Exact ground state and string tension in (1+1)-dimensional lattice gauge theories

Zheng Bo
Department of Physics, Zhongshan University, Guangzhou, People's Republic of China
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A new form of Hamiltonian of (1+1)-dimensional lattice gauge theory with fermions is proposed and the exact ground state is found. The string tension is calculated exactly. The result shows that for both massless and massive theories quarks are confined by a linear potential and no phase transition occurs when the lattice spacing goes to zero.

I. INTRODUCTION

In 1985 a new form of Hamiltonian of pure lattice gauge theory with exact ground state was proposed by Guo, Liu, and Chen.[1] Making use of the new form of Hamiltonian, they calculated glueball masses in (2+1)-dimensional theories with variational methods and extended the results to the deep weak-coupling region.[2] Even though the quantum continuum limit of the new form of Hamiltonian has not been investigated clearly, this work is still attractive because some exact things are important and useful in physics. Obviously it would be more interesting if we could work out similar results in the theory with fermions.

Recently there has been controversy on the question of whether the new Hamiltonian is in the same universality class as the conventional one.[3] This problem has not been settled yet. Even if the new Hamiltonian is not in the same universality class as the conventional one, the former is still of interest because it provides an alternative lattice model which may be more reliable for analytical study and may give new insight into lattice gauge theory.

On the other hand, although (1+1)-dimensional gauge theory has been studied extensively[4] since Schwinger solved his model in 1962 (Ref. 5), other (1+1)-dimensional models, such as the massive Schwinger model and non-Abelian models, have not been solved yet. Lattice gauge theory is promising in dealing with the low-energy properties of gauge theory. Some work in (1+1)-dimensional theories with fermions has been done with Monte Carlo methods.[6] But few exact analytical results are available.

In this paper an example of exact analytical calculation is given. In Sec. II a new form of Hamiltonian of lattice gauge theory with massless fermions in 1+1 dimensions is proposed and its exact ground state is found. In Sec. III we prove that the theory has the correct classical continuum limit. In Sec. IV the massive theory is studied. In Sec. V the string tension is calculated. Finally some discussion is given in Sec. VI.

II. MASSLESS FERMIONS

Now we concentrate our attention on (1+1)-dimensional lattice SU(N) theory with massless fermions to illustrate our approach. The simplest Hamiltonian of the lattice (1+1)-dimensional SU(N) theory is the naive one:[7]

$$H_{N_0}=\tfrac{1}{2}e^2a\sum_x E(x)^2+\frac{1}{2a}\sum_{\substack{x\\ k=\pm 1}}\bar{\psi}(x)\gamma_k U(x,k)\psi(x+k)\ . \tag{2.1}$$

Here $\gamma_{-1}=-\gamma_1$, a is the lattice spacing, $\bar{\psi}(x)$ and $\psi(x)$ are fermion fields, and $U(x,k)$ are gauge fields satisfying

$$\begin{aligned}&[U(x,1),E^\alpha(x)]=\Lambda^\alpha U(x,1),\\ &[U(x,-1),E^\alpha(x-1)]=-U(x,-1)\Lambda^\alpha\ ,\end{aligned} \tag{2.2}$$

where α denotes the color index and Λ^α is the generator of the SU(N) group. Unfortunately, it is very difficult to find the ground state of H_{N_0}. So we introduce a new Hamiltonian

$$H_N=\tfrac{1}{2}e^2a\sum_x e^{-CR_1}E(x)e^{2CR_1}E(x)e^{-CR_1}\ , \tag{2.3}$$

where C is a real constant to be determined and

$$R_1=\sum_{k=\pm 1}\bar{\psi}(x)\gamma_k U(x,k)\psi(x+k)\ . \tag{2.4}$$

We denote

$$\psi(x)=\begin{bmatrix}\xi(x)\\ \eta^\dagger(x)\end{bmatrix},\quad \psi^\dagger(x)=(\xi^\dagger(x)\ \ \eta(x)) \tag{2.5}$$

and choose the representation

$$\gamma_0=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad \gamma_1=\begin{bmatrix}0&i\\-i&0\end{bmatrix}\ . \tag{2.6}$$

