP indexes are downloaded from website ‘www.umindex.com’. The APP indexes represent the number of the active users for different kinds of mobile APPs. From ‘www.cpc.ncep.noaa.gov’, we collect five daily climate time series, i.e., North Atlantic Oscillation (NAO), Pacific North American (PNA), Western Pacific Oscillation (WPO), Eastern Pacific Oscillation (EPO), and Greenland Blocking Index (GBI). These climate time series characterize different modes of atmospheric circulation. We also retrieve the human brain EEG data. Our experimental data are of twenty healthy individuals with closed eyes. Measurements are performed every 1/250 s, and last for a total time $T = 130$ s. We first transform the EEG time series to the Fourier space, and filter out the α peak and possible β peak of the power spectrum in the Fourier space, and then transform it back to the configuration space [18, 43].

For financial markets, it has been well known that there is a long-term correlation of the fluctuation $|r(t')|$, and the correlating time is about some months [44, 45]. Within the correlating time, the auto-correlation function decays approximately by a power law. The variation–fluctuation correlation $L(t)$ is either negative for most stock markets in the world, or positive for the Chinese stock-market in the mainland, which is the so-called leverage effect or anti-leverage effect, respectively [36, 38, 46]. From the view of the dynamic evolution, the fluctuation–variation correlation is much more important. However, the usual fluctuation–variation correlation function defined in equation (4) fluctuates around zero.

We have examined these temporal correlation functions for the several dynamic systems considered in this paper, and similar dynamic behaviors are observed. In particular, a strong auto-correlation of the dynamic fluctuations is detected for all the mobile platforms, human brain and atmospheric systems, and the usual fluctuation–variation correlation function in equation (4) just fluctuates around zero. For example, the results for the APP indexes are shown in figure 1.

In figure 1(a), the auto-correlation function of $|r(t')|$ reveals that the fluctuation is rather strongly correlated in time. As displayed in figure 1(b), the variation–fluctuation correlation function $L(t)$ is positive within several days. Although the time window for the non-zero correlation is relatively small, it is indeed the anti-leverage effect, i.e., a positive variation tends to induce a large fluctuation. As shown in figure 1(c), however, $f(t)$ fluctuates around zero. For the human brain and atmospheric systems, both $L(t)$ and $f(t)$ do not indicate any non-zero correlations. Our understanding of the problem whether and how the past dynamic fluctuations affect the future motion of the dynamic variables is very limited.