Then we define $|0\rangle$ as

$$E(x)|0\rangle=0,\quad \xi(x)|0\rangle=0,\quad \eta(x)|0\rangle=0\ . \tag{2.7}$$

It is easy to understand that if setting

$$|\Omega_\theta\rangle=e^{CR_1}\sum_{n=0}^{\infty}e^{in\theta}C_n\left[\sum_x\xi^\dagger(x)\eta^\dagger(x)\right]^n|0\rangle\ , \tag{2.8}$$

where

$$C_n^{-2}=\left\langle 0\left|\left[\sum_x\eta(x)\xi(x)\right]^n e^{2CR_1}\left[\sum_x\xi^\dagger(x)\eta^\dagger(x)\right]^n\right|\right\rangle\ , \tag{2.9}$$

then we have

$$H_N|\Omega_\theta\rangle=0 \ . \tag{2.10}$$

That is, $|\Omega_\theta\rangle$ is an eigenstate of H_N with a zero eigenvalue. Because

$$[e^{CR_1}E(x)e^{-CR_1}]^\dagger=e^{-CR_1}E(x)e^{CR_1} \ , \tag{2.11}$$

H_N is positive definite. Therefore $|\Omega_\theta\rangle$ is a ground state. The additional parameter θ appeared in the ground state coincides with that in the continuum theory.[8]

In the same way, we can construct the corresponding model for Susskind fermions.[9] We take the new Hamiltonian

$$H_S=\tfrac{1}{2}e^2a\sum_x e^{-CR_{S_1}}E(x)e^{2CR_{S_1}}E(x)e^{-CR_{S_1}} \ , \tag{2.12}$$

where

$$R_{S_1}=\sum_{\substack{x\\ k=\pm1}}[\xi^\dagger(2x)i^kU(2x,k)\eta^\dagger(2x+k)+\eta(2x+1)i^kU(2x+1,k)\xi(2x+1+k)] \ . \tag{2.13}$$

Here we must notice that $\xi^\dagger(x)$, $\xi(x)$ are only defined on even sites and $\eta^\dagger(x)$, $\eta(x)$ on odd sites. Apparently, the ground state of H_S is

$$|\Omega_S\rangle=e^{CR_{S_1}}|0_S\rangle \ , \tag{2.14}$$

where $|0_S\rangle$ is defined according to

$$E(x)|0_S\rangle=0, \quad \xi(2x)|0_S\rangle=0, \quad \eta(2x+1)|0_S\rangle=0 \ . \tag{2.15}$$

Since H_S breaks chiral symmetry, the additional parameter θ does not appear in the ground state $|\Omega_S\rangle$.

III. CLASSICAL CONTINUUM LIMIT

As is well known, the ordering of fields is important when a classical Hamiltonian is quantized or the classical limit of quantum Hamiltonian is taken. Usually Weyl ordering is assumed. So in order to get the classical limit of the Hamiltonian of lattice gauge theory, we must arrange the field operators according to Weyl ordering before we let them tend to classical fields.

In $1+1$ dimensions, the dimensionless lattice fermion field ψ is related to the corresponding continuum field ψ_c by $\psi=\sqrt{a}\,\psi_c$. Naively, a lattice field ψ contributes a factor $\sqrt{a}$ when $a\to0$. However, for a product of ψ and $\psi^\dagger$ on the same site, the overall power of a is ambiguous because it depends on the ordering we adopt when the classical limit is taken. Therefore, Weyl ordering is not negligible in inferring the classical continuum limit.

Now we take H_N in (2.3) as an example for consideration. For convenience, we omit the space index of the fields if it would cause no confusion. At first, we introduce

$$\begin{aligned}A_1&=[R_1,E] \ ,\\ A_n&=[R_1,A_{n-1}]\\ &=\sum_{\substack{l=1\\ \{k_i=\pm1\}}}^{n}(-1)^{n-1}\frac{(n-1)!}{(n-l)!(l-1)!}\\ &\qquad\times\bar\psi\gamma_{k_1}U\cdots\gamma_0\gamma_{k_l}[U,E]\cdots\psi\end{aligned} \tag{3.1}$$

and rewrite H_N as

$$\begin{aligned}H_N&=\tfrac{1}{2}e^2a\sum_x\left[E+\sum_{n=1}^{\infty}(-1)^nA_n\frac{1}{n!}C^n\right]\\ &\qquad\times\left[E+\sum_{n=1}^{\infty}A_n\frac{1}{n!}C^n\right]\\ &=\tfrac{1}{2}e^2a\left[\sum_x E^2+T_1+T_2\right] \ ,\end{aligned} \tag{3.2}$$

where

$$\begin{aligned}T_1&=\sum_x\sum_{\nu=0}^{\infty}[E,A_{2\nu+1}]\frac{1}{(2\nu+1)!}C^{2\nu+1}\\ &\quad+\sum_x\sum_{\substack{n=1\\ \nu=0}}^{\infty}(-1)^n[A_n,A_{n+2\nu+1}]\\ &\qquad\times\frac{1}{n!(n+2\nu+1)!}C^{2n+2\nu+1} \ ,\end{aligned} \tag{3.3}$$

$$\begin{aligned}T_2&=\sum_x\sum_{\nu=1}^{\infty}(EA_{2\nu}+A_{2\nu}E)\frac{1}{(2\nu)!}C^{2\nu}\\ &\quad+\sum_x\sum_{\substack{n=1\\ \nu=0}}^{\infty}(-1)^n(1-\tfrac{1}{2}\delta_{\nu,0})(A_nA_{n+2\nu}+A_{n+2\nu}A_n)\\ &\qquad\times\frac{1}{n!(n+2\nu)!}C^{2n+2\nu} \ .\end{aligned} \tag{3.4}$$

If we denote

$$R_n=\sum_{\substack{x\\ \{k_i=\pm1\}}}\bar\psi(x)\gamma_{k_1}U(x,k_1)\cdots\gamma_0\gamma_{k_n}U\left[x+\sum_{i=1}^{n-1}k_i,k_n\right]\psi\left[x+\sum_{i=1}^{n}k_i\right] \ , \tag{3.5}$$

then

$$T_1=\sum_{i=0}^{\infty}\sigma_{2i+1}R_{2i+1} \ , \tag{3.6}$$

where σ_{2i+1} are real constants to be calculated. From (3.4) to (3.6) it is easy to understand that H_N has been in Weyl ordering except for a trivial term $\sim \sum_x \psi^\dagger(x)\psi(x)$.

Now we consider the continuum limit of H_N. Our methods are that we determine the constant C by demanding

$$\sigma_1 = \frac{1}{(ae)^2} \tag{3.7}$$

and then prove $\sum_{i=1}^{\infty} \sigma_{2i+1} R_{2i+1}$ and T_2 tend to zero at the continuum limit. That is,

$$H_N \to H_{N_0} \quad \text{when } a \to 0 \ . \tag{3.8}$$

Therefore H_N has correct continuum limit.

In fact, it is not too difficult to calculate

$$\sigma_1 = -2CC_N + C_N \sum_{\substack{n=0\\ \nu=0}}^{\infty} [3-2(2n+2\nu+1)] \frac{(2n+2\nu)!}{n!(n+2\nu+1)![(n+\nu)!]^2} C^{2n+2\nu+1} \ , \tag{3.9}$$

where C_N is the Casimir invariant of the SU(N) group. Making use of the formula

$$\frac{(2n+2\nu)!}{[(n+\nu)!]^2} = \frac{1}{2\pi} \int_{-\infty}^{+\infty} dx \frac{1}{\sqrt{1+x^2}} \left[\frac{2}{\sqrt{1+x^2}} \right]^{2n+2\nu+1} \tag{3.10}$$

and the property of Bessel function, we have

$$\sigma_1 = -2CC_N + 3C_N \int_0^C dC' I_0(-4C') - 2CC_N I_0(-4C) \ , \tag{3.11}$$

where $I_0(z)$ is the zero-order Bessel function. Thus (3.7) becomes

$$-2CC_N + 3C_N \int_0^C dc' I_0(-4C') - 2CC_N I_0(-4C) = \frac{1}{(ae)^2} \ . \tag{3.12}$$