3. Temporal correlation functions of non-stationary states

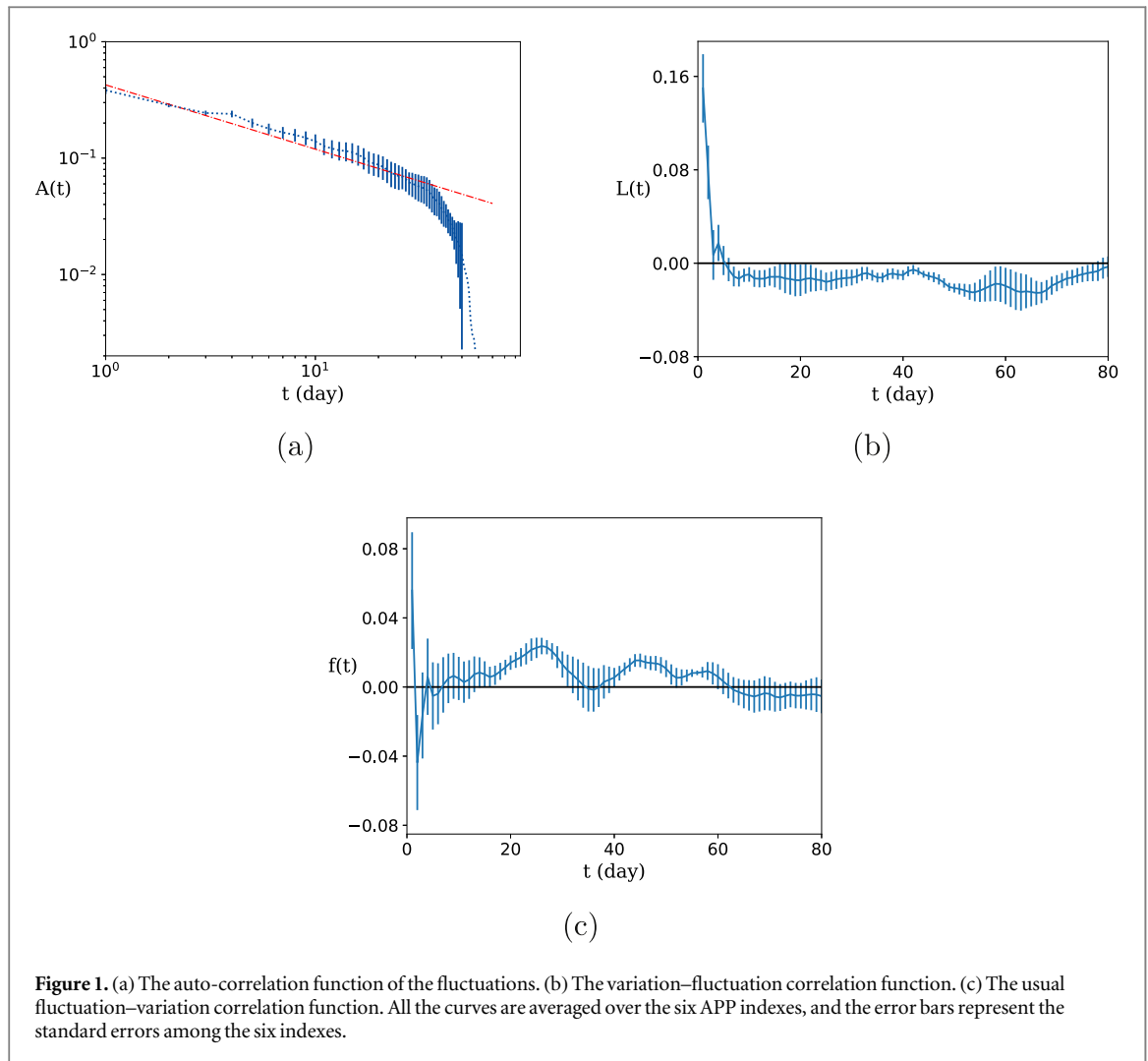
The analysis of the dynamic behaviors in the preceding section is based on the assumption that the dynamic system is in the stationary state. Otherwise, the dynamic effects of the non-stationary states should be taken into account in the computations of the temporal correlation functions. Here we focus on the non-stationary states induced by the long-term auto-correlation of the fluctuations. At first, we emphasize that $|r(t')|$ is only a simple quantity for describing the fluctuations. More comprehensively, we should introduce the standard fluctuation over a certain time window T ,

$$\nu(t') = \left[\frac{1}{T-1} \sum_{i=0}^{T-1} (r(t'-i) - \langle r(t'-i) \rangle_T)^2 \right]^{1/2}, \quad (5)$$

where $\langle \dots \rangle_T$ represents the average over i in the time window T .

In a large class of dynamic systems, the auto-correlation of the dynamic fluctuations is rather strong, as described in the preceding section. Let us denote the auto-correlating time of the dynamic fluctuations as T_b . In the time window T_b , the dynamic system is approximately in a quasi-stationary state. As time evolves, however, it switches among different quasi-stationary states. To compute the temporal correlation functions, we should consider the dynamic effects of the non-stationary states. For this purpose, we introduce the background fluctuation $\nu_b(t')$, which describes the average fluctuation in the time window T_b . With the definition of the fluctuation $\nu(t')$ in equation (5), a simple way to define $\nu_b(t')$ is just by the standard fluctuation in the time window T_b as defined in equation (5) by setting $T = T_b$. Different definitions of $\nu(t')$ and $\nu_b(t')$ may lead to similar results.