It is easy to realize that $C \to -\infty$ and the main part of σ_1 is $-2CC_N I_0(-4C)$ when $a \to 0$. Since

$$I_0(-4C) \to \frac{1}{\sqrt{-8\pi C}} e^{-4C} \quad \text{when } a \to 0 \ , \tag{3.13}$$

we get

$$C \to \tfrac{1}{2} \ln(ae) \quad \text{when } a \to 0 \ . \tag{3.14}$$

In a similar way, we work out

$$\sigma_3 = -2C_N \sum_{\substack{n=0\\ \nu=0\\ n+2\nu \geq 4}}^{\infty} a_3 \frac{(2n+2\nu-2)!}{n!(n+2\nu+1)![(n+\nu-1)!]^2} \times C^{2n+2\nu+1} \ , \tag{3.15}$$

where

$$a_3 = \tfrac{1}{12}(n+2\nu)(n+2\nu-1)(8n+4\nu-7) + \tfrac{1}{2}(n+2\nu)(n-1)^2 + \tfrac{1}{12}(n-1)(n-2)(2n-3) \ . \tag{3.16}$$

When $a \to 0$,

$$\sigma_3 \sim \sum_{\substack{n=0\\ \nu=0}}^{\infty} \frac{(2n+2\nu-2)!(2n+2\nu+1)(2n+2\nu)(2n+2\nu-1)}{n!(n+2\nu+1)![(n+\nu-1)!]^2} C^{2n+2\nu+1} \sim C^3 I_0(-4C) \sim \frac{C^2}{(ae)^2} \ . \tag{3.17}$$

Since $aR_3 \sim a^4$ when $a \to 0$, therefore,

$$a\sigma_3 R_3 \sim C^2 a^2 \quad \text{when } a \to 0 \ . \tag{3.18}$$

More generally,

$$a\sigma_{2i+1} R_{2i+1} \sim C^{2i} a^{2i} \ . \tag{3.19}$$

Because $a \to 0$ faster than $1/C$ does,

$$a \sum_{i=1}^{\infty} \sigma_{2i+1} R_{2i+1} \to 0 \quad \text{when } a \to 0 \ . \tag{3.20}$$

On the other hand, since

$$E \sim a^0, \quad A_n \sim a^n \quad \text{when} \quad a \to 0 \ . \tag{3.21}$$

we have

$$aT_2 \to 0 \quad \text{when } a \to 0 \ . \tag{3.22}$$

Thus we finished the proof of (3.8).

IV. MASSIVE FERMIONS

At first we concentrate on naive fermions again. For convenience, we denote

$$\psi(x)=\begin{bmatrix}\xi(x)\\ \eta^\dagger(x)\end{bmatrix},\quad \psi^\dagger(x)=[\xi^\dagger(x)\ \ \eta(x)] \tag{4.1}$$

and choose the representation

$$\gamma_0=\begin{bmatrix}1 & 0\\ 0 & -1\end{bmatrix},\quad \gamma_1=\begin{bmatrix}0 & i\\ -i & 0\end{bmatrix}. \tag{4.2}$$

Now we consider the classical continuum limit of

$$H_m=m\sum_x e^{-CR_1}\xi^\dagger(x)e^{2CR_1}\xi(x)e^{-CR_1}+m\sum_x e^{-CR_1}\eta^\dagger(x)e^{2CR_1}\eta(x)e^{-CR_1}. \tag{4.3}$$

Using (4.1) and

$$\begin{aligned}e^{CR_1}\psi(x)e^{-CR_1}&=\sum_{n=0}^{\infty}\frac{1}{n!}C^n\psi_n(x),\\ e^{-CR_1}\psi^\dagger(x)e^{CR_1}&=\sum_{n=0}^{\infty}\frac{1}{n!}C^n\psi_n^\dagger(x),\end{aligned} \tag{4.4}$$

where

$$\begin{aligned}\psi_0(x)&=\psi(x),\\ \psi_n(x)&=[R_1,\psi_{n-1}(x)]\\ &=\sum_{\{k_i=\pm1\}}(-1)^n\gamma_0\gamma_{k_1}U(x,k_1)\cdots\gamma_0\gamma_{k_n}U\left[x+\sum_{i=1}^{n-1}k_i,k_n\right]\psi\left[x+\sum_{i=1}^{n}k_i\right].\\ \psi_0^\dagger(x)&=\psi^\dagger(x),\\ \psi_n^\dagger(x)&=[-R_1,\psi_{n-1}^\dagger(x)]\\ &=\sum_{\{k_i=\pm1\}}(-1)^n\bar\psi\left[x-\sum_{i=1}^{n}k_i\right]\gamma_{k_n}U\left[x-\sum_{i=1}^{n}k_i,k_n\right]\cdots\gamma_0\gamma_{k_1}U(x-k_1,k_1).\end{aligned} \tag{4.5}$$

we rewrite H_m as

$$\begin{aligned}H_m=m\sum_x\bar\psi(x)\psi(x)&+m\sum_{n=0}^{\infty}\sum_{\substack{l=0\\ n+l\neq0}}^{\infty}\frac{1}{(2n)!(2l)!}C^{2n+2l}D_{2n+2l}-m\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{1}{(2n+1)!(2l+1)!}C^{2n+2l+2}D_{2n+2l+2}\\ &-2m\sum_{n=0}^{\infty}\sum_{l=0}^{\infty}\frac{1}{(2n)!(2l+1)!}C^{2n+2l+1}R_{2n+2l+1}+M_0,\end{aligned} \tag{4.6}$$

where

$$\begin{aligned}D_n&=\sum_{\substack{x\\ \{k_i=\pm1\}}}\bar\psi(x)\gamma_0\gamma_{k_1}U(x,k_1)\cdots\gamma_0\gamma_{k_n}U\left[x+\sum_{i=1}^{n-1}k_i,k_n\right]\psi\left[x+\sum_{i=1}^{n}k_i\right],\\ R_n&\doteq\sum_{\substack{x\\ \{k_i=\pm1\}}}\bar\psi(x)\gamma_{k_1}U(x,k_1)\cdots\gamma_0\gamma_{k_n}U\left[x+\sum_{i=1}^{n-1}k_i,k_n\right]\psi\left[x+\sum_{i=1}^{n}k_i\right]\end{aligned} \tag{4.7}$$

and M_0 is a constant related to normal ordering. Obviously H_m is in Weyl ordering except for a constant. When $a\to0$, according to (3.14),

$$C\sim\tfrac{1}{2}\ln(ae) \tag{4.8}$$

and at the classical level

$$D_n\sim(ae)^n,\quad R_n\sim(ae)^n; \tag{4.9}$$

therefore

$$H_m\sim m\sum_x\bar\psi(x)\psi(x)+M_0. \tag{4.10}$$

That is, H_m equals the fermion mass term except for a constant at the continuum limit. So we can take the Hamiltonian of SU(N) theory with a massive fermion as

$$H_N(m)=H_N+H_m \ . \tag{4.11}$$

Because

$$\begin{aligned}[e^{CR_1}E(x)e^{-CR_1}]^\dagger&=e^{-CR_1}E(x)e^{CR_1} \ ,\\ [e^{CR_1}\xi(x)e^{-CR_1}]^\dagger&=e^{-CR_1}\xi^\dagger(x)e^{CR_1} \ ,\\ [e^{CR_1}\eta(x)e^{-CR_1}]^\dagger&=e^{-CR_1}\eta^\dagger(x)e^{CR_1} \ .\end{aligned} \tag{4.12}$$

$H_N(m)$ is positive definite. Let

$$|\Omega\rangle=e^{CR_1}|0\rangle \ , \tag{4.13}$$

where $|0\rangle$ is defined as

$$E(x)|0\rangle=0, \quad \xi(x)|0\rangle=0, \quad \eta(x)|0\rangle=0 \ ; \tag{4.14}$$

then

$$H_N(m)|\Omega\rangle=0 \ . \tag{4.15}$$

$|\Omega\rangle$ is the ground state of $H_N(m)$. The degeneracy of the ground state in the massless fermion theory is removed by the fermion mass term.

Here we must note that because a different representation is adopted, the state $|0\rangle$ defined in this section is different from that in Sec. II.