The background time window T_b typically is much larger than the time window T for computing the instant fluctuation $\nu(t')$. In a non-stationary dynamic system, $\nu_b(t')$ evolves with time even for a large T_b . Thus, a direct computation of the usual fluctuation–variation correlation such as $f(t)$ defined in equation (4) is hindered by the dynamic effects of the non-stationary states represented by the time-dependent $\nu_b(t')$. To take into account the non-stationary dynamic effects of $\nu_b(t')$, we should rescale the instant fluctuation $\nu(t')$ to $\nu(t')/\nu_b(t')$, or subtract $\nu_b(t')$ from $\nu(t')$. The physical meaning is similar for both schemes. For example, we may redefine the



fluctuation–variation correlation function as,

$$C_{sr}(t) = \langle \text{sgn}[v(t') - v_b(t')] \cdot r(t' + t) \rangle. \quad (6)$$

Here $\langle \dots \rangle$ again represents the average over t' , and $\text{sgn}(x)$ is the sign function of x . In fact, there may be various definitions of the fluctuation–variation correlation function, which will be discussed at the end of this section. The results with the definition in equation (6) are relatively less fluctuating. In addition, the correlation provided by $C_{sr}(t)$ is somewhat more realistic in potential practical applications. We will mainly present the results for C_{sr} , and therefore denote C_{sr} as $C(t)$ hereinafter.

We note that $v(t') - v_b(t')$ can be seen as the spontaneous fluctuation, which is measured without the explicit external perturbations. If one believes that there may be a sort of fluctuation–dissipation theorems also in the non-stationary states, $v(t') - v_b(t')$ may drive the motion of the dynamic variable $r(t' + t)$. The dynamic effects of such non-stationary states are particularly important, in case the temporal or spatial correlation is zero or very weak when the dynamic effects of the non-stationary states are not considered. Recently, a dynamic observable nonlocal in time is constructed to explore the correlation between the past volatilities and future returns in financial dynamic systems, which is actually in a similar spirit [47].

In our approach to the dynamic effects of the non-stationary states, the time window T_b for computing the background fluctuation $v_b(t')$ is crucial. Although T_b qualitatively is the correlating time of the dynamic fluctuations, its precise value is unknown. On the other hand, the time window T for computing the instant fluctuation $v(t')$ must be much smaller than T_b . Therefore we may explore which parameters T and T_b yield the most prominent fluctuation–variation correlation. For each pair of T and T_b , we first determine whether $C(t)$ is non-zero, i.e., whether this pair of T and T_b is an effective one. We then count the number n of the effective pairs, and find out the maximum amplitude A_m of $C(t)$ and the corresponding time windows T and T_b . The length of the time series should be a few times T_b or more. As the length of the time series increases, $C(t)$ becomes less fluctuating.