For Susskind fermions, we just choose

$$\begin{aligned}H_S(m)=\tfrac{1}{2}e^2a\sum_x e^{-CR_{S_1}}E(x)e^{2CR_{S_1}}E(x)e^{-CR_{S_1}}\\ +m\sum_x e^{-CR_{S_1}}\xi^\dagger(2x)e^{2CR_{S_1}}\xi(2x)e^{-CR_{S_1}}\\ +m\sum_x e^{-CR_{S_1}}\eta^\dagger(2x+1)e^{2CR_{S_1}}(2x+1)e^{-CR_{S_1}} \ ,\end{aligned} \tag{4.16}$$

where $\xi^\dagger(x)$, $\xi(x)$ are only defined on even sites and $\eta^\dagger(x)$, $\eta(x)$ on odd sites.

V. STRING TENSION

We still take the naive fermion theory as an example to demonstrate our calculation. In fact, both naive and Susskind fermion theories give the same results.

A. String tension of an infinite string

At first we consider massive fermion theory. For a theory with dynamical fermions, the string tension is not well defined because any string will break due to the dynamical fermions. To give a definite meaning to the string tension, we define it as the expectation value of the energy per unit length in a translational invariant n-link state. Under this definition, we can calculate the string tension exactly. We adopt the representation in Sec. IV and denote the n-link state by

$$|M_n\rangle=e^{CR_1}M_n^\dagger|0\rangle \ , \tag{5.1}$$

where

$$\begin{aligned}M_n^\dagger&=\sum_{\substack{x\\ \Gamma=\pm n}}\xi^\dagger(x)i^\Gamma U(x,\Gamma)\eta^\dagger(x+\Gamma) \ ,\\ U(x,\pm n)&=U(x\pm 1)\\ &\quad\times U(x\pm 1,\pm 1)\cdots U(x\pm(n-1),\pm 1) \ .\end{aligned} \tag{5.2}$$

Then the string tension of an infinite string is defined as

$$\alpha_\infty=\lim_{n\to\infty}\frac{1}{(na)}\frac{\langle M_n|H_N(m)|M_n\rangle}{\langle M_n|M_n\rangle} \ . \tag{5.3}$$

For an arbitrarily given a,

$$\sum_{k=0}^{\infty}\left\langle 0\left|M_n\frac{1}{(2k)!}(2CR_1)^{2k}M_n^\dagger\right|0\right\rangle$$

converges uniformly for all n; therefore,

$$\begin{aligned}\lim_{n\to\infty}\langle M_n|M_n\rangle&=\lim_{n\to\infty}\lim_{K\to\infty}\sum_{k=0}^{K}\left\langle 0\left|M_n\frac{1}{(2k)!}(2CR_1)^{2k}M_n^\dagger\right|\right\rangle\\ &=\lim_{K\to\infty}\lim_{\substack{n\to\infty\\ n>2K}}\sum_{k=0}^{K}\left\langle 0\left|M_n\frac{1}{(2k)!}(2CR_1)^{2k}M_n^\dagger\right|0\right\rangle \ .\end{aligned} \tag{5.4}$$

When n is finite, $\langle M_n|M_n\rangle$ contains two kinds of configurations shown in diagrams (5.5) and (5.6):

$$\begin{array}{l}U(x,\Gamma)\\ \longrightarrow\\ \quad\longleftarrow\\ \quad U(x',-\Gamma) \ ,\end{array} \tag{5.5}$$

$$\begin{array}{l}U(x,\Gamma)\\ \longrightarrow\\ \quad\longrightarrow\\ \quad U(x',\Gamma) \ .\end{array} \tag{5.6}$$

Equation (5.4) means that, when $n\to\infty$, only the configuration shown in (5.5) contributes. In the same way, taking into account the fact that

$$\lim_{n\to\infty}\frac{1}{na}\langle M_n|H_m|M_n\rangle=0 \ , \tag{5.7}$$

we have

$$\lim_{n\to\infty}\frac{1}{(na)}\langle M_n|H_N(m)|M_n\rangle$$
$$=\tfrac{1}{2}e^2\lim_{K\to\infty}\lim_{\substack{n\to\infty\\ n>2K}}\frac{-1}{n}\sum_{k=0}^{K}\left\langle 0\left|[M_n,E]\frac{1}{(2k)!}(2C)^{2k}\times R_1^{2k}[M_n^\dagger,E]\right|0\right\rangle, \tag{5.8}$$

where the space index of $E(x)$ and the summation over x have been implied. From (5.5) it is not too difficult to understand

$$(n-2k)C_N\le\frac{-\langle 0|[M_n,E]R_1^{2k}[M_n^\dagger,E]|0\rangle}{\langle 0|M_nR_1^{2k}M_n^\dagger|0\rangle}\le nC_N \tag{5.9}$$

and then

$$\lim_{n\to\infty}\frac{-1}{n}\langle 0|[M_n,E]R_1^{2k}[M_n^\dagger,E]|0\rangle$$
$$=\lim_{n\to\infty}C_N\langle 0|M_nR_1^{2k}M_n^\dagger|0\rangle\ . \tag{5.10}$$

At last we get the result

$$\alpha_\infty=\tfrac{1}{2}e^2C_N\ . \tag{5.11}$$

In fact the same results can be obtained in massless fermion theory though an additional parameter θ appears in the ground state.

The result (5.11) shows that the potential between a quark and antiquark is linear. Therefore the string would break and new quark-antiquark pairs would be produced if we separate a quark-antiquark pair. So the quark and antiquark are confined in our theory.

B. String tension of a finite string when $a\to 0$

Here we consider massless fermion theory only. We still adopt the representation in Sec. IV. The string tension of a finite string in the continuum limit is defined as

$$\alpha_L=\lim_{\substack{a\to 0\\ na=L}}\frac{1}{na}\frac{\langle M_n|H_N|M_n\rangle}{\langle M_n|M_n\rangle}\ , \tag{5.12}$$

where L is the length of the string. According to (3.14), when $a\to 0$, C tends to infinity more slowly than $n=L/a$ and so we have

$$\lim_{\substack{a\to 0\\ na=L}}\langle M_n|M_n\rangle=\lim_{\substack{a\to 0\\ na=L}}\lim_{\substack{K\to\infty\\ n>2K}}\sum_{k=0}^{K}\left\langle 0\left|M_n\frac{1}{(2k)!}(2CR_1)^{2k}M_n^\dagger\right|0\right\rangle,$$
$$\lim_{\substack{a\to 0\\ na=L}}\frac{1}{na}\langle M_n|H_N|M_n\rangle=\frac{e^2}{2}\lim_{\substack{a\to 0\\ K\to\infty\\ na=L}}\lim_{2K/n\to 0}\frac{-1}{n}\sum_{k=0}^{K}\left\langle 0\left|[M_n,E]\frac{1}{(2k)!}(2CR_1)^{2k}[M_n^\dagger,E]\right|0\right\rangle. \tag{5.13}$$

Taking into account (5.9), again we get

$$\alpha_L=\tfrac{1}{2}e^2C_N\ .$$

VI. RESUITS AND DISCUSSIONS

(1) Our main idea is that, for (1+1)-dimensional lattice SU(N) gauge theory with naive fermions, if we introduce an additional term

$$\Delta H_N=\tfrac{1}{2}e^2a\left[\sum_{i=1}^{\infty}\sigma_{2i+1}R_{2i+1}+T^2\right], \tag{6.1}$$

which tends to zero at the classical continuum limit, and let

$$H_N=H_{N_0}+\Delta H_N\ , \tag{6.2}$$

then the ground state of H_N can be found. This result can be generalized to Susskind fermion and massive fermion theories.

(2) The string tension of an infinite string is calculated exactly in both massless and massive fermion theories. The result is $\frac{1}{2}e^2C_N$. This shows that for both massless and massive fermion theories the quark and antiquark are confined by a linear potential and no phase transition occurs when $a\to 0$. Although getting confinement is trivial in (1+1)-dimensional theories, it is still of interest to have a concrete calculation of the string tension using the new Hamiltonians.

Furthermore we calculate the string tension of a finite string of massless fermion theory in the continum limit and the result is also $\frac{1}{2}e^2C_N$. That is the potential between the quark and antiquark is exactly linear at any distance for massless fermions.

(3) We get the same result in the calculation of string tension in both naive and Susskind fermion theories. This simply means that naive fermions can produce the correct result in dealing with some kinds of problems.

(4) Even though ΔH_N in (6.1) vanishes in the classical continuum limit, the contribution of ΔH_N in the quantum theory is still unclear. We expect that the contribution of ΔH_N mainly concentrates in the high-energy region and is irrelevant to low-energy phenomena.

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