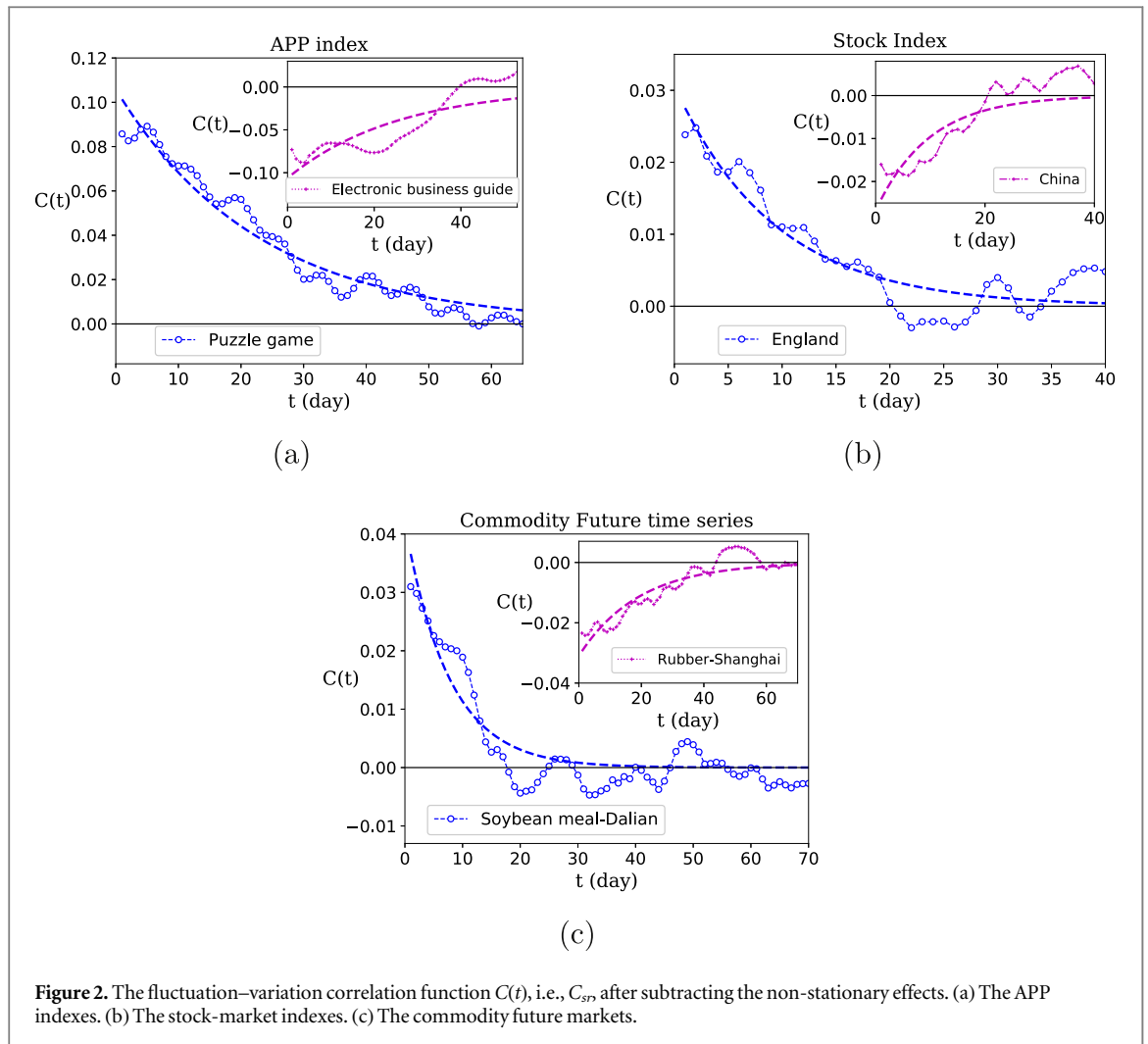


Figure 2. The fluctuation–variation correlation function $C(t)$, i.e., C_{st} , after subtracting the non-stationary effects. (a) The APP indexes. (b) The stock-market indexes. (c) The commodity future markets.

Table 1. The optimized time windows T and T_b , the maximum amplitude A_m and the number n of the effective time windows, for the APP indexes. There are totally 2880 different pairs of time windows.

Index	Period	T	T_b	A_m	n
Arcade action game	2012–2016	43	105	0.052 2	587
Video play	2012–2016	25	370	0.047 1	824
Book reading	2012–2016	40	140	0.056 3	696
Casual games	2012–2016	40	180	0.065 5	900
Puzzle game	2012–2016	12	365	0.079 2	999
Electronic busi- ness guide	2012–2016	9	400	−0.082 4	1102

In our calculations for the APP indexes, T ranges from 5 to 44 d, and T_b from 45 to 400 d. In figure 2(a), the fluctuation–variation correlation function $C(t)$ is displayed for the puzzle game and electronic business guide indexes. All the results are listed in table 1. For the APP indexes of the arcade action game, video play, book reading, casual game, and puzzle game, $C(t)$ is positive, which indicates that if the fluctuations of the usage for a certain mobile APP is large, the APP may attract more users in the future. However, $C(t)$ is negative for the APP index of the electronic business guide. The reason may be that the electronic business guide applications in China usually invest much money to attract users and cause large fluctuations. When the money is used up, however, users will automatically decline to a normal level. Our method provides a theoretical basis to handle the non-stationarity of the so-called big data, and predict the human behaviors.

The dynamic fluctuations of the APP indexes are long-range correlated in time, while the auto-correlation function of the dynamic variables fluctuates around zero. Although the variation–fluctuation correlation

function $L(t)$ is positive, indicating that a positive variation tends to induce a large fluctuation. However, our knowledge on what controls the movement of the dynamic variable itself is very limited. One may calculate the local fluctuation–variation correlation function, but it just fluctuates around zero. After taking into account the dynamic effects of the non-stationary states, however, we observe a non-zero fluctuation–variation correlation, which provides an important understanding of how the past dynamic fluctuations affect the future motion of the dynamic variables.

Besides the APP indexes, the dynamic effects of the non-stationary states can be also detected in financial systems, as shown in figure 2(b) for the stock markets in England and China, and in figure 2(c) for the commodity futures markets in China. It is interesting that the fluctuation–variation correlation exhibits opposite dynamic behaviors for the England and China stock markets. $C(t)$ of the soybean meal price is positive, while that of the rubber price is negative. For social systems such as the mobile platforms, the asymmetric human preference in volatile or and stable states may partially account for the fluctuation–variation correlation. Obviously, dynamic behaviors of the financial markets are more complicated. The main aim of this paper is to develop and present the methods for computing the temporal correlation functions, taking into account the non-stationary dynamic effects. The underlying mechanisms for the fluctuation–variation correlation in financial markets requires further investigation and will be presented elsewhere.

Besides C_{sr} defined in equation (6), we may still define other fluctuation–variation correlation functions such as

$$C_{rr}(t) = \langle [v(t') - v_b(t')] \cdot r(t' + t) \rangle, \quad (7)$$

and

$$C_{ss}(t) = \langle \text{sgn}[v(t') - v_b(t')] \cdot \text{sgn}[r(t' + t)] \rangle. \quad (8)$$

All $C_\alpha(t)$, with $\alpha = ss, sr, rr$, characterize the non-stationary dynamic effects of the past dynamic fluctuations on the future motion of the dynamic variables.

C_{sr} , C_{rr} and C_{ss} are related each other, but not fully equivalent. C_{ss} describes how the two states, i.e., $v(t') > v_b(t')$ and $v(t') < v_b(t')$, drive the future motion of $r(t' + t)$ to increase or decrease. C_{sr} and C_{rr} respectively take account into the amplitude of $r(t' + t)$ and the amplitudes of both $v(t') - v_b(t')$ and $r(t')$.

We may compare the results of different fluctuation–variation correlation functions. For instance, the sign of C_{ss} for the APP indexes of the arcade action game, video play and book reading, differs from that of C_{sr} and C_{rr} . Further, the ranges of the effective time windows T and T_b for different correlation functions are shown in figure 3. The ranges of the effective T_b for C_{sr} are somewhat wider than those for C_{rr} and C_{ss} .

4. Atmospheric and biological dynamic systems

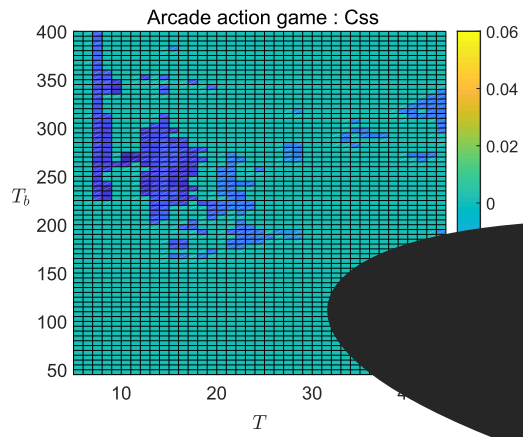
In the preceding section, we investigate the dynamic effects of the non-stationary states for the social and financial dynamic systems, and demonstrate that the past fluctuations may drive the future motion of the dynamic variables. To show further that the fluctuation–variation correlation is rather robust, we explore the non-stationary dynamic effects of the atmospheric and biological systems in this section.

Since the impact of long-term trends of $Y(t')$ is ignorable for the dynamic systems considered in this section, we simply define the dynamic variable as the difference $R(t') = Y(t') - Y(t' - 1)$. As displayed in figure 4, $C(t)$ is positive for both the climate indexes and the EEG time series of the human brain.

The climate indexes represent the teleconnection patterns recorded from different regions. Various patterns describe different preferred modes of the large scale variability, in both the tropics and extratropics. Teleconnection patterns received much attention since they stand for the dynamic transport of energy and the climate dynamics on global scales, typically thousands of kilometers. Understanding the dynamic behavior of teleconnection patterns is essential for the prediction of climate change [48, 49].

The results are summarized in table 2. In our calculations, T and T_b range, respectively, from 5 to 500 d and 500 to 1000 d. The fluctuation–variation correlation function $C(t)$ is positive for all five climate indexes. The results indicate that if the instant fluctuation of the climate index is larger than the background fluctuation, the dynamic variable will tend to increase in the future.

The human brain is a complex network of interacting non-stationary dynamic subsystems, whose complicated spatial-temporal dynamics is still poorly understood. The EEG is a major recording technique to assess the macroscopic brain dynamics in humans [43]. The time unit for the EEG data we collect is 1/125 s. In our computations, T ranges from 5 to 500 units, and T_b from 500 to 1000 units. There are 20 human subjects, 10 time series of different brain areas for each subject, and 200 EEG time series in total. We take the average over the 10 time series of different brain areas for each human subject. $C(t)$ is positive for 15 human subjects, negative for 3 human subjects, and zero for other two. Understanding of the possible mechanisms for these different dynamic behaviors may need further experimental study.



(a)



The human brain is an open, dissipative, and adaptive dynamical system and thus inherently non-stationary. For long-lasting recordings of the ongoing brain dynamics, the most common way of dealing with the non-stationarity is to apply a sliding-window analysis, i.e., cutting the time series into successive time windows during which the dynamic system can be regarded as approximately stationary. However, the length of each time

Table 2. The optimized time windows T and T_b , the maximum amplitude A_m and the number n of the effective time windows, for the atmospheric data. There are totally 300 different pairs of time windows.

Index	Period	T	T_b	A_m	n
Pacific Oscillation (EPO)	1948–2016	385	600	0.002	1
Greenland Blocking Index (GBI)	1948–2016	135	700	0.044	111
Pacific North American (PNA)	1948–2016	295	700	0.007	3
Western Pacific Oscillation (WPO)	1948–2016	25	800	0.013	57
North Atlantic Oscillation (NAO)	1948–2016	205	700	0.007	13

window in the sliding-window analysis is usually set artificially. In this sense, our method is a proper technique for tackling the non-stationarity of the human brain dynamics.

5. Conclusion

We propose novel methods to compute the temporal correlation functions of dynamic systems in non-stationary states. A large class of dynamic systems, for example, is in a quasi-stationary state within a time scale T_b , but the average fluctuation $v_b(t')$ in the time window T_b evolves with time. In computing the temporal correlation functions, we should take into account the dynamic effects of the non-stationary states. In various dynamic systems, we detect that the past dynamic fluctuations drive the future motion of the dynamic variables. This dynamic effect of the non-stationary states is a robust, intrinsic and important property of the complex dynamic systems. As important examples, we study the social, human brain and atmospheric systems. The method, however, can be applied to any dynamic systems in similar non-stationary states, and can be also extended to the computations of general temporal and spatial correlation functions.

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